PATTERN PROBLEM SOLVING TASKS AS A MEAN TO FOSTER CREATIVITY IN MATHEMATICS

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In this report we explore the implications of some challenging pattern problem solving tasks in the development of mathematical ability and creativity of students (future teachers). Solving challenging tasks usually requires creative thinking and our recent work in a project about patterns in the teaching and learning of mathematics showed that patterns can give a positive contribution to the development of mathematical ability and creativity for all students. So our main concern is to analyze, through some elementary classroom episodes, the contribution of pattern tasks to promote creative solutions by students.

INTRODUCTION

The major purpose of teachers is that students develop an increasing mathematical ability that allows them to solve the different problems they face inside and outside school. Innovation and creativity play an important role, being a dynamic characteristic that students must develop. Creativity begins with curiosity and engages students in exploration and experimentation tasks where they can translate their imagination and originality (Barbeau & Taylor, 2005). Research findings show that mathematical problem solving and problem posing are closely related to creativity (e.g. Pehkonen, 1997; Silver, 1997). So, learning environments with problem solving/posing activity should be used in our classes in order to develop students' creativity. Challenging tasks usually require creative thinking and our recent work in a project about patterns in the teaching and learning of Mathematics showed that patterns can contribute to the development of mathematical ability and creativity of students. So our purpose as mathematics educators is to provide all students (including future teachers) creative approaches for solving any problems and to think independently and critically. This way, future teachers should themselves develop these skills and go through the same type of tasks that they will offer their students.

THEORETICAL FRAMEWORK

Creativity, problem solving and patterns

Mathematical creativity is a rather complex phenomenon. Mann (2006), in an examination of the research about how to define mathematical creativity, found that there is a lack of an accepted definition for mathematical creativity since there are numerous ways to express it. But we can notice that there are some commonalities in the different attempts to define creativity that are: (1) it involves divergent and convergent thinking; (2) it has mainly three components/dimensions that are fluency, flexibility, and originality (novelty); and (3) it is related to problem solving and problem posing (including elaboration and generalization).

(1) Divergent and convergent thinking are both important aspects of intelligence, problem solving and critical thinking. Convergent thinking is a way of thinking oriented to obtain a single response to a situation. The solver is good at bringing material from a variety of sources to bear on a problem, in such a way as to produce the "correct" answer. It usually involves a thinking process that follows some set of rules or logic, while divergent thinking looks towards the problem, analyzing all the possible solutions and seeking the best solution to the problem. Here the solver is in broadly creative elaboration of ideas prompted by a stimulus. It is the opposite of convergent thinking, a creative process that involves trying to imagine as many possible solutions as one can. In contrast to convergent thinking, divergent thinking is usually more spontaneous and free-flow. People who have divergent thinking try to keep their mind open to any possibilities that are presented to them. The more possibilities they come up with, the better their divergent thinking is. Divergence is usually indicated by the ability to generate many, or more complex or complicated, ideas from one idea (Hudson, 1967).

(2) Components of Creativity: fluency, flexibility and originality. *Fluency* is the ability to generate a great number of ideas and refers to the continuity of those ideas, flow of associations, and use of basic knowledge. Silver (1997) defines it as apparent shifts in approaches taken when generating responses. *Flexibility* is the ability to produce different categories or perceptions whereby there is a variety of different ideas about the same problem or thing. It reflects when students show the capacity of changing ideas among solutions. *Originality* is the ability to create fresh, unique, unusual, totally new, or extremely different ideas or products. It refers to a unique way of thinking. With regard to mathematics classrooms, originality may be manifested when a student analyzes many solutions to a problem, methods or answers, and then creates another one different.

(3) Research has shown that the formulation and solution of problems in mathematics are closely related to creativity (Barbeau & Taylor, 2005; Silver, 1997). Tasks that can promote the above dimensions must be open-ended and ill structured, assuming the form of problem solving, problem posing (including elaboration and generalization) and mathematical explorations and investigations. Rather than closed problems with a single solution, students should be provided open-ended problems with a range of alternative solution methods (Fouche, 1993, as cited in Mann, 2006). Problem posing can be a powerful strategy to develop problem solving skills and to have good problem solvers; on the other hand, to formulate meaningful mathematical problems, it is necessary to be a good problem solver.

Patterns are a powerful tool in the mathematics classroom and can suggest several approaches, as well as they permeate all mathematics, and their study makes possible to get powerful mathematical ideas as generalization and algebraic thinking, where visualization can play an important role. Indeed, according to several authors, patterning tasks have creative potential as they may be open-ended, allow a depth and variety of connections with all topics of mathematics, both to prepare students for further learning and to develop skills of problem solving and posing, as well as communication (NCTM, 2000; Orton, 1999). Figure 1 summarized the ideas above.

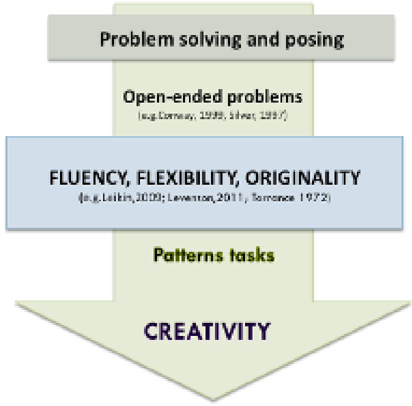


Figure 1: A path to creativity

Teachers and creativity

Learning heavily depends on teachers. One of the major obstacles to reforms is teachers' lack of familiarity with innovative instructional practices and tools. Teachers must have an in-depth understanding of fundamental mathematics and of the mathematical thinking of their students to support the development of their mathematical competence (Hiebert et al., 2007; Ma, 1999). The basic purpose of a math class is that students learn something about a particular topic that was planned by the teacher. Teachers must interpret the curriculum and select *good* curricular materials and strategies to use in the classroom. To achieve this, teachers should propose tasks involving students in a creative form, and also be mathematically competent to analyze their students' resolutions. Research shows that what students learn is greatly influenced by the tasks they are given (e.g. Doyle, 1988, Stein & Smith, 2009). Therefore, it is important to have good mathematical tasks. A task is good when it serves to introduce fundamental mathematical ideas, is an intellectual challenge for students and allows different approaches (NCTM, 2000). Tasks must develop new approaches and creative ideas, so they must provide multiple solutions in order to raise the student flow of mathematical ideas, flexibility of thought and originality in the responses. Teachers must encourage students to create, share and solve their own problems, as this is a very rich learning environment for the development of their ability to solve problems and their mathematical knowledge. Creativity is a dynamic characteristic that students can develop if teachers provide them appropriate learning opportunities (e.g. Leikin, 2009). Creativity is a topic that is often neglected within their mathematics teaching usually because they didn't realize its importance in mathematics and mathematics education. Creativity should be an intrinsic part of mathematics for all programs (Pehkonen, 1997).

METHODOLOGY

We adopted a qualitative exploratory approach with elementary pre-service teachers, to understand in what way a didactical experience through challenging tasks, grounded on figural pattern problems, is a suitable context for promoting creativity in students solutions, in particular in getting creativity ways of expression of generalization. This proposal emphasizes the figurative contexts of patterning as a way to reach generalization, through meaningful representations, in particular the algebraic (or numeric) expressions. The aim of this exploratory study is to note some of the diversified views from the perspectives of pre-service mathematics teachers on improving the creative thinking in solving patterning tasks, in figurative problem solving contexts. Our two main questions were: Did the pattern tasks in figurative contexts promote multiple solutions? How can we characterize creativity when students solve challenge pattern tasks in figurative contexts? The participants in the study were twenty-three elementary mathematics pre-service teachers during the didactics of mathematics classes of the 3rd academic year. The pattern didactical proposal has a sequence of tasks: counting, repeating and growing patterns, and pattern problems. However, in this paper we will only analyze the counting and growing pattern tasks. The data collected in a holistic, descriptive and interpretive way includes classroom observations, notes and documents (e.g. worksheets, tests, individual works).

The major difficulty that we found was how to measure creativity. As we are uncomfortable yet with a psychometric evaluation in this first approach, we chose to follow the basic ideas of the authors (Conway, 1999; Silver, 1997) without assigning a score to each student but making a global analysis. We followed the suggestion to measure creativity of students through the three dimensions according to Silver (1997) and Conway (1999). They suggested that fluency can be measured by the number of correct responses, solutions, obtained by the student to the same task in a process that Silver (1997) describes as multiple solution task. Flexibility can be measured with the number of different solutions that the student can produce organized in different categories or perceptions, whereby there are a variety of different ideas about the same problem or thing; that is, analyzing the number of different categories. And originality can be measured analyzing the number of responses in the categories that were identified as original, by comparison with the percentage of students in the same group that could produce the same solution. This mean can be assessed, as is the statistical infrequency of responses in relation to peer group responses. It is rather difficult to measure mainly the component of originality, which is for many authors the dimension that should out-top (Besemer & O'Quin, 1999). As these authors refer, this category can be highlighted by asking, "how often would this solution be found?" To overcome this difficulty where suggested to hear the opinion from a peer about the resolution. In this exploratory study we didn't measure the solutions but only analyzed them globally: the most common and the most original according to the frequency of the responses.

RESULTS AND DISCUSSION

During the didactical experience several tasks of different kinds were used. We present here three of those pattern tasks that require producing various and different responses.

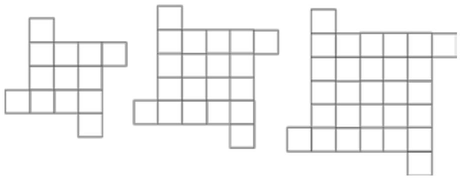
In this paper, we shall highlight the creative responses of the group of students involved in the modeling tasks. The description of the tasks is in Figure 2.

Task 1 - The shells

The sea girl organized the shells she caught yesterday like the figure shows. Can you find a quick process to count them? Discover as much ways as you can.

Task 2- Squares

Observe the growing pattern.



1. Draw the next figure.
2. Write the expression of the nth term.

Task 3 - Dots

Observe the dots in the figure.

1. Imagine that this is the 1st term of a sequence. Draw the next terms.
2. Write a numerical expression translating a way to calculate the nth number term of the sequence.
3. Imagine that the sequence you draw began with the 2nd term. Draw the 1st term.

Figure 2: Pattern tasks

Task 1. This type of task requires students to see the arrangement in different ways connecting previous knowledge about numbers relationships and their connections with basic geometric concepts. There are different ways to count the arrangement of the shells and each counting can be respectively written through a numerical expression that translates the students' thinking and seeing. Figure 3 illustrates the summary of the most common resolutions, with the expressions corresponding to each way of "seeing".

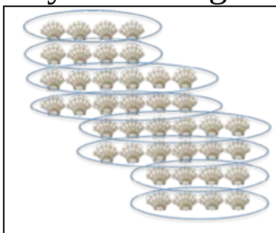
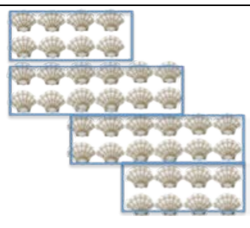
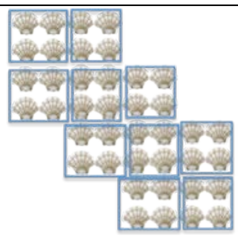
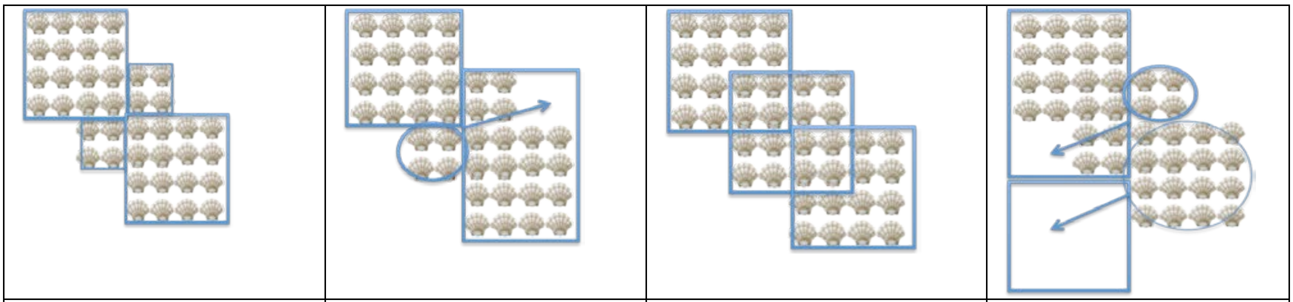
			
4+4+6+6+6+6+4+4 (20)	2x4+2x6+2x6+2x4 (16)	10 x (2x2) (10)	4x2+6x2+6x2+4x2 (9)

Figure 3: Summary of students' most common responses on task1

These expressions can be verbalized as the following: "I see the shells in horizontal rows each one with 4, 4, 6, 6, 6, 4 and 4 shells" or "One rectangle of 2 by 4, another rectangle of 2 by 6, another of 2 by 6 and a last one of 2 by 4" or "I see ten squares of 2 by 2". It is important that teachers allow students to discover that each expression illustrates one way of seeing but they are all equivalent and correspond to the same number of shells, 40. Figure 4 illustrates the most original responses.

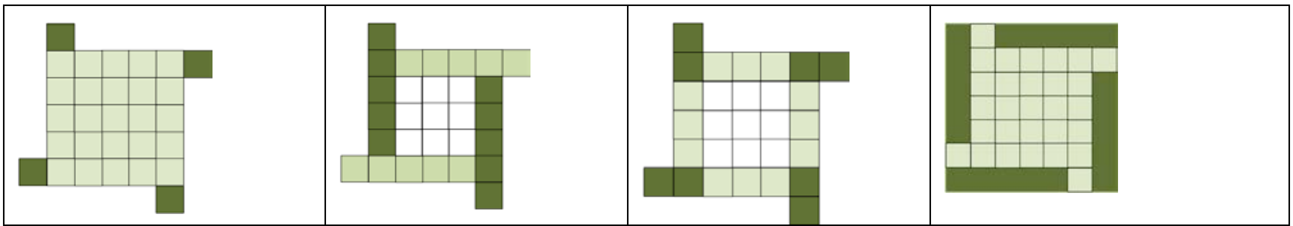


(4X4)+4+4+ (4X4) (4)	4x4+6x4 (2)	3x(4x4) – (4+4) (2)	10 x 4 (1)

Figure 4: Summary of students' most original responses on task1

Our expectations of students' creativity in this task lay in the different original ways of seeing/counting the number of shells. In this class we considered that these four students present the most original solutions, as has the statistical infrequency of responses in relation to peer group of responses. We claim that a previous work with counting tasks in figurative settings can be a particularly good way to develop skills of *seeing* (identification, decomposition, rearrangement) to facilitate similar processes in growing pattern tasks (Vale & Pimentel, 2011).

Task 2. We intended that students look for a pattern in a figurative sequence, describe it, and produce arguments to validate it using different representations. The previous work with visual counting may help to see a visual arrangement that changes in a predictable form and write numerical expressions translating the way of seeing, in order to make possible the generalization to distant terms. Students use different representations, more or less formal, to solve this task. They achieve a general rule through schemes and drawings or tables, but mainly using functional reasoning that allowed them to accomplish far generalization. We will regard only to the different ways of seeing the pattern to get far generalization, as we are convinced that is the most important aspect of solving this tasks in which students can be creative. Figure 5 illustrates different ways of seeing the 3rd term of the sequence.



nxn+4 (10)	nxn+4x(n+2) (3)	nxn+4x2+4xn (3)	$(n+4)^2$–4x(n+2) (1)

Figure 5: Summary of students' responses on task2

The same criterion of the previous task was used to analyze the students' work. The first solution was the most common and to get the general rule students used other representations, mainly a table to relate the number of the figure in the sequence and the number of squares, according to the way they saw the pattern. The last way was used only by one student, that applied deconstructive reasoning (Rivera, 2009).

Task 3. This task has the same objectives of the others but also formulating additional data, according to the solution presented for each student. Figure 6 synthesizes all the answers.

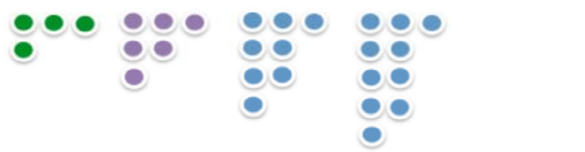
	
3+1+(n-1)x2 (8)	4+(n-1)x2 (7)
	
(n+1) (n+2)/2 (1)	$2+n^2$ (1)

Figure 6: Summary of students' responses on task3

This task wasn't completely solved by all the students. The table includes only 17 answers (n=21), from the students who completed the task. It was difficult for them not to invent the next terms starting from the given term, but they worked backwards to discover the new first term. The last two solutions were the most original since only two students presented them.

CONCLUDING REMARKS

Creativity is a field that we are just beginning to explore but this allowed us to experience the construction of some tasks that, in addition to the mathematical concepts and processes they involve, mainly generalization, allow students multiple solutions. We observed that two of the components of creativity, fluency and flexibility, were largely identified mainly in the counting tasks. Each task is intentionally not designed to assess only one component of mathematical creativity although, in some cases, one of the components is more relevant. We must look for ways to improve originality that in this class not had high results. Students need to be encouraged to seek unusual and original responses, since this strategy represents a way to get solutions to difficult problems or a path to creative solutions. The most successful problem solver is the individual who can apply diverse approaches (Conway, 1999). It is important that future teachers become themselves creative thinkers and they must be aware to act in the same way with their own students. They need to recognize that both flexibility and originality encourage divergent thinking, which promotes higher-level thinking. Classroom teachers should examine their teaching practices and seek out appropriate curricular materials to develop mathematical creativity. The challenge is to provide an environment of practice and problem solving that stimulates creativity that will enable the development of mathematically competence in all students. Our concern was not to categorize students but to identify potentialities in the tasks to develop creativity in students, detecting their mathematical strengths or weaknesses. This work obviously aimed also to identify potentially creative students.

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