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Patterns and Technology — A Creative Approach to Isometries

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Abstract: Today's math education basic syllabus gives an ever increasing emphasis to Geometry and Patterns. Curricula also advocate an approach that allows student to understand the concepts involved supported by dynamical computer tools. However, there is not much research work joining these paradigms. Therefore, a case study was developed, with 9th grade students (14-15 years old), to evaluate the impact of a creative approach to isometries and symmetries—friezes, rosaceas (rosettes) and wallpapers—centered in patterns (reproduction, continuation, completion, description and creation) and using Geometer's Sketchpad software to solve, mainly in pairs, and discuss, with the whole class, the challenging tasks proposed, involving the formulation of hypothesis, argumentation and justification of the reasoning. The statistical analysis of the quantifiable data and content analysis of the qualitative data, collecting trough enquiry, direct observation and documental analysis (involving questionnaires, field notes, logbook, pre-tests and post-test, other works of the students including those computer related, and internal documents of the school) enable to conclude positively regarding the main research question underlying the study. In fact, it led to the conclusion that the teaching strategy implemented has contributed to deepen the student's knowledge and skills on geometry, mathematical communication and autonomy as well as to develop a closer relation with the field of geometry itself. This article focuses on one of the cases studied. The pair was selected due to be representative of most students and due to their communication skills.

Key words: Isometries, patterns, geometer's sketchpad, mathematical creativity.

1. Introduction and Theoretical Framework

In many countries, the topic of isometric geometric transformations on the Euclidean plane [1] has assumed great importance in mathematics curricula for basic education [2]. In fact, it can play a major role in the process of developing the geometric sense, which is the main goal of studying Geometry [2].

Several authors suggest a learning process based on comprehension, supported by creative tasks, both open and complex as the problems themselves, as well as investigation schemes [3] that uncover the inherent structure of the geometrical entities, more than just the mechanical manipulation of the objects [4]. This kind of activities can promote flexibility, originality and fluency, the main dimensions underlying creativity [5, 6].

They also suggest a teaching that may promote the development of autonomy and communication skills in mathematics, as well as contribute to build a more positive view towards geometry [4].

The dynamical environments of dynamical geometry, like GSP (geometer's sketchpad), can be crucial to achieve such goals [7], that school is not always able to reach. In fact, if these environments are properly employed, especially within a social constructionist matrix [8], these open software, by making it possible to build, manipulate and explore different geometrical entities, make the learning process easier, more pleasant and more effective [9].

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On the other hand, recognizing the patterns and regularities underlying some structure (whether it is numerical, figurative, geometrical or sonorous) to understand the world around us is considered, by several authors, the essence of Mathematics itself [10]. The process will be favored if the students are faced with situations in which they have to reproduce patterns, continue them (forwards and backwards), complete them and identify repeating sets. Namely, it is fundamental to formulate and test conjectures, make generalizations and come up with justifications because it harnesses the mathematical power [11, 12]. Moreover, such a find as well as the creation of new models provide rich and motivating learning experiences and allow for the establishment of a more emphatic and affective relation with mathematics where multiple connections to aesthetics and creativity take place [13].

It's worth noting that the importance of creativity has been strongly enhanced lately [14]. It is not exclusively on the domain of arts, and mathematics must contribute to its development as well [15]. Creative tasks, implemented in a creative way, demanding creative solutions and letting student's imagination and actions grow free can be crucial to develop mathematical creativity [16, 17].

Once research work encompassing these various topics is scarce, we conducted a study whose main objective is to evaluate the impact of a creative approach to Isometries, centered in Patterns and using Geometer's Sketchpad, on the acquisition of knowledge on Geometry as well as on the development of mathematical communication and autonomy and a positive relation with Geometry.

2. Method

Given the objectives pursued and according to a constructivist paradigm [18], a qualitative case-study methodology was chosen [19]. In fact, it was intended to understand in sharp detail one particular phenomenon [20].

The study involved a class of 21 students of the 9th grade, from whose we selected representatives' pairs according different performances. Also the need for good communication skills was taken into account for selection.

The research design consisted of: characterization of the students by means of an IQ (initial questionnaire); planning of thematic unit; an introduction session to GSP; a pre-test; implementation of the thematic unit above mentioned adjusting the plan whenever needed; a post-test and a FQ (final questionnaire).

The main data collection techniques where enquiry, direct observation and documental analysis supported by questionnaires, field notes, logbook, tests (before and after), other works of the students including those computer related, and internal documents of the school.

Nine tasks were prepared, three of which consisted of diagnostics and introduction to GSP. They involve isometries and symmetries—friezes, rosaceas (rosettes) and wallpapers—and the reproduction, continuation, completion, description and creation of patterns. They are mainly exploratory tasks involving the formulation of hypothesis, argumentation and justification of the reasoning. They were developed during one month, encompassing nine classes of 90 min and three classes of 45 min. They were organized in four main stages: presentation of the tasks; resolution by the pairs of students and using GSP, oriented by the teacher; group discussion and synthesis of the main aspects. The latest stage, in some cases, was postponed to the following session.

To satisfy one of rules of the evaluation process—that of coherence on the work carried out [21]—the test was divided in three parts: A theoretical one that was individually solved with pencil and paper only; a practical one solved with GSP and another one solved in pairs.

For data treatment, we performed statistical analysis of the quantifiable data and content analysis of the qualitative data. This analysis was oriented by a system of categories [22] following from the research objectives: "geometrical knowledge"—"isometries" and "patterns" (identification of the smallest repeating set, continuation or completion, reproduction, construction and/or creation and characterization); "autonomy"; "communication" and signs of the development of a more positive view towards Geometry.

3. Results and Discussion

In this paper, due to space limitations, some results are discussed concerning one of the pairs. As it was said before, it was selected due to representative of most students and due to their communication skills.

The pair G2 was constituted by the friends A2 and A14, both 14 years old that used to work together. They usually showed, respectively, high and medium performance in mathematics. For A2, this was her favorite subject so she participated actively. She was attentive and organized, enjoyed learning new things showed a great sense of responsibility and used to help the other students. A14 was not participating much even though she talked a lot with her colleagues. She was less committed to work but still solved the proposed tasks. Both expressed great expectations towards the work with GSP because, although they liked to use the computer, both in school and at home, they had never used geometry software before.

3.1 Geometrical Knowledge—Isometries and Patterns

In what concerns to the initial tasks, it was verified that the dyad G2 found it easy to generalize, from the feedback given by the computer, properties of rotation, translation, reflection and glide reflection. For example, concerning rotation, they noted—"The image of an angle is a congruent angle and with the same orientation. A rotation transforms shapes in geometrically identical shapes. The image of a line segment is geometrically identical line segment".

On the other hand, the study of the composition of two isometries presented major difficulties for the group to extract and communicate relevant information, which suggests that they have seldom been faced with such tasks of high complexity and openness. Even after some tips from the teacher for exploring the situation, for instance in what respects to the composition of two parallel axis reflections, the students could only point out—"We concluded that the length of the vector is double the size of the distance between the axis r and s. The direction of the vector is vertical (perpendicular, forming a right angle) and the reflection axes are diagonal".

The following task involved the creation of a pattern using composition of all the isometries. And students took the chance to explore, on GSP, that regular convex polygons allow tiling. The constructions made by the pair of students (see Fig. 1) as well as the group discussion that followed, revealed that the group found the task straightforward.

In what concerns to the task involving the reproduction and construction of rosaceas (rosettes), using different amplitudes and rotation centers, the dyad worked committedly and the data collected suggest that GSP made the activity easier and favored collaborative learning, in the meaning of [23]. For example, with regard to the base set of one of the constructions, the following dialogue was registered:

Student A2: It may be… And now, it can let's build another triangle?

Student A14: Now I ask the program to measure the angle…

Student A2: Make successive rotations...

Student A14: Oh! But it is not possible… It doesn't remain the same;

Student A2: How are we going to do it?

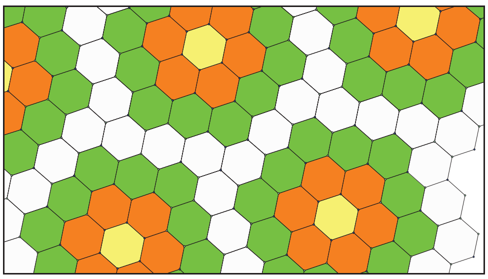

Fig. 1 Creation of a pattern by the pair G2.

Student A14: We have to choose an amplitude for the angle in such a way that it rotates… until it gets to the beginning again;

Student A2: Yes… start;

Student A2: How are we going to build the pinwheel?

Student A14: We have to build a triangle, then apply a rotation and get to the beginning;

Student A2: I do not understand how we are going to do it!

Student A14: Don't you? … We draw a triangle and, using that point as the center, you see… (pointing to the screen) we rotate!

Nevertheless, there were moments in which the students were not this confident in their reasoning and found some difficulties exploring but mainly in justifying. For example, regarding symmetry groups of a rosacea, they just wrote "dihedral" and "cyclic", as shown in Fig. 2.

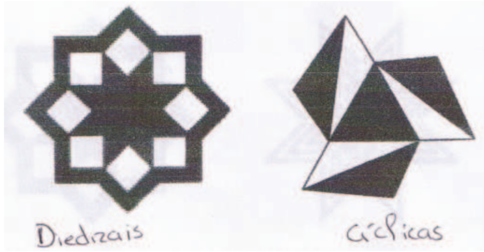

Fig. 2 Symmetry groups of rosaceas.

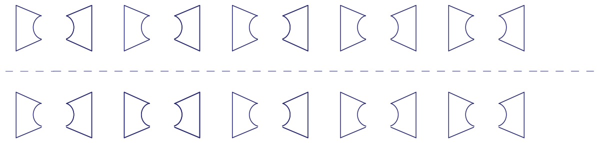

Fig. 3 Continuation of a frieze to the left and to the right.

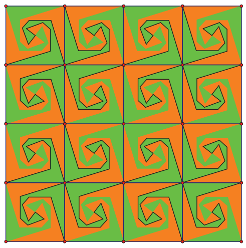

Fig. 4 Reproduction of a wallpaper.

In what concerns to the study of friezes, the teacher had to intervene a lot because the students were not familiarized with that kind of task, especially with using a dynamical environment.

Despite the initial difficulties, the dyad managed to continue (in both directions) and complete patterns on GSP (see Fig. 3). They revealed, though, more severe difficulties in describing the way they did it. For example, regarding the frieze below, the simply wrote "The pattern was generated by translation and reflection".

Concerning the wallpapers, the pair found some difficulties to identify the smallest repeating set of some of them and/or to describe the steps followed to build it. On the other hand, the reproduction task (see Fig. 4), which they enjoyed the most, raised no doubts.

Thus, it was realized that, during the empirical studies, the dyad improved its knowledge on the topic, noting that the learning context was determinant, as the students recognized on the FQ (final questionnaire).

It led to better performance on the post-test in relation to the pre-test (see Table 1), even though the final results have stayed below the expectations, especially in what concerns to A2 in the patterns task. The student A14 seems to have benefited the most of the experience, showing far more interesting relative global gains.

The setbacks encountered during the sessions (power failure, insufficient computers …), as well as the lack of time to deepen some of the subjects were an obstacle to achieve better results. It was also clear that some of the students didn't study enough to fix their difficulties, as they admitted on the FQ.

Regarding the sentence from the mentioned questionnaire—"The work centered in patterns allowed us to have a better perception of the geometrical concepts involved", A14 absolutely agreed and A2 partially agreed.

Surprisingly, concerning the GSP having contributed to the improvement of their knowledge, both students agreed but only partially. They considered the test coherent with the classes, which they consider "important" (A2) or "very important" (A14).

Table 1 Score on the pre and post-tests for G2.

Test	Pre					Post				
Part	Theor.	Pract.		Total	Total (%)	Theor.	Pract.		Total	Total (%)
	I	II	III			I	II	III		
Isometries										
Total score	12	31	14	57		12	31	14	57	
A2	3	16	0	19	33	12	8	11	31	54
A14	0	3		3	5	4	23		38	67
Patterns										
Total score	15	11	17	43		15	11	17	43	
A2	6	1	3	10	23	6	6	17	29	67
A14	0	2		5	12	4	7		28	65

3.2 Communication, Autonomy and a More Positive View Towards Geometry

During the empirical studies, it was found that A14 was more enthusiastic in working with the computer, assuming the leadership by possessing the mouse (functional activity), whilst A2's activity was more of an intentional nature. She supported the learning of her colleague, answering A14's questions and formulating other, hinting her during the resolution and explaining the procedures:

Student A2: For this question, what do you need to know? The figure only shows rotation symmetries!

Student A14: So it belongs to the... dihedral rosacea's group [taking over the mouse];

Student A2: And what do we have to do?

Student A14: Complete the rosacea;

Student A2: Yes, but what do we have to find out from the picture?

Student A14: I think it is the amplitude of rotation angle;

Student A2: Exactly. Measure the amplitude;

Student A14: But, which points should I consider?

Student A2: We have to find out... maybe we should consider these (pointing to the screen). What do you think?

Student A14: I'll measure and check if it works.

Gradually, they evolved to reciprocal negotiation, which led them to clarify their knowledge and increase the chances of conceptual growth. The idea that, by exchanging ideas, the concepts are more easily understood by each one was stronger in the end since, as A14 stated, "working in groups was very pleasant", because "I learned more". And both stated, on their FQ, that "the use of this software fosters the interaction between students".

It was discovered a sharp difficulty in communicating mathematical ideas, mainly in writing, even though some improvements were noted during the study. For example, regarding the identification of symmetry groups of the rosacea (Fig. 5), on a later stage, the group answered in a better way—"For the first shape we used isometries of rotation. And in the second one, we used isometries of rotation and reflection. The first one is cyclic and the second one is dihedral".

In what concerns to autonomy, on the first stage, the students would call for help even when faced with the smallest obstacle—"Teacher, I do not know what to do!", "What should I answer? We do not understand!". The teacher did nothing more than hinting them:

Teacher: Look at the figures you've just built and draw your conclusions;

Student A14: It's a rotation with the same center;

Teacher: What can you say about the amplitude of the rotation angle?

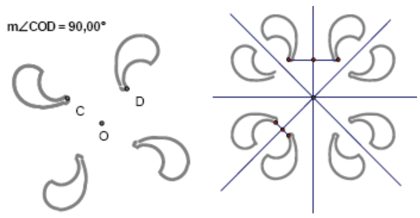

Fig. 5 Rosaceas for identification of the symmetry groups.

Student A14: We measured it already. It's equal to the sum of the remaining two;

Teacher: Ok. Try it with other amplitudes and check if the relations remain.

Such a dependence was notorious in the initial tasks involving patterns, an attitude that the students justified with the lack of experience with this kind of activity and with the fact that they are used to a teaching scheme in which "the teachers explain and after that they students give it a try". Gradually, they earned some autonomy, an improvement that they value—"it was important that the students drew their own conclusions". In fact, on the FQ, they affirmed to have enjoyed to use GSP, that the familiarization was easy and that "with this software it's preferable to work in pairs", which contributes to an active and dynamical learning of Geometry. A14 partially disagreed that GSP does not promote autonomy in learning and A2 agreed partially. The way the thematic unit was implemented contributed partially, in the opinion of both students, to the development of that attitude. In what concerns to foster a more positive view towards Geometry, according to the logbook notes, A14 stated that GSP motivated the learning and catalyzed her interest. To A2, "it made Geometry less boring". Both disagreed that "the study of isometries associated to patters and in a context of problem solving and investigations doesn't motivate the learning of Geometry". In general, they agreed that the didactic approach contributed "to a more positive view towards Geometry" as well as "for the development of an affective relation" with the subject.

4. Conclusions

The results show that the pair G2, in general, improved its geometry knowledge related to isometries, being able to transfer those skills to the analysis of friezes, rosaceas and tilings. And they did it in a very positive way, for which the teamwork was crucial—it allowed them to value what can be achieved together. GSP allowed them to explore interactively a wider range of situations. Thus, in a collaborative and computer mediated learning environment, they developed skills that go beyond Geometry. In addition to the technological skills, improvements in the relations between students and in autonomy were also noted. Concerning the relation with Geometry, they envisioned it as something dynamic, creative and useful, where they can investigate, experiment and explore. On the opposite end, improvements on communication skills were, by far, less clear. In summary, the way how G2 lived this didactic interaction seems to strengthen the conviction that such learning experiences are essential to the development of knowledge, skills and attitudes or, for short, of mathematical competence. That is why it is so important that they occur on a continued and diverse way, in order to promote significant learning processes that are, in turn, promoters of the student's mathematical power.

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