Bayesian Analysis of FIAPARCH Model: An Application to São Paulo Stock Market

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Abstract: In this paper, we develop a Bayesian analysis of a FIAPARCH(p,d,q) model for parameter estimation and conditional variance prediction. In order to study the inference problem we use the Metropolis-Hastings algorithm.This methodology is illustrated in a simulation study and it is applied to a set of observations concerning the returns of IBOVESPA values.

Key-words: Asymmetry, long memory, volatility

1 Introduction

It is well known that it is often an unrealistic situation the one which only the mean response could be changing with covariates while the variance remains constant over time. This is particularly obvious in financial time series where clusters of volatility can be detected visually. Considering financial time series it becomes particularly unlikely that positive and negative shocks have the same impact on volatility, leading to the volatility asymmetry concept.

However, there is no particular reason to consider the conditional variance process as a linear function of lagged squared residuals (Bollerslev, 1986) or of lagged absolute residuals (Taylor, 1986). The Asymmetric Power ARCH model, APARCH (Ding et al. (1993)) is obtained if the exponent of the residual power is different from 1 and 2. The long range dependence on conditional mean observed in financial series occurs also in the volatility or conditional variance. The integrated models whose features are similar to unit root processes where proposed to model theses characteristics. In particular, Baillie et al (1996) proposed the Fractionally Integrated GARCH, FIGARCH(p,d,q) model in order to accommodate long memory in volatility, accordingly to the most common definition of a long memory process.

In order to allow long memory and asymmetry in volatility, Tse (1998) proposed the “Fractionally Integrated Asymmetric Power ARCH”models, FI-APARCH.

So, due to the nature of FIAPARCH model, several volatility properties such as long memory, asymmetry, leverage effect and kurtosis are well captured. Despite the extreme importance of this model, it has not received much attention in the literature, specially in Bayesian context.

The proposal of this work is to extend the Bayesian analysis of the APARCH model (Silva, 2006) to a Fractionally Integrated APARCH. This paper is organized as follows. In section 2 we summarize some results presented in literature. Section 3 develops APARCH and FIAPARCH models. Section 4 gives the general Bayes setting for the particular FIAPARCH (1,d,1) model. In section 5 we carry on a simulation study and an application to a real set data. Finally, in section 6 we present some concluding comments.

2 Literature Review

The most common definition of long memory is the one where the autocovariance function decays at the hypergeometric rate k^{2d-1} $(0 < d < 0.5)$. Consequently, the autocovariance function of a long memory process is not absolutely summable. Series with long memory present persistence in sample autocorrelations, which means significative dependence between observations spaced by a long time period. In frequency domain the feature of d is detected by the behavior of the spectral density which tends to infinite as frequency tends to zero.

The analysis of the questions concerning the appropriate modeling of the long time dependency on conditional mean of financial series was extended to the conditional variance. This area of research leads to the formulation of the integrated GARCH process, IGARCH, by Engle e Bollerslev (1986), which has some features of the unit root processes, I(1). Therefore, the effect of the shocks on the optimal prediction of the future conditional variance leads to the convergency to a non null constant of the correspondent weights for the accumulated shocks. This implies the increase of the point previsions in a linear form with the prevision horizon.

Accordingly to Baillie et al. (1996), this would imply that long-term options and future contracts could exhibit an extreme dependency on initial conditions or on actual economy state. However, according to these authors, this extreme degree of dependency seems to contradict the observed behavior. Moreover, several studies (see Crato and Lima (1994) and Ding et al. (1993)) point to strong evidence of long memory on autocorrelations of squared or absolute returns. Motivated by these facts, Baillie et al.(1996) formulated the Fractionally Integrated GARCH process, that is, FIGARCH process. The differencing parameter d introduces a different behaviour: the effect of a shock to the forecast of the future conditional variance is expected to decay at a slow hyperbolic rate. In this sense and considering FIGARCH(1,d,0) Baillie et al (1996) prove the long memory in volatility of FIGARCH models. However the statistical properties, in particular the stationarity, remain unestablished.

One of the limitations of FIGARCH models concerns the symmetry of the shocks impact on volatility. Thus negative and positive shocks produce the same impact on conditional variance (or volatility). Now the majority of the applications of ARCH models are in finance where there is little probability that positive and negative shocks have the same impact. Due to this fact Isler (1999) suggests using other models that could incorporate asymmetry properties and could easily estimate parameters and forecast future observations.

Several time series models, such as Exponential GARCH (EGARCH) of Nelson (1990) and Threshold ARCH (TARCH) of Zakoian (1994) were proposed in literature. One of the most promising models is the Asymmetric Power ARCH model, APARCH, introduced by Ding et al. (1993).

Using a Monte Carlo study, Ding et al.(1993) conclude that there is no obvious reason to assume that, for one hand the conditional variance process has to be a linear function of the squared mudei lagged residuals, as it happens on the GARCH model of Bollerslev; and, for another hand the conditional standard deviation process has to be a linear function of the absolute lagged residuals - see Taylor (1986). This states the importance of considering models with different exponents of the residual powers. Based on these arguments Ding et al. (1993) propose a model which substantially generates ARCH class. Such model is called ARCH model with Asymmetrical Power, APARCH.

Long memory and asymmetry in volatility are exhibited on Fractionally Integrated Asymmetric Power ARCH (FIAPARCH) model, proposed by Tse (1998). An extension proposal of the univariate FIGARCH e FIAPARCH to a bivariate framework is given by Dark (2004).

Long memory in volatility has been documented across a range of equity indices: the SP500 (Ding et al, 1993; Bollerslev and Mikkelsen,1996; Ding et al, 1996; Granger and Ding, 1996; Andersen and Bollerslev, 1997; Lobato and Savin, 1998; Liu, 2000), the NYSE (Ding et al, 1993) the Nikkey (Ding et al, 1996).

3 APARCH and FIAPARCH Models

Ding et al.(1993) concluded there was no reason for the volatility to be a linear function of the squared residuals and introduce the APARCH(p,q) model, which allows the power δ of the heteroscedasticity equation to be estimated from the data;

$$\epsilon_t = z_t \sigma_t, \quad z_t \sim N(0,1) \quad or \quad z_t \sim t - Student \tag{1}$$

$$\sigma_t^\delta = \omega + \sum_{i=1}^{p} \alpha_i (|\epsilon_{t-i}| - \gamma_i \epsilon_{t-i})^\delta + \sum_{j=1}^{q} \beta_j \sigma_{t-j}^\delta, \tag{2}$$

where $\omega > 0$, $\delta \geq 0$, $\alpha_i \geq 0$ and $-1 < \gamma_i < 1$ for $i = 1, ..., p$ and $\beta_j \geq 0$, for $j = 1, ..., q$. The model introduces a Box-Cox power transformation on the conditional standard deviation process and on the asymmetric absolute innovation.

The APARCH model allows the flexible adjustment between exponential variation with the asymmetry coefficient. For this reason this model has the ability to detect asymmetrical shocks on the volatility. Specifically if leverage effect is positive, that is $\gamma > 0$, it is verified the gear shift effect; that is, a positive value of the γ_i means that past negative shocks have a deeper impact on current conditional volatility of the series, than past positive shocks, as would be expected in the analysis of financial time series.

Based on the observation that the volatility tends to change quite slowly over time, Tse (1998) modifies the FIGARCH(p,d,q) model in order to allow for asymmetries by introducing the Frationally Integrated Asymmetric ARCH, or FIAPARCH(p,d,q).

The fractional difference is given by:

$$(1 - B)^d = 1 - dB - \frac{d(1-d)}{2!} B^2 - \frac{d(1-d)(2-d)}{3!} B^3 - ...., \quad 0 < d < 1$$

where d is the fractional differential parameter. Long memory occurs for $0 < d < 0.5$.

Lets consider in (2)

$$g(\epsilon_t) = (|\epsilon_t| - \gamma_i \epsilon_t)^\delta$$

and

$$\xi_t = g(\epsilon_t) - \sigma_t^\delta.$$

Therefore, it can be rewritten as

$$1 - \alpha(B) - \beta(B)g(\epsilon_t) = \omega + (1 - \beta(B))\xi_t,$$

where $\alpha(B) = \sum_{i=1}^{p} \alpha_i B^i$ and $\beta(B) = \sum_{j=1}^{q} \beta_j B^j$ represent lag polynomials of p and q orders, respectively.

In order to allow that volatility shock has long memory we assume that there exists a polynomial $\phi(B)$ such that

$$\phi(B) = (1 - \alpha(B) - \beta(B)g(\epsilon_t));$$

then

$$(1 - B)^d \phi(B) g(\epsilon_t) = \omega + (1 - \beta(B))\xi_t,$$

with $0 < d < 0.5$ and the roots of $\phi(B) = 0$ are outside the unit circle.

The FIAPARCH (p,d,q) model satisfies the equation

$$\sigma_t^\delta = \frac{\omega}{1 - \beta(B)} + [1 - (1 - \beta(B))^{-1}\phi(B)(1 - B)^d](|\epsilon_t| - \gamma\epsilon_t)^\delta; \qquad (3)$$

which is strictly stationary and ergodic for $0 \leq d \leq 1$.

It is very interesting to observe that the FIAPARCH representation nests two major classes of ARCH models: The APARCH and FIGARCH models. It must be stressed that when $d = 0$ the FIAPARCH model reduces to the APARCH(1,1) model and when $\gamma = 0$ and $\delta = 2$, the process is the particular FIGARCH(1,d,1) model. Some advantages of FIAPARCH(p,d,q) models pointed out by Conrad et al (2008) are allowing:
(a) an asymmetric response of volatility to positive and negative shocks (and consequently being able to traduce the leverage effect);
(b) the data to determine the power of returns for which the predictable structure in the volatility pattern is the strongest,
(c) long-range volatility dependence (that is, are able to accommodate long memory in volatility, depending on the differencing parameter d).

Considering $p = 1$ and $q = 1$, $1-\beta(B)$ and $\phi(B)$ are polynomials of degree 1 and we let $\beta(B) = \beta B$ and $\phi(B) = 1-\phi B$, we obtain the FIAPARCH(1,d,1) model with

$$\sigma_t^\delta = \frac{\omega}{1 - \beta} + [1 - (1 - \beta B)^{-1}(1 - \phi B)(1 - B)^d]g(\epsilon_t) \qquad (4)$$

or equivalently,

$$\sigma_t^\delta = \frac{\omega}{1-\beta} + \lambda(B) g(\epsilon_t), \tag{5}$$

where

$$\lambda(B) = [1 - (1-\beta B)^{-1}(1-\phi B)(1-B)^d] = \sum_{i=1}^{\infty} \lambda_i B^i.$$

Bollerslev and Mikkelsen (1996) establish the sufficient conditions in order to endsure the FIGARCH (1,d,1) model to be well-defined and with positive conditional variance. These inequality constrains in the FIAPARCH(1,d,1) model are:

$$\omega > 0, \quad \delta \geq 0, \quad \phi \geq 0, \quad -1 < \gamma < 1, \quad \beta \geq 0 \tag{6}$$
$$\text{and} \quad \lambda_i \geq 0, \quad \text{for} \quad i = 1, \ldots.$$

Note that $\lambda_i \geq 0$ means

$$\beta - d \leq \phi \leq \frac{2-d}{3} \quad \text{and} \quad d(\phi - \frac{(1-d)}{2}) \leq \beta(\phi - \beta + d).$$

The series coefficients $\lambda_i \geq 0$ can be obtained recursively, considering

$$\lambda_i = \beta \lambda_{i-1} + [\frac{(i-1-d)}{i} - \phi] v_{i-1},$$

where

$$v_i = v_{i-1}(i-1-d)/i, i = 2, \ldots \infty$$

with $v_1 = d$ and $\lambda_1 = \phi - \beta + d$.

Once the λ_i coefficients are the same in FIGARCH and FIAPARCH models, Tse(1998) states that the effects of the past residuals on future conditional volatility has the same hiperbolic decay pointed out by Baillie et al (1996)in FIGARCH models. It can be noticed that the FIGARCH model is a case where the lag coefficients decline hyperbolically, rather than geometrically, to 0. According to this, Davidson (2004) emphasized the term "hyperbolic memory"is preferable to distinguish FIGARCH from the geometric memory case such as GARCH.

Some properties, like strictly stationarity and ergodicity, still remain open questions. Considering FIAPARCH model, the parameter estimation and the study of estimators asymptotic properties , have not received much attention in the literature. Accordingly to Bollerslev and Wooldrigde (1992) the Quasi

Maximum Likelihood Estimation (QMLE) is generally consistent, asymptotically normal distributed ansd with standard errors which are valid under non-normality. Baillie et al (1996) state the same properties considering the FIGARCH model. However the impact of violations in conditional normality is unknown for FIGARCH and FIAPARCH models.

4 Bayesian Analysis of FIAPARCH Models

In this section we consider a Bayesian analysis of the parameters of the model (3). Assuming that $z_t \sim N(0,1)$, the likelihood (L) can be expressed as

$$L(\epsilon|\theta) \quad = \quad \prod_{t=1}^{n} \frac{1}{\sqrt{2\pi\sigma_t^2}} exp\{-\frac{\epsilon_t^2}{2\sigma_t^2}\}, \tag{7}$$

where $\theta = (\omega, \alpha, \gamma, \beta, \delta, d)$, n is the number of observations and $\epsilon = \{\epsilon_1, ..., \epsilon_n\}$.

An approximation of the likelihood function conditioned on the p first observations can be written as

$$L(\epsilon|\theta) \quad \alpha \quad \prod_{t=p+1}^{n} \sigma_t^{-1} exp\{-\frac{\epsilon_t^2}{2\sigma_t^2}\}. \tag{8}$$

In order to perform posterior analysis we denote the log-likelihood function by $\pounds$ and we assume the prior distribution $P(\theta)$ satisfying the constrains given in (6).

Hence the posterior distribution is

$$P(\theta|\epsilon) \quad \alpha \quad \pounds(\epsilon|\theta)P(\theta)$$

$$\alpha \quad \prod_{t=p+1}^{n} \sigma_t^{-1} exp\{-\frac{\epsilon_t^2}{2\sigma_t^2}\} I_{(\omega>0)} I_{(\alpha\geq 0)} I_{(\beta\geq 0)} I_{(-1<\gamma<1)} I_{(0<d<0.5)} I_{(\delta\geq 0)}. \tag{9}$$

Besides the inequality constrains in the priors used previously, we also have $\lambda_i \geq 0$.

It should be noted that standard Gibbs sampling methods are not easy to use in FIAPARCH models. Their full conditional densities have nonstandard forms. So, we use the Metropolis-Hastings algorithm with multivariate t-Student as proposal distribution.

Samples are obtained by the algorithm as follow: according to Dellaportas et al. (2000), a sample is attained after running a chain sufficiently long, in which the accumulated quantiles behavior corresponding to each parameter are tracked - this fits to the burn-in period. Afterwards we look at the autocorrelation function in order to choose the appropriate space between the observations to get a sample non correlated approximately. Finally the convergence of chain is analyzed by the R criterium of Gelman e Rubin (1992), Z-score test of Geweke (1992) and graphic methods.

Once the estimates of the posterior parameters are obtained, and noting that σ_t^δ only depends on parameters and observations, the evaluation of posterior conditional variances can be obtained without further difficulties. Considering FIAPARCH(1,d,1) model we obtain:

$$E(\sigma_t^\delta|\epsilon_t) \approx \frac{E(\omega|\epsilon_t)}{1-E(\beta|\epsilon_t)} + \sum_{i=1}^{\infty} E(\lambda_i|\epsilon_t)B^i(|\epsilon_t| - E(\gamma|\epsilon_t)\epsilon_t)^{E(\delta|\epsilon_t)}, \quad (10)$$

where,

$$E(\lambda_i|\epsilon_t) \approx E(\beta|\epsilon_t)E(\lambda_{i-1}|\epsilon_t) + [\frac{i-1-E(d|\epsilon_t)}{i} - E(\phi|\epsilon_t)]v_{i-1},$$

and $v_i = v_{i-1}(i-1-E(d|\epsilon_t))/i$ for $t = 2, ..., n$.

The predictive density of ϵ_{t+1},

$$P(\epsilon_{t+1}|\epsilon) = \int P(\epsilon_{t+1}|\theta, \epsilon)P(\theta|\epsilon)d\theta, \quad (11)$$

can be used to evaluate the predictive accuracy of the model under consideration.

In the context of the FIAPARCH model, the steps required to estimate the k one-step-ahead predictive densities $\hat{P}(\epsilon_{t+1}|\epsilon)$, given by

$$\hat{P}(\epsilon_{t+1}|\epsilon) = \frac{\sum_{s=1}^{S} \hat{P}(\epsilon_{t+1}|\epsilon, \theta^s)}{S}, \quad (12)$$

where $t = n, n+1, \ldots, n+k-1$ are the following:

1. Generate samples θ^s $(s = 1, \ldots, S)$ from the joint posterior distribution $P(\theta|\epsilon)$, using Metropolis-Hastings algorithm.

2. Generate samples of size S from the conditional variances σ_{t+1}^δ, using the samples θ^s $(s = 1, \ldots, S)$.

3. Calculate $\hat{P}(\epsilon_{t+1}|\epsilon, \theta^s)$, for each $\sigma_{t+1}^\delta(s)$.

4. Estimate the predictive density, by

$$\hat{P}(\epsilon_{n+1}|\epsilon) = \frac{1}{S}\sum_{s=1}^{S}\hat{P}(\epsilon_{n+1}|\theta^{s},\epsilon). \tag{13}$$

Since Monte Carlo Markov chain (MCMC) methods produce observations from the joint posterior distribution $P(\theta|\epsilon)$, the simulations results $\theta_s, (s = 1, \ldots, S)$ can be used to compare the models. Therefore, according to Gelfand et al. (1992) proposal we can use,

$$\prod_{t=n}^{n+k-1}\hat{P}(\epsilon_{t+1}|\epsilon). \tag{14}$$

It is chosen the model which makes greater the value of $\prod_{t=n}^{n+k-1}\hat{P}(\epsilon_{t+1}|\epsilon)$. Specificaly, to compare M_1 and M_2 models, Gelfand et al. (1992) propose the following quantity,

$$D = \log\left(\frac{\prod_{t=n}^{n+k-1}\hat{P}(\epsilon_{t+1} = \epsilon_{t+1}|\Psi_t,\theta,M_1)}{\prod_{t=n}^{n+k-1}\hat{P}(\epsilon_{t+1} = \epsilon_{t+1}|\Psi_t,\theta,M_2)}\right), \tag{15}$$

where ϵ_{t+1} is a realization of the stochastic process at time $t+1$, Ψ_t is the available information until time t, and θ is the posterior mean vector of the specified model. It is chosen the model M_1 (M_2) if $D > 0 (D < 0)$. Gelfand et al. (1992) called $\exp(D)$ as Bayes pseudo-Factor.

For the FIAPARCH (p,d,q) model with z_t normal or t-Student, the residuals standardized

$$\tilde{\epsilon_t} = \frac{\epsilon_t}{\sqrt{\sigma_t^2}}$$

are random variables i.i.d. with normal distribution or t-Student. So one way to see if the model is appropriate is to calculate the statistical Q Lung-Box for $\tilde{\epsilon_t}$. Moreover, we can calculate the coefficients of asymmetry and kurtosis and make a qq-plot to verify the assumption of normality.

5 Results

5.1 Simulated data

We simulated 50 samples of size 500 from the FIAPARCH model with different values for the parameters. We calculated Bayesian estimates using the Metropolis-Hastings algorithm with multivariate t-Student as the proponent distribution. Due to convergence requirements we run the sampler algorithm 155100 iterations.

For each iteration we obtained the mean, mode or median of the posterior distribution and considering all the 50 replications we calculated the corresponding sample means and standard deviations. We studied several combinations of parameters. The results of this simulation study, for four particular sets of parameters which illustrate well the overall behavior of the estimates, are displayed in Tables (1), (2), (3). Real values of the parameters of the simulated model are indicated in the first column.

Generally, mean and median estimates are similar and behave better than mode estimate of the parameters. A closer look at the tables reveals that there is some difficulty in estimating the value of β. Depending on the magnitude of the parameters the correspondent real values are underestimated or overestimated.

TABLE 1: Inference simulated results - gaussian model FIAPARCH(1,d,1).

Parameter	mean	mode	$Q_{50\%}$	R
ω=0.8	0.752	0.841	0.730	1.154
sd	0.177	0.524	0.121	0.085
ϕ=0.37	0.315	0.320	0.305	1.049
sd	0.099	0.021	0.129	0.071
γ=0.76	0.722	0.583	0.727	1.146
sd	0.144	0.062	0.160	0.184
β=0.52	0.276	0.295	0.274	1.046
sd	0.013	0.018	0.003	0.09
δ=1.4	1.319	1.387	1.310	1.079
sd	0.209	0.201	0.221	0.056
d=0.20	0.339	0.256	0.357	1.043
sd	0.108	0.008	0.124	0.043

Note: *sd* is the standard deviation, $Q_{50\%}$ is the posterior median and R is the value of Gelman and Rubin´s criterium.

5.2 Returns of IBOVESPA data

The proposed Bayesian approach is illustrated using daily returns from S. Paulo Stock Market rates, IBOVESPA, for the period from 01/03/1994 to 30/12/2007. (www.ibovespa.com.br)

The returns were calculated as usual, this is, $\epsilon_t = ln(I_t/I_{t-1})$, $t = 1, ..., n$.

TABLE 2: Inference simulated results - gaussian model FIAPARCH(1,d,1).

Parameter	mean	mode	$Q_{50\%}$	R
ω=0.1	0.119	0.154	0.113	1.164
sd	0.033	0.036	0.035	0.117
ϕ=0.3	0.267	0.322	0.319	1.176
sd	0.095	0.016	0.135	0.350
γ=0.8	0.751	0.573	0.761	1.127
sd	0.097	0.086	0.116	0.045
β=0.52	0.277	0.301	0.274	1.089
sd	0.035	0.029	0.045	0.028
δ=1.2	1.168	1.355	1.136	1.233
sd	0.251	0.281	0.321	0.185
d=0.3	0.287	0.254	0.284	1.290
sd	0.014	0.014	0.121	0.298

Note: sd is the standard deviation, $Q_{50\%}$ is the posterior median and R is the value of Gelman and Rubin´s criterium.

TABLE 3: Inference simulated results - gaussian model FIAPARCH(1,d,1).

Parameter	mean	mode	$Q_{50\%}$	R
ω=0.14	0.140	0.243	0.125	1.053
sd	0.035	0.041	0.038	0.072
ϕ=0.20	0.321	0.322	0.323	1.031
sd	0.038	0.008	0.053	0.029
γ=0.21	0.233	0.302	0.225	1.034
sd	0.193	0.302	0.183	0.043
β=0.38	0.267	0.366	0.253	1.034
sd	0.047	0.028	0.053	0.041
δ=1.28	1.294	1.253	1.239	1.085
sd	0.312	0.298	0.342	
d=0.27	0.143	0.241	0.123	1.032
sd	0.049	0.009	0.089	0.039

Note: sd is the standard deviation, $Q_{50\%}$ is the posterior median and R is the value of Gelman and Rubin´s criterium.

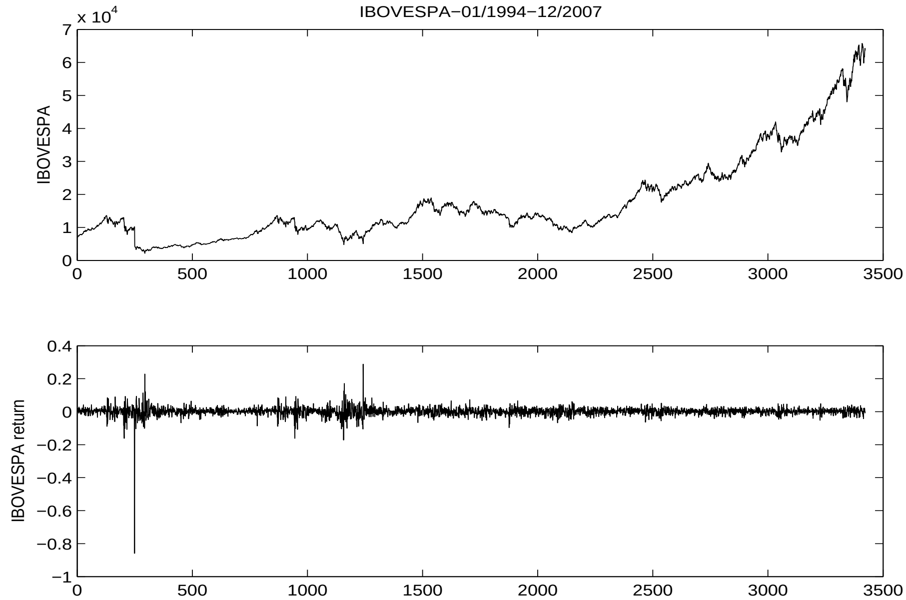

FIGURE 1: IBOVESPA data and IBOVESPA returns.

The Figure (1) shows the IBOVESPA data and the IBOVESPA returns ϵ_t. The plot of autocorrelation function to ϵ_t (not presented here) shows that we need to fit a AR(10) model to the return , so we have ϵ_t given by

$$\epsilon_t = -0.0506\epsilon_{t-4} + 0.082\epsilon_{t-10} + y_t.$$

In the remaining of this work, the residuals y_t will be denoted by returns of IBOVESPA. Figure (2) shows the gaussian qq-plot to y_t and the autocorrelation functions of y_t, y_t^2 and $|y_t|$, respectively . Some of the typical regularities of financial time series are captured in this data set, such as: weak dependence without any evident pattern on the series level, and significative dependence on squared and absolute returns. In particular, the gaussian qq-plot presents the leptocurtosis of returns distribution. Considering the long memory and the heteroscedasticity shown in Figure (2) the FIGARCH model would be indicated for fit the IBOVESPA returns. Looking at the present autocorrelation in the series y_t^2 and $|y_t|$ is prudent considering a model whose power of volatility could be any positive value. So, the FIAPARCH model can be useful to fit the IBOVESPA returns.

Moreover, Figure (2) shows a significative correlation on observations with large delay, so the binomial term $(1-B)^d$ will be truncated in M=100 terms.

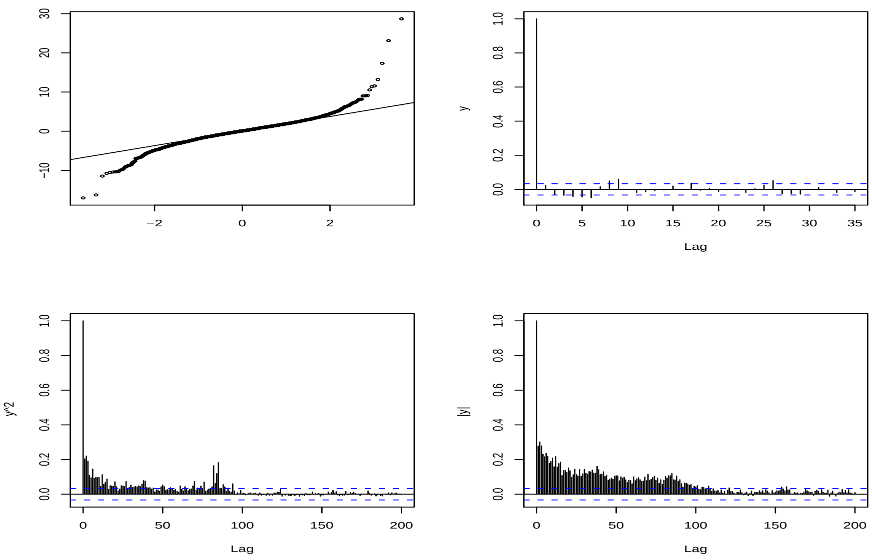

FIGURE 2: Gaussian qq-plot and acf for y_t, y_t^2 and $|y_t|$, respectively.

For the t-Student distribution (ν, μ, Σ) used in Metropolis-Hastings algorithm we consider $\nu = 6$,

$$\mu = (0.1922, 0.0396, 0.7659, 0.8545, 0.3000, 1.6513)'$$

and

$$\Sigma = 0.8diag(0.0259, 0.0229, 0.2289, 0.0188, 0.012, 0.1116),$$

where the 0.8 multiplicative constant was chosen by empirical work in order to achieve a greater acceptance rate of Metropolis-Hastings algorithm.

To calculate the Bayesian estimates we run 400,000 iterations, among these we discarded the first 30% as burn-in time. To reduce autocorrelation between MCMC samples we considered only samples from every 250 iterations. Consequently we use 14,400 samples for posterior inference.

Table (4) provides numerical results for the parameter estimates. Note that the posterior means of γ and d take the values 0.6346 and 0.3331, respectively; these results show that past negative shocks have a deeper impact on current conditional volatility of the series as also the presence of long memory in IBOVESPA series returns. It is worth to mention that the simulation study presented in previous section contains the particular set of parameters, obtained by the means of the posterior distribution, as the true values of model parameters.

TABLE 4: Inference results - gaussian model FIAPARCH(1,d,1) - returns of Ibovespa.

Parameter	mean	mode	$Q_{50\%}$	MC	L.B	U.B	z-score	R
ω	0.4915	0.4800	0.4740	0.011	0.3785	0.7311	0.9584	1.0523
sd	0.0839							
ϕ	0.2091	0.2070	0.2220	0.0047	0.1006	0.2736	0.1191	1.0011
sd	0.0489							
γ	0.6346	0.5543	0.6077	0.001	0.5040	0.7797	0.0966	1.0468
sd	0.0787							
β	0.4139	0.3555	0.4229	0.0072	0.2491	0.5064	0.6553	1.0290
sd	0.0605							
δ	1.2811	1.2235	1.2484	0.012	1.1600	1.4438	0.6868	1.0208
sd	0.0997							
d	0.3331	0.3237	0.3344	0.006	0.2462	0.4579	0.8973	1.0873
sd	0.0518							

Note: sd is the standard deviation, z-score is the p-value of Geweke´s test, $Q_{50\%}$ is the posterior median, MC error is the Monte Carlo error,Highest Probability Density Intervals (HPD)- Lower Bound (LB) , Upper Bound (UB) and R is the value of Gelman and Rubin´s criterium.

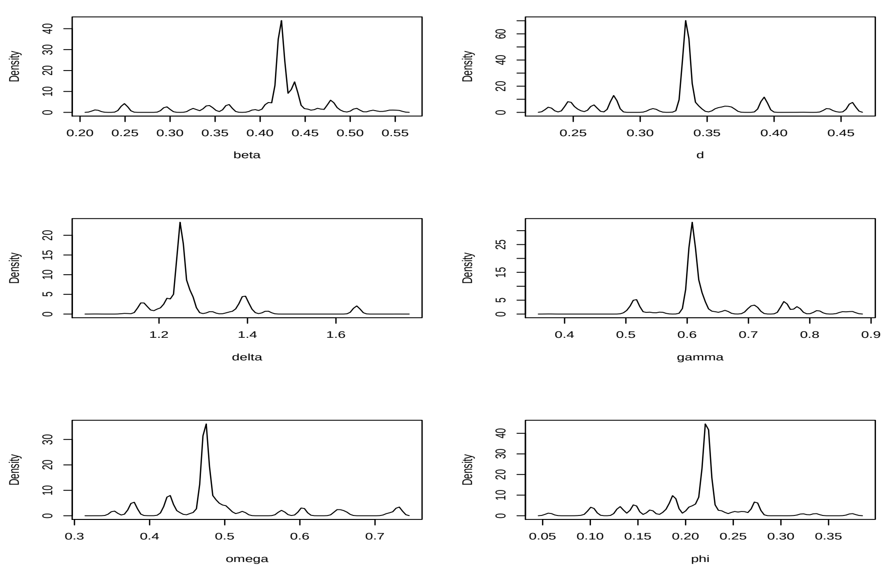

FIGURE 3: Density of the posterior marginal function .

The density of the marginal posterior samples of the parameters of the FIAPARCH(1,d,1) model are illustrated in Figure (3). Since the graphics show that marginal posterior densities are asymmetric maybe we could use asymmetric priors. It is interesting to notice that these facts are compatible with the results of Bawens and Lubrano (1998) and Dellaportas et al. (2000). These authors point out that the use of asymmetric densities such as gamma or log-normal distributions could generate good results.

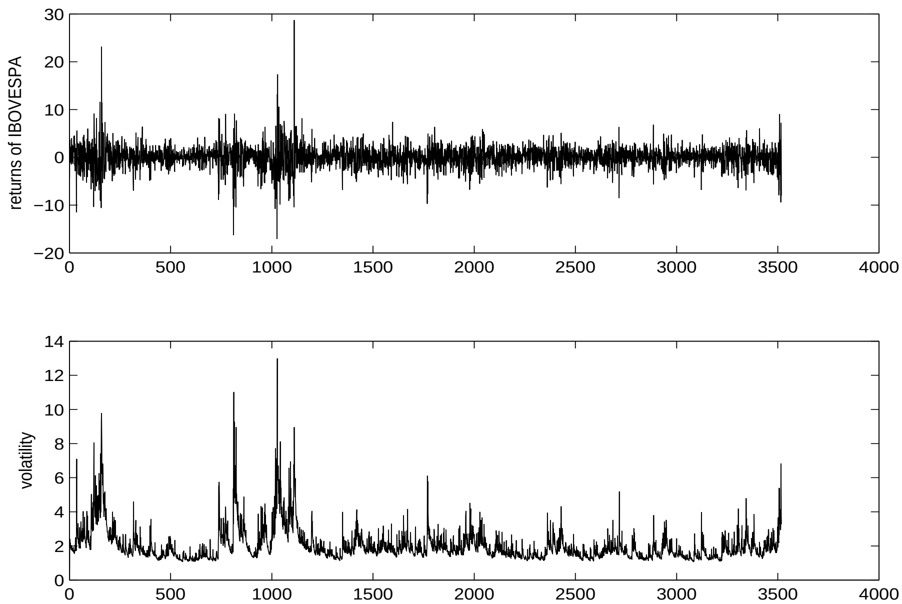

FIGURE 4: (a) Returns and (b) Posterior mean values of σ_t^δ - Ibovespa

In Figure (4) we represent graphically the posterior mean of σ_t^δ, $E(\sigma_t^\delta|\epsilon_t)$. It is possible to see that all periods of great variability of the return of the IBOVESPA were captured by volatility estimated considering $\delta = 1.2811$ and long memory with $d = 0.3331$.

The good fit of the model is shown in Figure (5) where the residuals standardized are white noise. The absence of heteroskedasticity is noted in the acf for the quadratic residuals standardized.

6 Conclusion

FIAPARCH model is extremely important because it includes several models: long memory models and seven special cases of APARCH models. The performance of the proposed Bayesian approach is good. We notice the

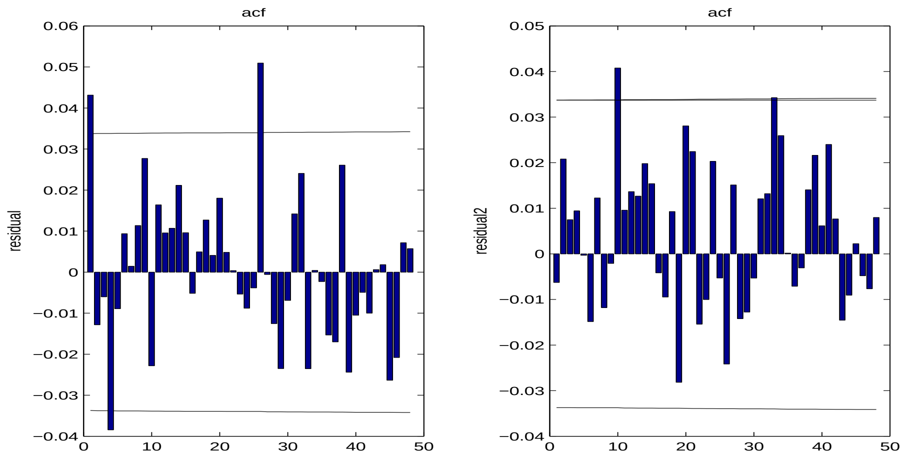

FIGURE 5: ACF of residual, ACF of quadratic residual.

possibility of detecting several particularities such as the long memory and asymmetry.

According to the results obtained by the simulation study and by using IBOVESPA series, the value of the posterior mean estimate of the volatility shows that this analysis has caught extremely well the variability.

In a future research we intend to investigate the effect of using heavy-tailed distributions for the error, for instance, t-Student or asymmetric t-distributions.

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