Single-Photon Source Based on FWM With Adjustable Linear SOP

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Abstract – We present a setup able to generate and detect single photons in optical fibers using the stimulated four-wave mixing (FWM) process. The results show an accurate generation of single photons at four different linear states of polarization (SOPs), with angles 0, 45, 90 and -45 degrees. The detection was performed in back-to-back configuration and after transmission over an optical fiber with a length up to 10 m.

Keywords – Single-Photon Source, Polarization Control, Four-Wave Mixing, State of Polarization.

I. INTRODUCTION

The security of public-key cryptographic systems could become vulnerable due to advances in mathematics or computation [1]. Moreover, quantum cryptography offers us a practical solution to transmit secure information between two identities, usually called Alice and Bob [2]. The security of the transmitted information is assured by quantum mechanics properties such as no-cloning theorem and the Heisenberg uncertainty principle [3]. This means that quantum cryptography is not vulnerable to theoretical or technological advances [1]. The main application of quantum cryptography is the Quantum Key Distribution (QKD). In terms of encoding methods, QKD can be implemented using polarization- or phase-encoding [3].

Stephen Wiesner first proposed quantum cryptography in 1970, although more than 10 years have passed before his paper was finally published [4]. Using Wiesner’s work, in 1984, Bennett and Brassard created the first quantum protocol, the well known BB84 [5]. The first QKD experiment using polarization-encoding for the BB84 protocol took place in 1989 and was performed in free-space over 32.5 cm [6], [7]. Besides BB84 many other quantum protocols were designed. One of them is the 2-state protocol, also known as B92 that needs only two non-orthogonal states to implement secure QKD [8]. A more secure, but also more difficult to implement, quantum protocol, is the 6-state protocol, that uses three non-orthogonal bases that Alice and Bob randomly alternate [9], [10]. A very suitable QKD protocol called Differential-Phase-Shift Quantum Key Distribution (DPS-QKD) is now emerging as a technology that can be deployed in real fiber optical networks, and it is inherently secure against strong eavesdropping attacks called Photon Number-Splitting (PNS) attacks [1], [11].

Most of the quantum protocols require single-photon sources in order to transmit information between two different locations [12]. Single-photon sources can be obtained from many devices, such as color centers in diamond, quantum dots, or single atoms and molecules [13]. However, these sources have some practical problems, such as low collection efficiency, the technological complexity or the stability of the molecules [13]. Recently, stimulated FWM has been used to generate single-photons in optical fibers [14]-[18]. The stimulated FWM process is a third-order nonlinear process that occurs when light of two or more frequencies (known as pump and signal fields) are launched into an optical fiber given rise to a new wave, known as idler field [19]. One advantage of using the stimulated FWM is the possibility of generating the photons already inside the optical fiber [14], [15], [17]-[19]. Polarization encoded photons have been proposed in some quantum experiments to implement quantum cryptography protocols in optical fibers [12].

In this paper, we use the stimulated FWM process to obtain a source of single photons with adjustable linear SOP. This source also has the advantage of using only one optical field to obtain four different linear SOPs.

This paper is organized in the following way: in section II we describe the experimental setup used to generate and detect single photons in four different linear SOPs. In section III we present a theoretical description of the experiment. In section IV the experimental results are presented and discussed. Finally, in section V we present our conclusions.

II. EXPERIMENTAL DESCRIPTION OF THE SINGLE-PHOTON SOURCE

In the experimental setup, see Fig. 1, a pump at $\lambda_1$ from an external cavity laser (ECL), passes through a polarization controller (PC1), before being coupled to another optical signal, $\lambda_2$, from a tunable laser source (TLS). This second optical signal, $\lambda_2$, is externally modulated to produce optical pulses with a width at half maximum of 1.6 ns and a repetition rate of 555.6 kHz. The two optical fields, $\lambda_1$ and $\lambda_2$, pass through a linear polarizer, P1, and are launched into a dispersion-shifted fiber (DSF), with a length equal to 305 m. Due to the stimulated FWM process, a new optical field, called idler, is generated inside the DSF at $\lambda_3 = \lambda_1 - \lambda_2$ [19]. At the fiber output, the three optical fields pass through the PC4 in order to align the SOP of the idler photons with the linear polarizer P2. A filter, F1, blocks the pump and signal waves. At the exit of P2 the angle of the linear SOP, $\theta_1$, is controlled using a...
rotatable key connector (RKC1). After passing through the quantum channel (optical fiber), the SOP of the idler photons is analyzed using another linear polarizer, P3, whose orientation, \( \theta_2 \), is tuned using a second RKC (RKC2). The idler photons from P3 are detected with a Single-Photon Detector Module (SPDM) operating in the Geiger mode. The dark count probability of the SPDM for a time gate, \( t_g = 2.5 \text{ ns} \), is \( P_{dc} = 5 \times 10^{-5} \), and the quantum detection efficiency is \( 10 \% \) \cite{20}. Photon counting measurements were performed with 0.1 photons per pulse at the input of the quantum channel, during a period of time of 10 s.

### III. Theoretical Description

In order to assess if the SOPs are correctly generated and detected, we present here a theoretical description that allows to evaluate the obtained experimental data.

From the definition of Stokes polarization parameters \( (S_0, S_1, S_2 \) and \( S_3) \) for a plane wave, we can write the total optical power as,

\[
S_0 = \sqrt{S_1^2 + S_2^2 + S_3^2},
\]

The parameter \( S_1 \) is the difference between the optical power that passes through a linear horizontal polarizer and a linear vertical polarizer. The parameter \( S_2 \) is the difference between the optical power that passes through a linear polarizer at 45° and a linear polarizer at -45°, and \( S_3 \) is the difference between the optical power that passes through a circular right polarizer and circular left polarizer \cite{21}.

The SOP at P3 output (see Fig. 1) can be written as,

\[
S'(2\theta) = M_P(2\theta) S_i,
\]

where \( M_P(2\theta) \) is the Mueller matrix for a rotated linear polarizer in Stokes space, and \( S_i \) is the Stokes vector corresponding to the SOP at the fiber link output. The Mueller matrix for a rotated ideal linear polarizer is given by \cite{21},

\[
M_P(2\theta) = \frac{1}{2} \begin{bmatrix}
\cos(2\theta) & \cos(2\theta) & \sin(2\theta) \\
-\sin(2\theta) & \cos(2\theta) & \sin(2\theta) \\
0 & 0 & 0
\end{bmatrix}.
\]

where \( \theta \) is the angle of the analyzer, see Fig. 1, which can vary between 0 and \( \pi \). The Stokes vector corresponding to the SOP at the fiber link output can be written as \cite{21},

\[
S_i(2\alpha, 2\beta) = \begin{bmatrix}
S_0 \\
S_1 \\
S_2 \\
S_3
\end{bmatrix} = S_0 \begin{bmatrix}
\cos(2\alpha) & \sin(2\alpha) \\
\cos(2\alpha) & -\sin(2\alpha) \\
\cos(2\alpha) & \sin(2\alpha) \\
-\sin(2\alpha) & \cos(2\alpha)
\end{bmatrix},
\]

where \( \alpha \) represents the ellipticity of the SOP, that can vary between -45° and 45°, and \( \beta \) is the orientation of the semi-major axis of the ellipse, which can vary between 0° and 180°. Note that \( \alpha \) and \( \beta \) angles are referred to the Stokes space. Substituting (3) and (4) in (2), we obtain,

\[
S'(2\theta, 2\alpha, 2\beta) = \frac{S_0}{2} \begin{bmatrix}
1 + \cos(2\alpha) \cos(2(\beta - \theta)) \\
\cos(2\theta) [1 + \cos(2\alpha) \cos(2(\beta - \theta))] \\
\sin(2\theta) [1 + \cos(2\alpha) \cos(2(\beta - \theta))] \\
0
\end{bmatrix}.
\]

From (5), we can obtain the four Stokes polarization parameters. Using (1), we can write the total optical power at P3 output, as,

\[
S_0'(2\theta, 2\alpha, 2\beta) = \frac{S_0}{2} \left[ 1 + \cos(2\alpha) \cos(2(\beta - \theta)) \right].
\]

In terms of the average number of counts, (6) can be written as,

\[
\bar{N}(2\theta, 2\alpha, 2\beta) = \bar{N} \left[ 1 + \cos(2\alpha) \cos(2(\beta - \theta)) \right],
\]

where \( \bar{N} \) is the average number of counts for each SOP, and \( \bar{N} \) represents the average number of counts for a linear SOP, i.e., \( \alpha = 0^\circ \), aligned with the polarizer angle, i.e., \( \beta = \theta_2 \).

### IV. Analysis of the Experimental Results

In this section we present and discuss the experimental results obtained for the three different schemes: (i) back-to-back configuration, and (ii) 1 m of transmission fiber and (iii) 10 m of transmission fiber.

#### A. Back-to-Back

First, we performed a measurement without the quantum channel, where the RKC1 was in back-to-back with the RKC2, i.e., the two RKCs was directly linked.

In Fig. 2 we plot the average number of counts as a function of \( \theta_2 \), for the four different linear SOPs generated by our single-photon source. The results presented in Fig. 2 show that it exists a maximum and a minimum, separated of about 90°, for each linear SOP. The mean phase difference between each linear SOP is about 45°. The fact that the maximums do not present the exact same value is because the losses are different for different angles of the RKCs.

The theoretical fits given by (7) are also plotted in Fig. 2. We can see a good adjustment to the experimental results, which means that the experimental data for this scheme...
were well achieved. However, some minor deviations between theoretical and experimental results are observed. These are mainly due to the wear of the RKCs that are directly linked, and consequently lead to different losses for different angles of the RKCs.

From this analysis we can conclude that the generated single-photon were detected efficiently, and we can establish a good correspondence between the emitter and the receiver.

B. Propagation on a Single-Mode Fiber

In this subsection we report the obtained experimental data in the case where a SMF was used as a quantum channel. We have used two different fibers lengths, 1 and 10 m.

B.1 Length equal to 1 m

In Fig. 3 we present the average number of counts obtained with a SMF with a length equal to 1 m as a function of \( \theta_2 \), for the four different linear SOPs. From Fig. 3, we can see that exists a maximum and a minimum separated of about 90° for each SOP. In this scheme, the mean phase difference between the different linear SOPs, is about 48°. The minimum value obtained for 0 and -45 degrees is 0, 45, 90 and -45 degrees, while the other three present some ellipticity due to polarization changes inside the optical fiber. The theoretical fits approximate well to the experimental data in the four different SOPs.

B.2 Length equal to 10 m

In Fig. 4 we present the average number of counts obtained with a SMF with a length equal to 10 m as a function of \( \theta_2 \), for the four different linear SOPs. The presence of a maximum and a minimum separated of about 90° for each SOP also maintains, as the decrease of about 1500 counts. The mean phase difference between the different linear SOPs is about 46°. However, between the SOPs of 0 and 45 degrees, that difference is about 60°. In terms of minimum values, we can see that only one SOP remains linear (45°), while the other three present some ellipticity due to polarization changes inside the optical fiber. The theoretical fits approximate well to the experimental data in the four different SOPs.
For a SMF with a length longer than 10 m the results begin to deviate considerably from the theoretical description, since the polarization mode dispersion (PMD) assumes a major importance. PMD is a well known problem in optical communication systems [22]-[24]. This is due to the fact that birefringence varies randomly in an optical fiber, changing the SOP also randomly [25], [26]. This so-called modal birefringence, \( b_{mn} = |n_x - n_y| \), where \( n_x \) and \( n_y \) are the modal refractive indices for the two orthogonally polarized states, with \( n_x \neq n_y \), randomly change the orientation of \( x \) and \( y \) axes over a length scale of \(~10 \text{ m}\), unless polarization-maintaining fibers (PMFs) were used [19]. This is the major issue of polarization-encoding in optical fibers. Due to this impairment we are unable to obtain a secure transmission in a fiber with a length longer than 10 m. To mitigate this impairment, some birefringence compensator will be needed, in order to control the SOP over the optical fiber [25], [26].

C. Visibility, Ellipticity and Orientation of the Ellipse

In order to perform a better evaluation of the experimental results, we can introduce a quantity that is called visibility. The visibility of the average number of counts, for each SOP, can be calculated as [27],

\[
\psi' = \frac{M - m}{M + m},
\]

where \( M \) and \( m \) are, respectively, the maximum and minimum average number of counts for each SOP. Since the visibility can assume values between 0 and 1, the results for a linear SOP are considered ideal when it is equal to 1. The visibility values for each SOP, given by (8), are presented in Table I. In the table we also present the ellipticity and the orientation of the semi-major axis of the ellipse. These values were obtained from the theoretical fits to the experimental results.

Analyzing the Table I, we can conclude, from the visibility and ellipticity values, that for back-to-back case, all the SOPs presented at Fig. 2 have similar values for maximum and minimum average number of counts, and are all linearly polarized.

From the values obtained with the SMF with a length equal to 1 m we can see that the SOPs of 0 and -45 degrees are not perfectly linear but present some ellipticity different than 1. This is due to some random polarizations changes inside the optical fiber, as mentioned before.

From the values obtained with the SMF with a length equal to 10 m we can see that three of the SOPs present ellipticity values greater than 15. It traduces the fact that these SOPs are slightly elliptically polarized, mainly viewed for the SOP of 90\(^\circ\), as can be seen from the visibility values.

In respect to the orientation of the semi-major axis of the ellipse, \( \beta \), we can see that the mean phase difference between the four SOPs of each scheme is about 45\(^\circ\). However, for the third scheme (SMF, \( L = 10 \text{ m} \)) we have two SOPs separated of about 60\(^\circ\), namely the SOPs of 0 and 45 degrees. This is due to the fact that in a fiber with such a length, the correlation between the SOPs begins to lose. The fact that the values of \( \beta \) for the SMF with a length equal to 10 m are so different from the other two, it is because in this case the SOPs suffered a random translation.

If we compare the three schemes tested in terms of the three parameters presented at Table I, we can see that there are a degradation in the results obtained with the increase of the length of the fiber. First, for the SMF with a length equal to 1 m we have two linearly polarized and two slightly elliptically polarized SOPs. For the SMF with a length equal to 10 m we have only one linearly polarized SOP, and three slightly elliptically polarized SOPs. Second, the maximum value of ellipticity obtained with the SMF with a length equal to 1 m is smaller than the one for the SMF with a length equal to 10 m.

These results are in good agreement with other experiments reported in the literature [19].

Based on the above discussion, we can say that we are able to implement a 4-states protocol with our single-photon source using a SMF with a length up to 10 m as a quantum channel.

V. CONCLUSIONS

We have shown that it is possible to generate and detect single photons in well defined four linear SOPs using a SMF with a length up to 10 m, as a quantum channel. This setup can be used to implement a quantum protocol to authenticate classical messages with enhanced security.

Our single-photon source permits to select four linear SOPs separated of 45\(^\circ\), and has the advantage of generate the photons already inside the optical fiber using only one optical field.

To extend the transmission length behind 10 m, active polarization-tracking devices must be used in order to surpass the PMD limitation.

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