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Modelling the problem of food distribution by the Portuguese food banks

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Abstract: A food bank is a non-profit, social solidarity organisation that typically distributes the donated food among a wide variety of local non-profit, social solidarity institutions which in turn feed the low-income people. The problem presented by the Portuguese Federation of Food Banks is to determine, for a specific food bank, the quantities of the donated products that must be assigned to each local social solidarity institution in order to satisfy the needs of the supported people as much as possible, without favouring any institution. We propose a linear programming model followed by a rounding heuristic to obtain a solution to the problem described. Computational results are reported.

Keywords: linear programming; heuristics; integer programming; Portugal.

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1 Introduction

Food banks are non-profit, social solidarity organisations that typically distribute the donated food among local non-profit, social solidarity institutions that in turn distribute the food among the people in need. The first food bank in Portugal was launched in Lisbon in 1992 and ever since many other food banks have been opened. Today, the Portuguese Federation of Food Banks represents a network of 17 food banks across the country. Each food bank runs a centralised warehouse which serves as a single collection and distribution point for food donations. The largest portion of the donated food comes from food leftover from the normal processes found among profit-making companies. This food can come from any part of the food chain, e.g., from farmers who have produced too much or whose products are not that visually appealing, from manufacturers who have overproduced, or from retailers who have over-ordered. All these donated products are delivered to the food bank's warehouses by the companies themselves. Often the products are approaching or beyond their expiry dates and, in these cases, the food banks must ensure that the food is safe for consumption. The other portion of the donated products comes from the general public through the national seasonal campaigns where each food bank collects food at various local supermarkets. A social solidarity institution must be registered in a food bank to receive food from it and all the institutions must go to the food banks to collect the products.

The problem raised by the Portuguese Federation of Food Banks is to determine, for a particular food bank, the quantities of the donated products that have to be assigned to each local social solidarity institution in order to satisfy the needs of the supported people as much as possible, without favouring any institution. Details of this problem follow and then we describe how, in practice, Lisbon's food bank, the largest of its kind in Portugal, solves the issue. We should add that this procedure is also followed by the majority of the other national food banks.

Supply of the donated products to the institutions is planned according to the types of products involved, i.e., dry or fresh. A *dry product* is considered to be every product that is non-perishable in the short term (e.g., rice, pasta, canned food and UHT milk), whereas a *fresh product* is the reverse (e.g., yogurt, cheese, butter, pasteurised milk, fruit, fresh vegetables and frozen food). Each institution receives one package (the so-called box) of dry products once a month and one box of fresh products once a week at the most. There is a schedule (date and time) for picking up the dry and the fresh products. The day agreed for the dry products is also used to collect fresh products. An institution might not be able to go to the food bank for the fresh products every week (e.g., there may be no vehicle or staff). Some food banks do not collect food requiring refrigeration or freezing, such as yogurt, butter, cheese and frozen products, because their warehouses have no refrigerator facilities.

Let IT be the set of institutions that receive products. By the end of every month, each institution $j \in IT$ sends a form to the food bank containing the following information relative to the next month:

a number of individuals that apply for packages (the so-called baskets) with dry products, nbd_j

b number of individuals that apply for baskets with fresh products, nbf_j

c number of days the institution serves meals of type m, nd_{mj}, where $m \in M = \{\text{breakfast, lunch, snack, dinner}\}$

d number of individuals that take meals of type $m \in M$, nm_{mj}

e list of products not wanted, U_j

f numbers of families and children that receive baskets with dry products

g numbers of families and children that receive baskets with fresh products

h numbers of children, elderly, other individuals and employees that take meals of type $m \in M$

i daily number of customers

j specific observations.

Data a to e are used to find a solution to the food distribution problem, while the remaining data are only used for statistical purposes.

To plan food distribution among the institutions, the food bank classifies the products into two categories: products for breakfast and snacks and products for lunch and dinner. A *product for breakfast and snacks* is considered to be every product that is typically consumed at breakfast-time and snacks (e.g., milk, cookies and sugar), and a *product for lunch and dinner* is considered to be every product that is typically consumed at lunches and dinner-times (e.g., meat, canned fish, pasta and rice). An institution either receives products from both categories (an Agreement A institution) or just receives products for breakfast and snacks (an Agreement B institution). In general, an institution of Agreement B category serves only breakfasts and snacks or has inadequate credibility according to the food bank to receive all kinds of products. Institutions also exist that receive only a surplus of the fresh products (Agreement C institutions), since, as a rule, the food bank does not have enough food to meet the needs of the other institutions (Agreements A and B institutions). Agreement C institutions wait for one of the other agreements.

At the end of each month, the food bank plans the supply of dry products in stock for the next month among all institutions pertaining to Agreement A or B. The supply of the fresh products in stock is planned on a daily basis for all institutions scheduled for the day. We then describe the procedure followed by the food bank to plan the dry food distribution. Distribution of the fresh food is planned in a similar way, except that one part of the fresh products in stock is reserved for the Agreement C institutions.

1.1 The usual procedure of the food bank to plan the dry food distribution for a specific month

For each institution $j \in IT$ of Agreement A do the following:

- define the number of individuals that apply for baskets with dry products according to the risk of hunger of the supported people, C_j^I, where $C_j^I = nbd_j \times ris_j$ with $ris_j = 1.15$ for a higher risk (the so-called Agreement AI), $ris_j = 0.85$ for a lower risk (Agreement AIII) or $ris_j = 1$ for an intermediate level (Agreement AII)
- define the number of individuals that apply for baskets with products for breakfast and snacks, $C_j^{I,PBS}$, and the number of individuals that apply for baskets with

For each institution $j \in IT$, calculate:

- the mean daily number of 'meals that use products for breakfast and snacks only', $C_j^{II,PBS}$

$$C_j^{II,PBS} = \frac{0.3\sum_{m\in M_1} nm_{mj}nd_{mj} + 0.2\sum_{m\in M_2} nm_{mj}nd_{mj}}{30},$$

where $M_1 = \{\text{breakfast, snack}\}$ and $M_2 = M \setminus M_1$ (it is assumed that 30% and 20% of the products consumed at each meal of type M_1 and M_2, respectively, provide one meal that uses products for breakfast and snacks only)

- the mean daily number of 'meals that use products for lunch and dinner only', $C_j^{II,PLD}$

$$C_j^{II,PLD} = \frac{0.5\sum_{m\in M_2} nm_{mj}nd_{mj}}{30}$$

(it is assumed that 50% of the products consumed at each meal of type M_2 provide one meal that uses products for lunch and dinner only)

- the sums $C_j^{PBS} = C_j^{I,PBS} + C_j^{II,PBS}$ and $C_j^{PLD} = C_j^{I,PLD} + C_j^{II,PLD}$.

Calculate the sums $C^{PBS} = \sum\limits_{j\in IT} C_j^{PBS}$ and $C^{PLD} = \sum\limits_{j\in IT} C_j^{PLD}$.

Let P be the set of donated products. For each institution $j \in IT$, calculate the quantity of product $p \in P \setminus U_j$ to supply:

$\frac{C_j^{PBS}}{C^{PBS}} d_p$, if p is a product for breakfast and snacks,

$\frac{C_j^{PLD}}{C^{PLD}} d_p$, if p is a product for lunch and dinner,

where d_p is the quantity of product p in stock (by the end of the previous month), measured in weight or capacity units.

The food bank makes some slight adjustments to the plan resulting from this procedure (e.g., the institutions that support more children and elderly people can receive larger quantities of baby food than the others).

The main drawback is that the dietary needs of the individuals in need are not explicitly catered for. Additionally, the compatibilities between the expiry dates of the products and the pick up days of the institutions are not targeted to the maximum. To solve the food distribution problem presented by the Portuguese Federation of Food Banks, we propose a linear programming (LP) approach where these issues are taken into account. The optimal linear solution obtained is then rounded to integer by a heuristic. For general discussions on linear and integer programming, we refer those interested to Dantzig and Thapa (1997) and Hillier and Lieberman (2005). The problem considered here is a variation of the classical 'diet problem' (Stigler, 1945) in the sense that its aim is to distribute food products taking into account nutrition and food restrictions. Since there is no cost associated with the products, the focus is on maximising the global satisfaction of the needs for the nutrients among the institutions, through the products that each one receives. There is a vast literature on the diet problem and its extensions. See, for example, Garille and Gass (2001), Anderson and Earle (1983), Darmon et al. (2002), Lancaster (1992) and Cadenas et al. (2004) on the planning of menus and diets for both humans and animals, and (Johnson and Behrens, 1982) for the study of the diet of a population.

In this paper, we propose a LP model followed by a rounding heuristic for the food distribution problem described above. In Section 2, we describe the LP formulation. In Section 3, we describe the heuristic. In Section 4, we report on computational experience. In the last section, we present some conclusions.

2 Formulation

The food distribution problem is solved in two stages, where the dietary needs of the supported people are considered. The distribution of the dry products is modelled in the first stage. The second stage concerns distribution of the fresh products bearing in mind the needs already met by the previous distributions over the month. In both stages, it is assumed that each individual receives either meals or baskets, and an individual who applies for a basket containing dry products also applies for a basket with fresh food and vice-versa. An institution can serve *breakfasts*, *lunches*, *snacks* or *dinners*. As for the meals, we consider three types of individuals, each with specific dietary needs: the *child*, the *elderly* and the *other*. Account is taken of the need for the main macro nutrients and energy for each type of individual per type of meal. For the micro nutrients, it is meaningless to consider the needs per type of meal and thus, we bear in mind the needs for the main micro nutrients for each type of individual per day. In the case of the baskets, we consider two types of individuals: the *child* and the *adult*. Account is taken of the need for the main macro nutrients, energy and micro nutrients for each of these types of individuals per month. This approach ensures that the food assigned to each institution is safe for consumption over specific periods of time.

The LP model that we shall consider consists of assigning to each institution an amount of each product subject to several constraints. Constraints A1 will state that the quantity of each nutrient received by each institution cannot exceed the needs for this nutrient. The aim of Constraints A2 together with the objective function is to convey the sense of justice and equality sought by the food bank. They will ensure that the values of the ratio between the quantity of each nutrient received and the needs for this nutrient are close for all institutions. There are donated products that are similar in nutritional terms (e.g., oil and olive oil). Constraints A3 will, for each set of similar products, prevent each institution from receiving only one product (e.g., neither oil alone nor olive oil). There are products that are specially recommended for individuals of a certain type (e.g., baby food can be particularly recommended for children and the elderly). The aim of Constraints A4 is to obtain even distributions of these products among the individuals with special needs. Constraints A4 will ensure, for each special product, that the values of the ratio between the quantity received for the individuals with special needs and the mean number of these individuals are close for all institutions. Constraints A5 will guarantee that those values are not less than the values of the ratio between the quantity received for the remaining individuals and the mean number of these individuals. The aim of these constraints is to prevent people with no special needs from being favoured in relation to the others. There are products that are similar in functional terms (e.g., rice and pasta are used with the same purposes at lunches and dinners). Constraints A6 aim to guarantee the assignment of one product at least among the similar products (functionally) to each institution. These constraints will state that the values of the amount received of all similar products per individual are close for all institutions. Constraints A7 will, for each product, prevent the amount assigned to all institutions from exceeding the quantity in stock. The objective is to maximise global satisfaction of the needs for the nutrients.

- *General data*

IT	set of institutions that receive products
P	set of donated products
NM	set of main macro nutrients and energy
Nm	set of main micro nutrients
Ib	set of types of individuals concerning the baskets
Im	set of types of individuals concerning the meals
I	set of types of individuals, $I = Ib \cup Im$
M	set of types of meals, $M = \{\text{breakfast, lunch, snack, dinner}\}$
M_1	set of types of 'small' meals, $M_1 = \{\text{breakfast, snack}\}$
M_2	set of types of 'large' meals, $M_2 = \text{M} \setminus M_1$
MP_p	set of types of meals where product $p \in P$ is typically consumed $MP_p = M$ or $MP_p = M_2$
a_{np}	quantity of nutrient $n \in NM \cup Nm$ per unit of product $p \in P$ (the unit p can be either one metric unit of weight, e.g., 1 kg, or capacity, e.g., 1 l, or one package)
qm_{nmi}	mean quantity of nutrient $n \in NM$ that a meal of type $m \in M$ should contain for an individual of type $i \in Im$
q'_{ni}	mean quantity of nutrient $n \in Nm$ that an individual of type $i \in Im$ should consume per day
q_{ni}	mean quantity of nutrient $n \in NM \cup Nm$ that an individual of type $i \in Ib$ should consume per month
δ_j	mean proportion of each nutrient that every basket delivered by institution $j \in IT$ should contain, depending on the risk of hunger of the supported people ($\delta_j = 1, 0.87, 0.74$ if $ris_j = 1.15, 1, 0.85$, respectively, and $\delta_j = 0$ if institution j does not deliver baskets)
Γ	partition of P where each element is a maximal set of products that are similar in nutritive terms
nsi	cardinality of Γ
SiP_g	element of $\Gamma, g = 1, \cdots, nsi$
SpP	set of products $p \in P$ that are specially recommended for individuals of some type

SpI_p — set of types of individuals for whom $p \in SpP$ is specially recommended

Π — set of subsets of products $p \in P$ such that each subset contains products that are similar in functional terms and is maximal

nfu — cardinality of Π

FuP_h — element of $\Pi, h = 1, \cdots, nfu$

$MFuP_h$ — set of types of meals where the products in FuP_h are typically consumed, $h = 1, \cdots, nfu$

- *General data* for a specific month

U_j — set of products not wanted by institution $j \in IT$

nb_{ij} — number of individuals of type $i \in Ib$ that apply for baskets from institution $j \in IT$

nd_{mj} — number of days when institution $j \in IT$ serves meals of type $m \in M$

nm_{mij} — number of meals of type $m \in M$ served each day to individuals of type $i \in Im$ by institution $j \in IT$

MI_j — set of types of meals that institution $j \in IT$ provides, directly or through the baskets,

$$MI_j = M, \text{ if } \sum_{i \in Ib} nb_{ij} + \sum_{m \in M_2} \sum_{i \in Im} nm_{mij} > 0$$

$$MI_j = M_1, \text{ otherwise}$$

IP_j — set of products that institution $j \in IT$ should not receive because they are typically consumed at meals that the institution does not provide,

$$IP_j = \{p \in P : |MP_p| = 2\}, \text{ if } |MI_j| = 2$$

$$IP_j = \emptyset, \text{ otherwise}$$

Q_{nj} — quantity of nutrient $n \in NM \cup Nm$ needed by institution $j \in IT$,

$$Q_{nj} = \sum_{i \in Ib} \delta_j nb_{ij} q_{ni} + \sum_{m \in M} \sum_{i \in Im} nm_{mij} qm_{nmi} nd_{mj}, \text{if } n \in NM$$

$$Q_{nj} = \sum_{i \in Ib} \delta_j nb_{ij} q_{ni} + \sum_{m \in M} \left(\sum_{i \in Im} \frac{q'_{ni}}{4} nm_{mij} \right) nd_{mj}, \text{ otherwise}$$

$\bar{A}^{\Omega}_{pj}$ — "mean" monthly number of individuals that may consume product $p \in S_pP$ supplied by institution $j \in IT$,

$$\bar{A}^{\Omega}_{pj} = \sum_{i \in \Omega \cap Ib} \delta_j nb_{ij} + \frac{\frac{\sum_{m \in MP_p} \left(\sum_{i \in \Omega \cap Im} nm_{mij} \right) nd_{mj}}{|MP_p|}}{30}$$

for whom p is specially recommended, if $\Omega = S_pI_p$
with no special needs regarding p, if $\Omega = I \setminus S_pI_p$

$\bar{B}_{hj}$ 'mean' monthly number of individuals that may consume products from set FuP_h, $h = \cdots, nfu$

$$\bar{B}_{hj} = \sum_{i \in Ib} \delta_j nb_{ij} + \frac{\frac{\sum_{m \in MFuP_h} \left(\sum_{i \in Im} nm_{mij} \right) nd_{mj}}{|MFuP_h|}}{30}.$$

In the next subsections, we describe the LP models for the dry and fresh product distributions.

2.1 *Dry product distribution*

Every institution has a specific day per month to go to the food bank to pick up the dry products. It is assumed that a dry product can be donated to an institution provided its expiry date has not yet lapsed by l days after the pick-up day. Dry product distribution for each month is planned by considering the dietary needs of the supported people from each institution for a period of one month.

- *Data* for a specific month

  DP set of dry products available

  d_p quantity of product $p \in DP$

  val_p expiry date of product $p \in DP$

  DB_p proportion of the stock of product $p \in SiP_g \cap DP$ relative to the stock of all products in $SiP_g \cap DP$, $g = 1, \cdots, nsi$,

  $$DB_p = \frac{d_p}{\sum_{p' \in SiP_g \cap DP} d_{p'}}$$

  $datD_j$ date when institution $j \in IT$ picks up the dry products

  $DNVal_j$ set of dry products whose expiry dates have lapsed by l days after the pick-up day of institution j,

  $$DNVal_j = \{p \in DP : \ val_p < datD_j + l\}$$

  DNA_j set of dry products that institution $j \in IT$ should not receive,

  $$DNA_j = ((U_j \cup IP_j) \cap DP) \cup DNVal_j.$$

- *Variables*

  xd_{pj} quantity of product $p \in DP$ assigned to institution $j \in IT$

  yd^1_{pj} quantity of product $p \in SpP \cap DP$ assigned to institution $j \in IT$ for the individuals with special needs regarding product p

  yd^2_{pj} quantity of product $p \in SpP \cap DP$ assigned to institution $j \in IT$ for the individuals with no special needs regarding product p

  wd_n minimum proportion of nutrient $n \in NM \cup Nm$ assigned to an institution in IT

zd_p^{Sp} target level on the relative quantity of product $p \in SpP \cap DP$ assigned to every institution in IT for the individuals for whom p is specially recommended

zd_h^{Fu} target level on the relative quantity of all products in $FuP_h \cap DP$ assigned to every institution in IT, $h = 1, ..., nfu$.

The quantities of product p represented by d_p, xd_{pj}, yd_{pj}^1, yd_{pj}^2, zd_p^{Sp} and zd_p^{Fu} can be measured either in metric units of weight or capacity or in the number of packages, and should be consistent with the unit of p used in a_{np}.

- *Formulation*

$$\max \sum_{n \in NM \cup Nm} wd_n \tag{1}$$

subject to

$$\sum_{p \in DP \setminus DNA_j} a_{np} xd_{pj} \leq Q_{nj}, \forall n \in NM \cup Nm, \forall j \in IT \tag{2}$$

$$\sum_{p \in DP \setminus DNA_j} a_{np} xd_{pj} \geq wd_n Q_{nj}, \forall n \in NM \cup Nm, \forall j \in IT \tag{3}$$

$$\begin{aligned} xd_{pj} \leq (1 + \Delta^{Si}) DB_p \sum_{p' \in SiP_g \cap DP \setminus DNA_j} xd_{p'j}, \\ g = 1, ..., nsi, \forall p \in SiP_g \cap DP, \forall j \in IT: \ p \notin DNA_j \end{aligned} \tag{4}$$

$$yd_{pj}^1 \leq (1 + \Delta^{Sp}) zd_p^{Sp} \bar{A}_{pj}^{SpI_p}, \forall p \in SpP \cap DP, \forall j \in IT: \ p \notin DNA_j \tag{5}$$

$$yd_{pj}^1 \geq (1 - \Delta^{Sp}) zd_p^{Sp} \bar{A}_{pj}^{SpI_p}, \forall p \in SpP \cap DP, \forall j \in IT: \ p \notin DNA_j \tag{6}$$

$$yd_{pj}^2 \leq (1 - \Delta^{Sp}) zd_p^{Sp} \bar{A}_{pj}^{I \setminus SpI_p}, \forall p \in SpP \cap DP, \forall j \in IT: \ p \notin DNA_j \tag{7}$$

$$xd_{pj} = yd_{pj}^1 + yd_{pj}^2, \forall p \in SpP \cap DP, \forall j \in IT: \ p \notin DNA_j \tag{8}$$

$$\sum_{p \in FuP_h \cap DP \setminus DNA_j} xd_{pj} \leq (1 + \Delta^{Fu}) zd_h^{Fu} \bar{B}_{hj}, h = 1, ..., nfu, \forall j \in IT \tag{9}$$

$$\sum_{p \in FuP_h \cap DP \setminus DNA_j} xd_{pj} \geq (1 - \Delta^{Fu}) zd_h^{Fu} \bar{B}_{hj}, h = 1, ..., nfu, \forall j \in IT \tag{10}$$

$$\sum_{j \in IT:\, p \in DP \setminus DNA_j} xd_{pj} \leq d_p, \forall p \in DP \tag{11}$$

$$xd_{pj} \geq 0, \forall p \in DP, \forall j \in IT: \ p \notin DNA_j \tag{12}$$

$$yd^1_{pj} \geq 0, \forall p \in SpP \cap DP, \forall j \in IT : \ p \notin DNA_j \quad (13)$$

$$yd^2_{pj} \geq 0, \forall p \in SpP \cap DP, \forall j \in IT : \ p \notin DNA_j \quad (14)$$

$$wd_n \geq 0, \forall n \in NM \cup Nm \quad (15)$$

$$zd^{Sp}_p \geq 0, \forall p \in SpP \cap DP \quad (16)$$

$$zd^{Fu}_h \geq 0, \ h = 1, ..., nfu, \quad (17)$$

where Δ^{Si}, Δ^{Sp}, Δ^{Fu} are constants between 0 and 1.

With Constraints (2), it is intended that the quantity of each nutrient assigned to each institution does not exceed the needs (Constraints A1). Constraints (3) state that wd_n is the minimum proportion of nutrient n assigned to an institution (Constraints A2). It is clear that one seeks to obtain an even distribution of each nutrient among the institutions by increasing as much as possible the minimum proportion of nutrient quantity that has been attributed. In Constraints (4), we consider the sets of products that are nutritionally similar. For every set, the proportion of the amount of each product assigned to each institution (considering the assigned amounts of all products from that set) does not exceed $1 + \Delta^{Si}$ times the similar proportion regarding the food bank (Constraints A3). With $\Delta^{Si} = 0$, an institution either receives no products from the same set or receives each product in the same proportion as that of the food bank. It is clear that increasing values of Δ^{Si} allow more differences between these two proportions. With these constraints we try to avoid solutions where institutions receive only one product from the sets with several products. In Constraints (5), (6) and (7) we consider the products that are special for certain types of individuals. According to Constraints (5) and (6), the ratio of the amount of each product assigned to each institution over the mean number of individuals with special needs is allowed to range from $1 - \Delta^{Sp}$ to $1 + \Delta^{Sp}$ times zd^{Sp}_p, where zd^{Sp}_p is a target value defined by the formulation (Constraints A4). As zd^{Sp}_p is the same for all institutions, the maximum difference between the ratios of each product p is $2\Delta^{Sp}$ times zd^{Sp}_p. By imposing Constraints (7), we try to obtain a distribution where the relative assigned amount of each special product for the individuals with special needs is greater than or equal to the similar ratio regarding the remaining individuals (Constraints A5). Hence, these constraints require that the ratio of the amount of each special product assigned to each institution over the mean number of individuals with no special needs does not exceed $1 - \Delta^{Sp}$ times zd^{Sp}_p. In Constraints (9) and (10) we consider the sets of products that are similar in functional terms. For every set, the amount of all products assigned to each institution over the mean number of individuals is allowed to range from $1 - \Delta^{Fu}$ to $1 + \Delta^{Fu}$ times zd^{Fu}_h, where zd^{Fu}_h is a target value defined by the formulation (Constraints A6). As zd^{Fu}_h is the same for all institutions, the maximum difference between the ratios of each product p is $2\Delta^{Fu}$ times zd^{Fu}_h. With these constraints our intention is not to obtain an even distribution of the products among the institutions but to avoid solutions where the institutions do not receive any product from the same set. Constraints (11) ensure that the amount of each product assigned to all institutions does not exceed the stock (Constraints A7). The remaining restrictions state non-negativity requirements on variables.

Expression (1) states the objective of maximising the sum of the minimum proportions of the nutrients assigned to the institutions over the month. Notice that the distribution generated by this formulation may be such that some institutions receive large proportions of certain nutrients and small proportions of others. Nevertheless, if that happens all institutions are affected in view of the objective function [together with Constraints (3)].

An even distribution of the nutrients among the institutions could alternatively be achieved by the following constraints [instead of Constraints (3)]:

$$\sum_{p \in DP \backslash DNA_j} a_{np} xd_{pj} \leq (1 + \Delta^N) wd_n Q_{nj}, \forall n \in NM \cup Nm, \forall j \in IT \qquad (18)$$

$$\sum_{p \in DP \backslash DNA_j} a_{np} xd_{pj} \geq (1 - \Delta^N) wd_n Q_{nj}, \forall n \in NM \cup Nm, \forall j \in IT, \qquad (19)$$

where Δ^N is a constant between 0 and 1. These constraints allow the proportion of each nutrient n assigned to each institution to range from $1 - \Delta^N$ to $1 + \Delta^N$ times wd_n, where wd_n is a target value defined by the formulation. As wd_n is the same for all institutions, the maximum difference between the proportions of each nutrient n is $2\Delta^N$ times wd_n. It is clear that one seeks to obtain an even distribution of the nutrients among the institutions by decreasing the value of Δ^N as much as possible. Now, expression (1) states the objective of maximising the sum of the target levels on the proportions of the nutrients assigned to the institutions over the month. Again, the distribution generated by the formulation with these constraints may be such that some institutions receive large proportions of certain nutrients and small proportions of others. Nevertheless, if that happens all institutions are affected because of the equilibrium Constraints (18) and (19).

2.2 *Fresh product distribution*

Every institution has a specific day each week to go to the food bank to pick up the fresh food. It is assumed that a fresh product can be donated to an institution provided its expiry date is greater than or equal to k days after the pick-up day. The formulation is similar to the one described for the dry product distribution.

- *Data* for a specific day d

FP set of fresh products available at the beginning of the day
d_p quantity of product $p \in FP$
val_p expiry date of product $p \in FP$
FB_p proportion of the stock of product $p \in SiP_g \cap FP$ relative to the stock of all products in $SiP_g \cap FP$, $g = 1, \cdots, nsi$,

$$FB_p = \frac{d_p}{\sum_{p' \in SiP_g \cap FP} d_{p'}}$$

IF set of institutions that receive fresh products during the day

$FNVal_j$ set of fresh products whose expiry dates have lapsed by k days after the day,
$FNVal_j = \{p \in FP : \ val_p < d + k\}$

FNA_j set of fresh products that institution $j \in IF$ should not receive,
$FNA_j = ((U_j \cup IP_j) \cap FP) \cup FNVal_j$.

- *Variables*

xf_{pj} quantity of product $p \in FP$ assigned to institution $j \in IF$

yf_{pj}^1 quantity of product $p \in SpP \cap FP$ assigned to institution $j \in IF$ for the individuals with special needs regarding product p

yf_{pj}^2 quantity of product $p \in SpP \cap FP$ assigned to institution $j \in IF$ for the individuals with no special needs regarding product p

wf_n minimum proportion of nutrient $n \in NM \cup Nm$ assigned to an institution in IF

zf_p^{Sp} target level on the relative quantity of product $p \in SpP \cap FP$ assigned to every institution in IF for the individuals for whom p is specially recommended

zf_h^{Fu} target level on the relative quantity of all products in $FuP_h \cap FP$ assigned to every institution in IF, $h = 1, ..., nfu$.

- *Formulation*

$$\max \sum_{n \in NM \cup Nm} wf_n \tag{20}$$

subject to

$$\sum_{p \in FP \setminus FNA_j} a_{np} xf_{pj} \leq Q_{nj}(1 - pp_{nj}), \forall n \in NM \cup Nm, \forall j \in IF \tag{21}$$

$$\sum_{p \in FP \setminus FNA_j} a_{np} xf_{pj} \geq wf_n Q_{nj}(1 - pp_{nj}), \forall n \in NM \cup Nm, \forall j \in IF \tag{22}$$

$$\begin{aligned} xf_{pj} \leq (1 + \Delta^{Si}) FB_p \sum_{p' \in SiP_g \cap FP \setminus FNA_j} xf_{p'j}, \\ g = 1, ..., nsi, \forall p \in SiP_g \cap FP, \forall j \in IF : \ p \notin FNA_j \end{aligned} \tag{23}$$

$$yf_{pj}^1 \leq (1 + \Delta^{Sp}) zf_p^{Sp} \bar{A}_{pj}^{SpI_p}, \forall p \in SpP \cap FP, \forall j \in IF : \ p \notin FNA_j \tag{24}$$

$$yf_{pj}^1 \geq (1 - \Delta^{Sp}) zf_p^{Sp} \bar{A}_{pj}^{SpI_p}, \forall p \in SpP \cap FP, \forall j \in IF : \ p \notin FNA_j \tag{25}$$

$$yf_{pj}^2 \leq (1 - \Delta^{Sp}) zf_p^{Sp} \bar{A}_{pj}^{I \setminus SpI_p}, \forall p \in SpP \cap FP, \forall j \in IF : \ p \notin FNA_j \tag{26}$$

$$xf_{pj} = yf^1_{pj} + yf^2_{pj},\ \forall p \in SpP \cap FP, \forall j \in IF :\ p \notin FNA_j \tag{27}$$

$$\sum_{p \in FuP_h \cap FP \backslash FNA_j} xf_{pj} \leq (1 + \Delta^{Fu}) zf^{Fu}_h \bar{B}_{hj},\ h = 1, ..., nfu, \forall j \in IF \tag{28}$$

$$\sum_{p \in FuP_h \cap FP \backslash FNA_j} xf_{pj} \geq (1 - \Delta^{Fu}) zf^{Fu}_h \bar{B}_{hj},\ h = 1, ..., nfu, \forall j \in IF \tag{29}$$

$$\sum_{j \in IF:\ p \in FP \backslash FNA_j} xf_{pj} \leq d_p,\ \forall p \in FP \tag{30}$$

$$xf_{pj} \geq 0,\ \forall p \in FP, \forall j \in IF :\ p \notin FNA_j \tag{31}$$

$$yf^1_{pj} \geq 0,\ \forall p \in SpP \cap FP, \forall j \in IF :\ p \notin FNA_j \tag{32}$$

$$yf^2_{pj} \geq 0,\ \forall p \in SpP \cap FP, \forall j \in IF :\ p \notin FNA_j \tag{33}$$

$$wf_n \geq 0,\ \forall n \in NM \cup Nm \tag{34}$$

$$zf^{Sp}_p \geq 0,\ \forall p \in SpP \cap FP \tag{35}$$

$$zf^{Fu}_h \geq 0,\ h = 1, ..., nfu, \tag{36}$$

where pp_{nj} is the proportion of nutrient n already received by institution j. This value can be obtained by totaling the proportions that were delivered during the month through the dry products and the fresh ones.

The right-hand sides of Constraints (21) define the amount of each nutrient still required by each institution, that is the difference between the needs for the month and the needs already met by the previous distributions of dry and fresh products. In Constraints (22), the proportion of each nutrient assigned to each institution is relative to the amount still required. Expression (20) and Constraints (21) to (36) have the same meaning for the fresh products as the objective function (1) and Constraints (2) to (17) for the dry products, respectively.

3 Rounding heuristic

In general, all the donated products are packed. Since the models described above are in linear programming, the optimal values of the xd_{pj} (xf_{pj}) variables expressed as a number of packages can be non integer, i.e., the quantity of a packed product assigned to an institution can be a fractional number of packages. To obtain integer values of packages, we propose a rounding heuristic. Note that we could formulate the distribution problems in mixed integer programming with integer requirements on variables xd_{pj} (xf_{pj}) expressing the number of packages. Nevertheless, requirements as to distribution levels (Constraints (5), (6), (9), (10) or the corresponding constraints for the fresh products) may increase the difficulty of solving the MIP formulations and thus, each MIP model may be far more difficult to solve than the LP formulation together with the rounding heuristic.

We shall now describe the heuristic for the dry products (lines 1 to 21 of Heuristic), although the heuristic for the fresh ones is similar. In simpler terms, consider that all products are packed. The heuristic assigns an integer number of packages of each product p to each institution. Let xd^*_{pj} be the optimal number of packages of product p for institution j obtained by solving the LP model, and consider y_{pj} to be the number of packages given by the whole approach (model and heuristic). In a first step, $y_{pj} = \lfloor xd^*_{pj} \rfloor$ packages of product p are assigned to institution j (lines 1 to 3). Then, a procedure is applied to decide how many further packages are assigned or, equivalently, the final value of y_{pj} (lines 4 to 21).

Let $R_p = d_p - \sum_{j \in IT} \lfloor xd^*_{pj} \rfloor$ be the quantity of product p that has not yet been assigned, and $\delta_{nj} = \dfrac{\sum_{p \in DP \setminus DNA_j} a_{np}(xd^*_{pj} - \lfloor xd^*_{pj} \rfloor)}{Q_{nj}}$ be the relative needs of institution j for nutrient n that have not yet been satisfied by the first step. Let $\xi = \{p \in DP : R_p > 0\}$ be the set of all non-completed assigned products and $\wp_n = \{p \in DP : a_{np} > 0\}$ the set of all products containing nutrient n.

The procedure can be summarised as follows: select a nutrient and an institution by the decreasing order of δ_{nj}, n_1 and j_1, respectively; select the richest product in nutrient n_1 among the set of available products that are required by institution j_1, p_1; assign to j_1 one more package of p_1; update δ_{nj_1} for each nutrient n, Φ, R_{p_1} and ξ; repeat the whole process whenever it is possible to satisfy the needs of some institution for some nutrient.

Algorithm 1 Heuristic

Input $xd^*_{pj}, \forall p \in DP, j \in IT : p \notin DNA_j; \delta_{nj}, \forall n \in NM \cup Nm, j \in IT; \xi;$
$R_p, \forall p \in DP; \wp_n, \forall n \in NM \cup Nm;$

1 **for** $j \in IT$ **do**
2 $\quad y_{pj} \longleftarrow \lfloor xd^*_{pj} \rfloor, \forall p \in DP \setminus DNA_j;$
3 $\quad \chi_j \longleftarrow DP \setminus DNA_j$
4 $\Phi \longleftarrow \{(n,j); n \in NM \cup Nm, j \in IT : \delta_{nj} > 0, \chi_j \cap \xi \neq \emptyset\};$
5 **while** $\Phi \neq \emptyset$ **do**
6 $\quad$ select (n_1, j_1), the element of Φ such that $\delta_{n_1 j_1}$ is maximum;
7 $\quad$ **if** $\wp_{n_1} \cap \chi_{j_1} \cap \xi \neq \emptyset$ **then**
8 $\quad\quad$ select p_1, the element of $\wp_{n_1} \cap \chi_{j_1} \cap \xi$ such that $a_{n_1 p_1}$ is maximum;
9 $\quad\quad y_{p_1 j_1} \longleftarrow \lfloor y_{p_1 j_1} \rfloor + 1;$
10 $\quad\quad \delta_{nj_1} \longleftarrow \delta_{nj_1} - \frac{a_{np_1}}{Q_{nj_1}}, \forall n \in NM \cup Nm : p_1 \in \wp_n;$
11 $\quad\quad \Phi \longleftarrow \Phi \setminus \{(n, j_1); n \in NM \cup Nm : \delta_{nj_1} \leq 0\};$
12 $\quad\quad R_{p_1} \longleftarrow R_{p_1} - 1;$
13 $\quad\quad$ **if** $R_{p_1} = 0$ **then**
14 $\quad\quad\quad \xi \longleftarrow \xi \setminus \{p_1\};$
15 $\quad\quad\quad$ **for** $n \in NM \cup Nm$ **do**
16 $\quad\quad\quad\quad$ **if** $\wp_n \cap \xi = \emptyset$ **then**
17 $\quad\quad\quad\quad\quad \Phi \longleftarrow \Phi \setminus \{(n,j); j \in IT : \delta_{nj} > 0\};$
18 $\quad\quad\quad$ **for** $j \in IT$ **do**
19 $\quad\quad\quad\quad$ **if** $\chi_j \cap \xi = \emptyset$ **then**
20 $\quad\quad\quad\quad\quad \Phi \longleftarrow \Phi \setminus \{(n,j); n \in NM \cup Nm : \delta_{nj} > 0\};$
21 $\quad$ **else** $\Phi \longleftarrow \Phi \setminus \{(n_1, j_1)\};$

In line 3, the heuristic initialises χ_j, the set of all products that institution j wants to receive. In line 4, the heuristic initialises Φ, the set of all nutrient/institution pairs such that the needs of the institution for the nutrient have not yet been satisfied and there still are products that the institution may receive and have not yet been completely assigned. In line 7, the heuristic verifies if there are available products with nutrient n_1 that institution j_1 may receive. If there are no such products, Φ is updated (line 21). Otherwise, the richest product in nutrient n_1 is selected (line 8), one package of this product is assigned to the institution (line 9) and several updatings are made (lines 10 to 12). If the selected product is over, it is necessary to verify

a for each nutrient, if there are products available that contain the nutrient (line 16)

b for each institution, if there are products available that the institution may receive (line 19).

In both cases a and b, if there are no such products, updatings on Φ are made (lines 17 and 20, respectively).

4 Computational experience

The principal aim of the computational tests is to assess the ability of the LP model together with the rounding heuristic to generate food distributions of a certain quality in a reasonable amount of time (up to two hours). A secondary aim involves comparing the abilities of the whole approach and the model with integer requirements on the xd_{pj} variables. We used Xpress-IVE 1.20.01 (XPRESS, 2010) as a LP solver, to implement the heuristic and as a MIP solver. Xpress-IVE default parameters of the LP and MIP solvers were used throughout. Computations were performed on a desktop computer with an Intel Core 2 – 2.53 GHz processor and with 4.00 GB RAM. In the next subsections, we describe the data and present the results.

4.1 Data

The Lisbon's food bank has provided the following information concerning one specific month: the list of the dry products in stock at the end of the previous month (to the number of 33) and their quantities (Table A1 in the Appendix), and the list of the registered Agreement A or B institutions (to the number of 313) and their characteristics (the number of meals of each type a day, the number of days serving meals, the number of adults and the number of children receiving baskets). Specific data for the fresh product distribution were not provided nor was the following information: the number of meals of each type per type of individual (child, elderly and other), the expiry dates of the products and the numbers and types of packages of each product. For this reason, we defined the *person* as being the only type of individual regarding the meals and assumed that every product could be delivered throughout the month. Furthermore, due to the lack of information about the packages, we considered that the quantities represented by the decision variables xd_{pj}, yd^1_{pj}, yd^2_{pj}, zd^{Sp}_p and zd^{Fu}_h of the formulation (1) to (17) are measured in Kg or l and the parameter a_{np} is the quantity of nutrient n per Kg or l of product p. As for the heuristic, we assumed that, for each product, the weights/capacities of all packages were identical and one weight/capacity was chosen. The optimal quantities of the products obtained by solving the LP model were translated into the numbers of packages and a_{np} was replaced by a'_{np}, the quantity of nutrient n per package of product p.

We considered three macro nutrients, carbohydrate, protein and fat, and five micro nutrients, calcium, iron, magnesium, phosphorus and vitamin B12. Information about food composition was taken from Tabela da Composição dos Alimentos (2008) and the calculation of the needs for nutrients (Tables A2 to A4 in the Appendix) was based on data from Dietary Guidance (2010). We considered an individual up to the age of eight (inclusive) to be a child.

We partitioned set P into 21 sets of similar products in nutritive terms, SiP_g, $g = 1, \cdots, 21$. Set SpP is defined by two products that are both specially recommended for children ($SpI_p = \{\text{child}\}$, $p = 1, 2$). We defined two sets of similar products in functional terms, FuP_h, $h = 1, 2$, one with two products and the other with five.

The numbers of variables and constraints of the formulation (1) to (17) for that month are 11,594 and 15,390, respectively. Due to the lack of data, we did not use the model (20) to (36).

4.2 Results

Dry food distribution generated by the approach proposed in this work is analysed on the strength of five main criteria: pattern of the proportion of the needs that have been met (Criteria 1); distribution patterns of specific products, the ones classified as nutritionally similar (Criteria 2), specially recommended for individuals of some type (Criteria 3) and similar in functional terms (Criteria 4); the distribution pattern of all products together (Criteria 5). Criteria 1 is related to the senses of justice and equality sought by the food bank. Criteria 2 to 4 are related to specific constraints of the LP model and are influenced by the values of the corresponding parameters: Criteria 2 with Constraints (4) and parameter Δ^{Si}; Criteria 3 with Constraints (5) to (7) and parameter Δ^{Sp}; Criteria 4 with Constraints (9) and (10) and parameter Δ^{Fu}. We analyse the distribution generated by the LP formulation and the one provided by the LP formulation together with the rounding heuristic. The criteria are evaluated using the indicators that we now describe.

- *Data* for the indicators

xd^*_{pj}	optimal quantity (in kg or l) of product p assigned to institution j, obtained by solving the model
y_{pj}	number of packages of product p assigned to institution j, obtained after running the heuristic
yd^{1*}_{pj}	linear optimal quantity of product p assigned to institution j for the individuals with special needs
yd^{2*}_{pj}	linear optimal quantities of product p assigned to institution j for the individuals with no special needs
pk_p	weight/capacity (in Kg/l) of one package of product p

$\bar{C}_{pj}$ 'mean' monthly number of individuals that may consume product $p \in P$ supplied by institution $j \in IT$,

$$\bar{C}_{pj} = \sum_{i \in Ib} \delta_j nb_{ij} + \frac{\frac{\sum_{m \in MP_p} \left(\sum_{i \in Im} nm_{mij} \right) nd_{mj}}{|MP_p|}}{30}.$$

- *Indicators*

1 Criteria 1: for each nutrient $n \in NM \cup Nm$,

$\overline{wd'_n}$ mean value of wd'_{nj}, where

$$wd'_{nj} = \frac{\sum_{p \in DP \setminus DNA_j} a_{np} xd^*_{pj}}{Q_{nj}}, \forall j \in IT$$

$\overline{wd''_n}$ mean value of wd''_{nj}, where

$$wd''_{nj} = \frac{\sum_{p \in DP \setminus DNA_j} a_{np} y_{pj} pk_p}{Q_{nj}} \forall j \in IT;$$

wd'_{nj} and wd''_{nj} are the proportions of the needs of institution j for nutrient n that have been met, obtained by the model and the whole approach, respectively

2 Criteria 2: for each product $p \in SiP_g \cap DP$, $g = 1, \cdots, nsi$,

$\overline{rsi'_p}$ mean value of rsi'_{pj}, where

$$rsi'_{pj} = \frac{xd^*_{pj}}{\sum_{p \in SiP_g \cap DP \setminus DNA_j} xd^*_{pj}}, \forall j \in IT : \textstyle\sum_{p \in SiP_g \cap DP \setminus DNA_j} xd^*_{pj} > 0,$$

$$p \notin DNA_j$$

$\overline{rsi''_p}$ mean value of rsi''_{pj}, where

$$rsi''_{pj} = \frac{y_{pj} pk_p}{\sum_{p \in SiP_g \cap DP \setminus DNA_j} y_{pj} pk_p}, \forall j \in IT : \textstyle\sum_{p \in SiP_g \cap DP \setminus DNA_j} y_{pj} > 0,$$

$$p \notin DNA_j;$$

rsi'_{pj} is the proportion of the quantity of product p relative to the similar products for institution j that receives some of these products, obtained by the model; rsi''_{pj} is the corresponding proportion obtained by the whole approach

3 Criteria 3: for each product $p \in SpP \cap DP$,

$\overline{rsp^{1'}_p}$ mean value of $rsp^{1'}_{pj}$, where

$$rsp^{1'}_{pj} = \frac{yd^{1*}_{pj}}{\bar{A}^{SpI_p}_{pj}}, \forall j \in IT : \bar{A}^{SpI_p}_{pj} \neq 0, p \notin DNA_j;$$

$rsp^{1'}_{pj}$ is the ratio of the assigned quantity of product p for the individuals with special needs over the mean number of these individuals, for each institution with those needs, obtained by the model

$\overline{rsp^{2'}_p}$ mean value of $rsp^{2'}_{pj}$, where

$$rsp^{2'}_{pj} = \frac{yd^{2*}_{pj}}{\bar{A}^{I\setminus SpI_p}_{pj}}, \forall j \in IT: \bar{A}^{I\setminus SpI_p}_{pj} \neq 0, p \notin DNA_j;$$

$rsp^{2'}_{pj}$ is the ratio of the assigned quantity of product p for the individuals with no special needs over the mean number of these individuals, for each institution, obtained by the model

$\overline{r'_p}$ mean value of r'_{pj}, where

$$r'_{pj} = \frac{xd^*_{pj}}{\bar{C}_{pj}}, \forall j \in IT: p \notin DNA_j$$

$\overline{r''_p}$ mean value of r''_{pj}, where

$$r''_{pj} = \frac{y_{pj}pk_p}{\bar{C}_{pj}}, \forall j \in IT: p \notin DNA_j;$$

r'_{pj} is the ratio of the assigned quantity of product p over the mean number of individuals that may consume this product, for each institution, obtained by the model; r''_{pj} is the corresponding ratio obtained by the whole approach

4 Criteria 4: for each set FuP_h, $h = 1, \cdots, nfu$,

$\overline{rfu'_h}$ mean value of rfu'_{hj} where

$$rfu'_{hj} = \frac{\sum\limits_{p \in FuP_h \cap DP \setminus DNA_j} xd^*_{pj}}{\bar{B}_{hj}}, \forall j \in IT$$

$\overline{rfu''_h}$ mean value of rfu''_{hj} where

$$rfu''_{hj} = \frac{\sum\limits_{p \in FuP_h \cap DP \setminus DNA_j} y_{pj}pk_p}{\bar{B}_{hj}}, \forall j \in IT;$$

rfu'_{hj} is the ratio of the assigned quantity of all products from $FuP_h \cap DP$ over the mean number of individuals that may consume these products, for each institution, obtained by the model; rfu''_{hj} is the corresponding ratio obtained by the whole approach

5 Criteria 5: we extend the ratios r'_{pj} and r''_{pj} to all products $p \in DP$.
For Criteria 2 and 5, we also show

$\overline{nid^{0'}}$ mean value of $nid^{0'}_p$, where

$nid^{0'}_p = \left|\{j \in IT : p \notin DNA_j, xd^*_{pj} = 0\}\right|, \forall p \in SiP_p \cap DP$

$\overline{nid^{0''}}$ mean value of $nid^{0''}_p$, where

$nid^{0''}_p = |\{j \in IT : p \notin DNA_j, y_{pj} = 0\}|, \forall p \in DP;$

$nid^{0'}$ and $nid^{0''}$ are the mean number of institutions that do not receive anything regarding each product p, both obtained by the model and the whole approach.

We show results for $\Delta^{Si} = 0.1$, $\Delta^{Sp} = 0.1$ and $\Delta^{Fu} = 0.5$. For Δ^{Si}, the value allows rsi'_{pj} proportions slightly different comparing with the proportion of product p in the food bank. For Δ^{Sp}, the value allows reasonable ranges on $rsp^{1'}_{pj}$ ratios from the target level zd^{Sp}_p. For Δ^{Fu}, the value is suited insofar as it merely guarantees that at least one product from each set FuP_h is assigned to each institution.

The LP solver was able to solve the LP model within eight minutes and the heuristic run within one second. It was impossible to find a feasible solution to the MIP formulation within two hours.

Figure 1 and Table 1 display the distribution pattern of the nutrients. For each nutrient, the smallest and largest values non-outliers are very close. Although there are several outliers for the majority of the nutrients, the mean proportions $\overline{wd'_n}$ of the needs that have been met are small and very close to the corresponding minimum values wd'_n. The heuristic increases the number of outliers, but maintains those mean proportions. Figure 2 shows the distribution pattern of the nutrients provided by the food bank. For each nutrient, the mean proportion $\overline{wd'''_n}$ of the needs that have been met through the food bank varies far more compared with that of the LP approach.

Tables 2 to 4 indicate the distribution patterns of specific products, the ones classified as nutritionally similar (Table 2), special (Table 3) and similar in functional terms (Table 4), respectively. The LP model guarantees that each institution receives more than one product from each set of similar products (Table 2), except for P18 and P33. In this case, Constraints (4) do not restrict the corresponding rsi'_{pj} proportions $\left(rsi'_{\text{P18},j} = \frac{xd_{\text{P18},j}}{xd_{\text{P18},j}+xd_{\text{P33},j}} < (1+0.1)DB_{\text{P18}}\right)$. For each special product (Table 3), the mean of the relative quantity for the individuals with special needs is small and very close to the corresponding target value (it could assume a value up to 1.1 times the target value). The heuristic slightly changes the distribution generated by the LP model. For each set FuP_h (Table 4), the mean of the relative quantity is also small and lower than the corresponding target value (it could assume a value up to 1.5 times the target value). For all these products, the heuristic generally decreases the number of institutions that do not receive anything (Tables 2 and 5).

Figure 1 Boxplots of the wd'_{nj} and wd''_{nj} values, along the 313 institutions, for each nutrient, with $\Delta^{Si} = 0.1$, $\Delta^{Sp} = 0.1$ and $\Delta^{Fu} = 0.5$

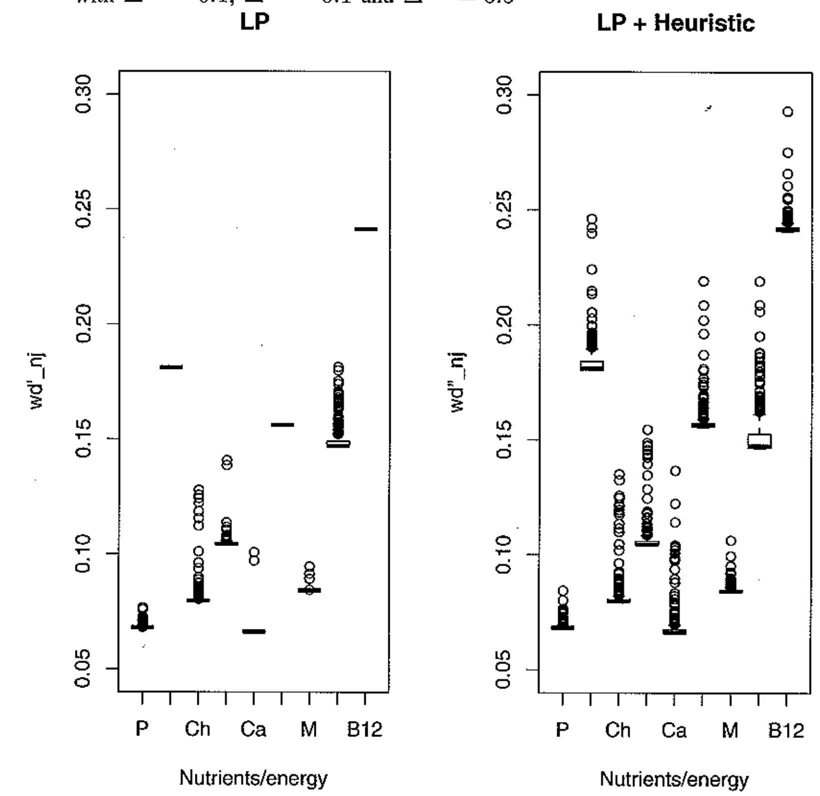


Table 1 Distribution pattern of the nutrients, with $\Delta^{Si} = 0.1$, $\Delta^{Sp} = 0.1$ and $\Delta^{Fu} = 0.5$

Nutrient n	*LP*		*LP + Heuristic*
	wd_n	$\overline{wd'_n}$	$\overline{wd''_n}$
Protein	0.0681846	0.0685	0.0688
Fat	0.1811340	0.1811	0.1840
Carbohydrate	0.0798556	0.0819	0.0824
Energy	0.1042820	0.1059	0.1071
Calcium	0.0663066	0.0675	0.0688
Iron	0.1561830	0.1562	0.1583
Magnesium	0.0842475	0.0843	0.0849
Phosphorus	0.1470000	0.1508	0.1521
Vitamin B12	0.2414930	0.2415	0.2427

Note: wd_n is the minimum proportion.

Figure 2 Boxplots of the r'''_{pj} values, along the 313 institutions, for each product, by the food bank distribution

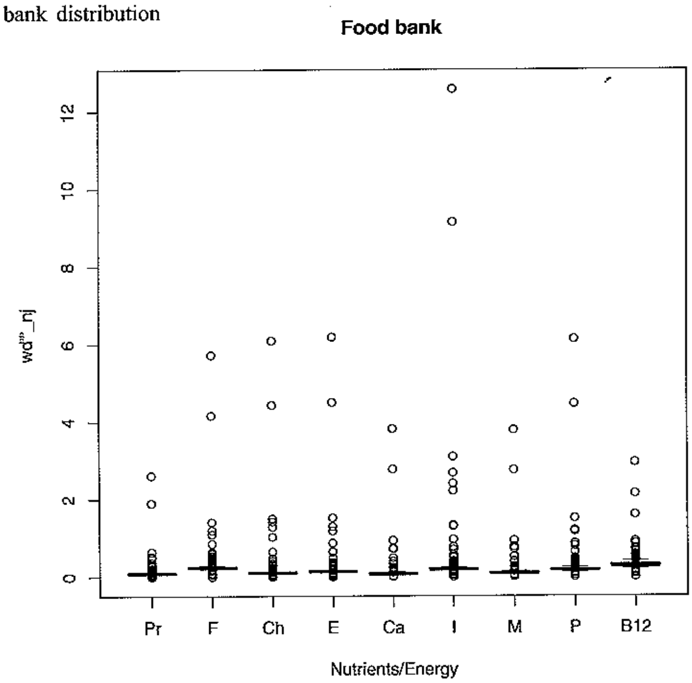


Table 2 Distribution pattern of the products that are similar in nutritive terms ($|SiP_g| > 1$), with $\Delta^{Si} = 0.1$, $\Delta^{Sp} = 0.1$ and $\Delta^{Fu} = 0.5$

Product	*Food bank*	*LP*		*LP + Heuristic*	
p	DB_p	$\overline{rsi'_p}$	$nid_p^{0'}$	$\overline{rsi''_p}$	$nid_p^{0''}$
P1	0.855	0.8591	174	0.8504	152
P24	0.117	0.1136	174	0.1518	152
P25	0.027	0.0273	186	0.0785	160
P3	0.503	0.4909	12	0.5170	8
P5	0.497	0.4725	12	0.4535	18
P4	0.559	0.5371	11	0.5108	13
P9	0.441	0.4265	11	0.4561	11
P8	0.221	0.2109	213	0.4065	185
P13	0.779	0.7891	213	0.8278	187
P12	0.607	0.6043	228	0.5504	207
P31	0.237	0.2379	228	0.2783	210
P32	0.156	0.1578	228	0.2525	202

Table 2 Distribution pattern of the products that are similar in nutritive terms ($|SiP_g| > 1$), with $\Delta^{Si} = 0.1$, $\Delta^{Sp} = 0.1$ and $\Delta^{Fu} = 0.5$ (continued)

Product	*Food bank*	*LP*		*LP + Heuristic*	
p	DB_p	$\overline{rsi'_p}$	$nid_p^{0'}$	$\overline{rsi''_p}$	$nid_p^{0''}$
P27	0.515	0.4678	181	0.4833	152
P28	0.186	0.1727	181	0.2302	155
P29	0.116	0.1086	181	0.1365	155
P30	0.183	0.1675	181	0.1997	158
P18	0.924	0.9259	268	0.9507	252
P33	0.076	0.0741	273	0.2460	261
P22	0.690	0.6962	151	0.6943	115
P23	0.309	0.3038	151	0.3360	120

Table 3 Distribution pattern of the special products, with $\Delta^{Si} = 0.1$, $\Delta^{Sp} = 0.1$ and $\Delta^{Fu} = 0.5$

Product for children	*LP*				*LP + Heuristic*
p	zd_p^{Sp}	$\overline{rsp_p^{1'}}$	$\overline{rsp_p^{2'}}$	$\overline{r'_p}$	$\overline{r''_p}$
P15	0.0252621	0.0250	0.0070	0.0099	0.0097
P19	0.0529894	0.0526	0.0168	0.0259	0.0304

Note: zd_p^{Sp} is the optimal target value.

Table 4 Distribution pattern of the products that are similar in functional terms, with $\Delta^{Si} = 0.1$, $\Delta^{Sp} = 0.1$ and $\Delta^{Fu} = 0.5$

FuP_h	*LP*		*LP + Heuristic*
	zd_h^{Fu}	$\overline{rfu'_h}$	$\overline{rfu''_h}$
P2/P6	0.314991	0.2627	0.2612
P26/P27/P28/P29/P30	0.267336	0.2175	0.2142

Note: zd_p^{Fu} is the optimal target value.

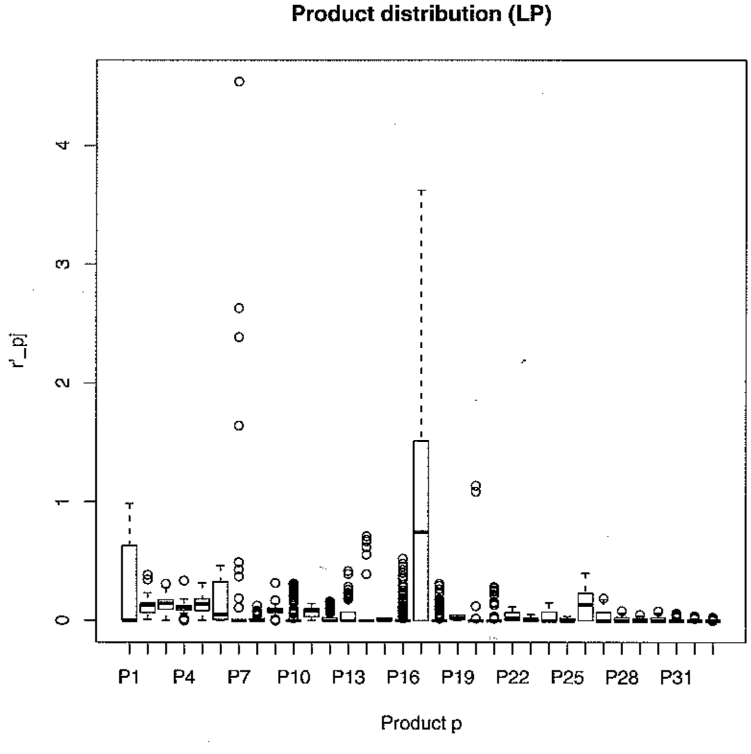
Table 5 Number of institutions that do not receive anything regarding certain products, with $\Delta^{Si} = 0.1$, $\Delta^{Sp} = 0.1$ and $\Delta^{Fu} = 0.5$

Product	*LP*	*LP + Heuristic*
p	$nid_p^{0'}$	$nid_p^{0''}$
P2	0	0
P6	39	39
P7	293	288
P10	243	218
P11	62	48

Table 5 Number of institutions that do not receive anything regarding certain products, with $\Delta^{Si} = 0.1$, $\Delta^{Sp} = 0.1$ and $\Delta^{Fu} = 0.5$ (continued)

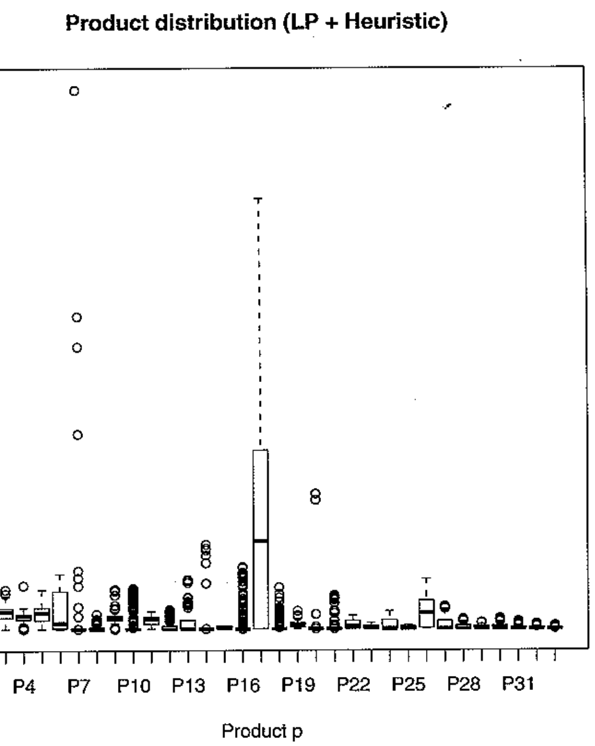
Product	*LP*	*LP + Heuristic*
p	$nid_p^{0'}$	$nid_p^{0''}$
P14	296	292
P15	74	82
P16	260	236
P17	149	99
P19	53	18
P20	300	295
P21	294	293
P26	96	68

Figure 3 Boxplots of the r'_{pj} values, throughout the 313 institutions, for each product, with $\Delta^{Si} = 0.1$, $\Delta^{Sp} = 0.1$ and $\Delta^{Fu} = 0.5$



Figures 3 and 4 provide an overview of the distribution of each product throughout the institutions. The LP model generally produces small ranges in the r'_{pj} ratios of the assigned quantity over the mean number of individuals. For the majority of the products, the smallest and largest values non-outliers are very close and there are some outliers. P1 and P17 are an obvious exception. For each of these products, the box plot is skewed to the top but 50% of the institutions show very small values. The heuristic does not change this tendency.

Figure 4 Boxplots of the r''_{pj} values, throughout the 313 institutions, for each product, with $\Delta^N = 0.15$, $\Delta^{Si} = 0.1$, $\Delta^{Sp} = 0.1$ and $\Delta^{Fu} = 0.5$



One drawback to the approach described is that there are many institutions without several products (Tables 2, 5 and Figure 5). We changed the values of the parameters Δ^{Si}, Δ^{Sp} and Δ^{Fu} so as to compare the results in terms of the number of institutions receiving nothing in terms of each product (Table 6). No significant differences were registered. We also tested the alternative formulation (1), (2), (18), (19), (4) to (17). For Δ^N, it is considered a value that allows reasonable ranges on wd'_{nj} proportions from the target level wd_n. For $\Delta^N = 0.15$, the LP model assigns all quantities in stock. Lower values of Δ^N produce more even nutrient distributions, but some products may not be completely assigned. The results obtained with $\Delta^N = 0.15$, $\Delta^{Si} = 0.1$, $\Delta^{Sp} = 0.1$ and $\Delta^{Fu} = 0.5$ are similar to those described above, apart from the fact that, for many products, the number of institutions that do not receive anything increases somewhat and, in the case of some nutrients, the distribution pattern is slightly less even.

Figure 5 Histogram of the number of non-assigned products, throughout the 313 institutions, with $\Delta^{Si} = 0.1$, $\Delta^{Sp} = 0.1$ and $\Delta^{Fu} = 0.5$

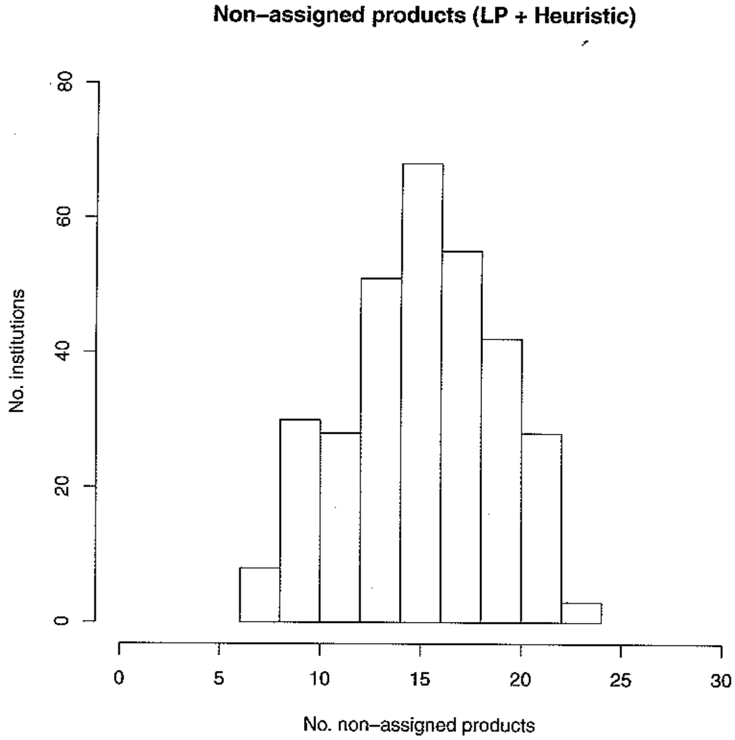


Table 6 Mean number of institutions receiving nothing with regard to each product, according to the values of Δ^{Si}, Δ^{Sp} and Δ^{Fu}

Δ^{Si}	Δ^{Sp}	Δ^{Fu}	*LP* $\overline{nid^{0'}}$	*LP + Heuristic* $\overline{nid^{0''}}$
0.15	0.1	0.5	161.7	146.4
0.20	0.1	0.5	164.1	149.5
0.30	0.1	0.5	167.3	150.4
0.1	*0.15*	0.5	159.8	144.0
0.1	*0.20*	0.5	161.7	146.4
0.1	*0.30*	0.5	161.8	145.2
0.1	0.1	*0.4*	158.7	142.3
0.1	0.1	*0.3*	165.8	148.0
0.1	0.1	*0.2*	154.5	139.0

5 Conclusions

This paper presents a LP approach to the problem of food distribution by the Portuguese food banks among the social solidarity institutions. The paradigm of this approach is that the dietary requirements of the individuals in need are borne in mind. The models proposed for the dry and fresh product distributions are designed to distribute the donated products according to the needs for nutrients without favouring any particular institution. These models are similar, apart from the fact that the fresh product model considers the requirements already met by the previous month's distributions. As, in general, all the products are packaged and since the models described are in linear programming, we propose a heuristic to round the optimal linear solutions. The approach was tested with real data, using 33 dry products and covering 313 institutions.

The results indicate that the approach successfully solves the problem involving the distributions of the nutrients and specific products. Additionally, an optimal solution to the LP model and a solution to the heuristic were found in useful time. However, the main drawback to the approach lies in the fact that there are many institutions with several non-assigned products. Any modification of the parameter values or addition of constraints to the model so as to avoid this situation may change the even distribution of the nutrients and distribution patterns of specific products.

It is our opinion that more detailed data of the products and institutions (e.g., more types of individuals per basket or meal) may enable one to use the information about food composition and dietary needs in a more accurate way and thus, to more successfully address both the distribution patterns of the products and the nutrients.

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Tabela da Composição dos Alimentos (2008) Instituto Nacional de Saúde Dr. Ricardo Jorge, Lisbon.

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Appendix

Table A1 Available quantity of dry products

Product p		
Code	*Designation*	d_p *(kg or l)*
P1	Milk – UHT	18,847
P2	Fish – canned	7,500
P3	Legumes – dry	8,893.5
P4	Oil	7,100
P5	Legumes – canned	8,999.8
P6	Meat – canned	8,000
P7	Tomato – bald	1,750
P8	Flours – others than wheat	764
P9	Olive oil	5,600
P10	Cereals	3,244
P11	Cakes – manufactured	4,500
P12	Biscuits – various	2,050
P13	Flour – wheat	2,698
P14	Sauces – various	269
P15	Desserts – baby	782
P16	Chocolate and confection	1,550
P17	Refreshers and juices	60,000
P18	Sugar	1,000
P19	Milk – powder	1,878.8
P20	Desserts – rice and milk	151
P21	Butter	300
P22	Cheese – slices	1,900
P23	Cheese – triangles	849.9
P24	Milk – vanilla	2,583.5
P25	Milk – chocolate	601.5
P26	Rice	8,554
P27	Pasta – spaghetti	2,365.5
P28	Pasta – elbows	856

Table A1 Available quantity of dry products (continued)

Product p		
Code	*Designation*	d_p *(kg or l)*
P29	Pasta – risone	531
P30	Pasta – macaroni	838
P31	Biscuits – maria	801
P32	Biscuits – water and salt	526
P33	Honey	82

Table A2 Mean dietary needs of an individual for macro nutrients and energy, per type of meal (qm_{nmi})

Nutrient	*Breakfast*	*Lunch*	*Snack*	*Dinner*
Protein (g)	16.4	28.6	16.4	20.4
Fat (g)	11.4	20	11.4	14.3
Carbohydrate (g)	51.4	67.4	63.7	64.2
Energy (Kcal)	373.3	653.3	373.3	466.7

Table A3 Mean dietary needs of an individual for micro nutrients, per day (q'_{ni})

Nutrient	*Daily*
Calcium (mg)	935
Iron (mg)	8.7
Magnesium (mg)	264.3
Phosphorus (mg)	651.3
Vitamin B12 (μg)	1.8

Table A4 Mean dietary needs for nutrients and energy per type of individual, per month (q_{ni})

Nutrient	*Monthly*	
	Child	*Adult*
Protein (g)	2,232	2,560.5
Fat (g)	1,560	1,788
Carbohydrate (g)	7,017	7,593
Energy (Kcal)	51,000	58,500
Calcium (mg)	13,350	35,400
Iron (mg)	213	285
Magnesium (mg)	2,364	10,710
Phosphorus (mg)	10,014	24,300
Vitamin B12 (μg)	24	70.5