State Space Modelling to Increase the Accuracy of Weather Radar Estimation of Mean Area Precipitation

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Abstract: This work compares some state space models for correcting the bias of the weather radar in Cruz do Leão, at Coruche, Portugal. Particular attention was given to the statistical adjustment of models, because precipitation data clearly deviated from the normal distribution. This work is based on 17 storms occurred between September of 1998 and November of 2000, in an area that includes part of the Alenquer river hydrographical basin.

Keywords: State Space Models; Weather Radar Calibration; Area Rainfall Estimation.

1 Introduction

Many problems in Meteorology and Hydrology need an accurate measurement of the total rainfall amount in a certain area. For this purpose, the best results are achieved using both rain gauges and weather radar measurements. However, although rain gauges may provide good point rainfall estimates, they fail to depict the rainfall spatial distribution. Alternatively, weather radar may outline accurate rainfall isopleths (Figure 1), but their point estimates are not so good, due to errors of either meteorological or instrumental nature. This can be reduced in some extent if the radar is carefully calibrated.

There are several approaches to combine the rain gauges and radar estimates, taking into consideration the different but complementary nature of the two sensors. Krajewski (1987) and Severino and Alpuim (2005) apply an optimal interpolation method based on Kriging and CoKriging. Calheiros and Zawaadzki (1993) and Rosenfeld et al. (1993) use a probability matching method. In this work we follow Ahnert et al. (1986), Anagnostou et al. (1998), Alpuim and Barbosa (1999) and Chumchean et al. (2004), who use the approach of modelling the relationship between the two types of measurements through a state space model and consequent application of the Kalman filter algorithm.

The data available for this study correspond to 17 storms which occurred between September of 1998 and November of 2000 in a 10×10 Km2 area

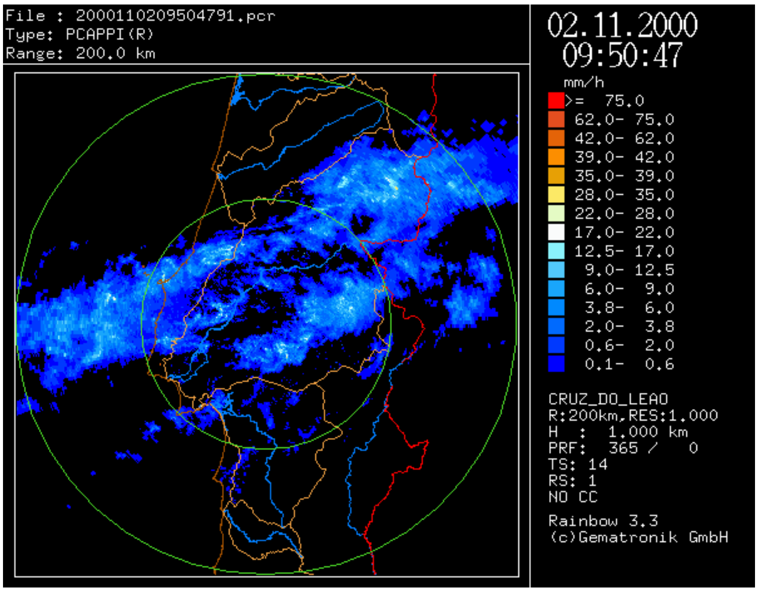

FIGURE 1. Representations of (G)-rain gauge measurement, (R)-radar estimates (no calibrated), (ML)-radar calibrated estimates with parameter gaussian maximum likelihood estimation and (D-F)-radar calibrated estimates with distribution-free parameters estimation.

located in the Alenquer river basin, 40 Km north of Lisbon. There are five rain gauges located in this area, which correspond to a reasonably high density of 20Km^2/rain gauge.

2 Methodology

There are several possible state space models that can be used to describe the relationship between radar and rain gauges measurements. In this work we consider that this relationship verifies the equations

$$\begin{aligned} \mathbf{G}_t &= \mathbf{R}_t\boldsymbol{\beta}_t + \mathbf{e}_t \\ \boldsymbol{\beta}_t &= \boldsymbol{\Phi}\boldsymbol{\beta}_{t-1} + \boldsymbol{\varepsilon}_t \end{aligned}$$

where $\boldsymbol{\beta}_t$ is a vector autoregressive, VAR(1), process of unobservable calibration factors, called states, and $\mathbf{G}_t$ is a vector of rain gauges measurements. The matriz $\mathbf{R}_t$ is a diagonal matrix with radar measurements and the vectors $\mathbf{e}_t$ and $\boldsymbol{\varepsilon}_t$ are uncorrelated white noises with covariance matrices $\boldsymbol{\Sigma}_{\mathbf{e}}$ and $\boldsymbol{\Sigma}_{\boldsymbol{\varepsilon}}$, respectively.

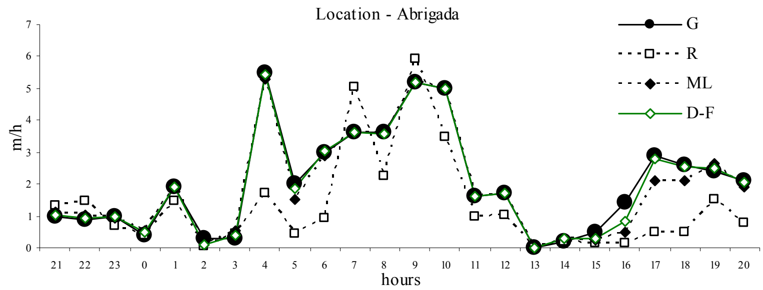

FIGURE 2. Representations of (G)-rain gauge measurement, (R)-radar estimates (no calibrated), (ML)-radar calibrated estimates with parameter gaussian maximum likelihood estimation and (D-F)-radar calibrated estimates with distribution-free parameters estimation.

In each time t, the vector of states $\boldsymbol{\beta}_t$ is predicted by the recursive equations of the Kalman filter algorithm, which corresponds to the best linear unbiased predictor.
The vector of parameters $\{\mathbf{\Phi}, \mathbf{\Sigma_e}, \mathbf{\Sigma_{\varepsilon}}\}$ was estimated by the maximum likelihood method, thus assuming that the error sequences follow a normal distribution. In this case, the log-likelihood of a sample can be written through conditional distributions and evaluated by the Newton-Raphson method or the EM-algorithm (Shumway and Stoffer, 1982).
However, as precipitation data clearly deviated from the normal curve, in addition to maximum likelihood, we consider also distribution-free estimators for the model parameters. We compare the final results produced by the maximum likelihood and distribution-free methods, both for point and mean area precipitation estimation.

3 Results

These methodologies reduce bias of radar measurement of precipitation, mainly in mean area precipitation estimation. It is clear (Figure 2) that a dynamic model can be successful in calibration radar estimates. In the example of Figure 2, for a particular storm in a rain gauge location, we get a good performance, reducing the radar bias significantly.
The quality of the adjustment process in the sites that are not used in parameters process is evaluated by the Error Variance at Gauges (EVG) and the Root Mean Square Error (RMSE), respectively. The results indicate that models adjusted through distribution-free estimation produce better results both for point and mean area estimation of precipitation.

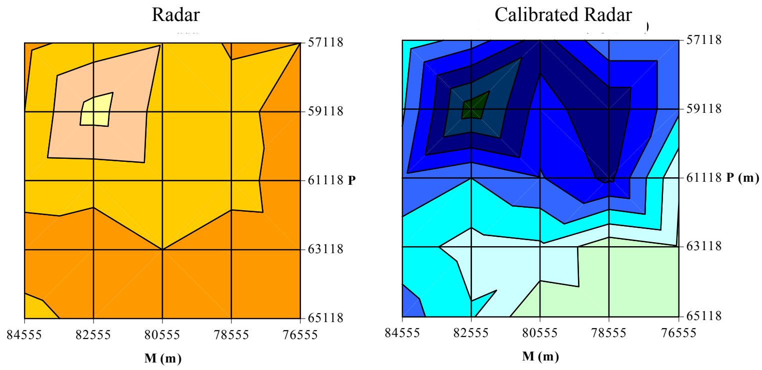

FIGURE 3. Radar (left) and calibrated radar (right) estimation of mean area precipitation.

Acknowledgments: Special thanks to Sérgio Barbosa from Portuguese Meteorological Institute for the data and valid contributions.

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