Sensitivity analysis of the of the square cup stamping process using a polynomial chaos expansion

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Abstract. The stochastic modelling and quantification of the various sources of uncertainty associated with sheet metal forming processes, usually requires a large computational cost to obtain accurate results. In this work, a polynomial chaos expansion metamodel is used in order to reduce the computational cost of the uncertainty quantification (through Sobol's indices). The metamodel allows to establish mathematical relationships between the square cup forming results and the uncertainty sources associated with the material behaviour and process conditions. Then, sensitivity indices are estimated with the trained metamodel, without resorting to additional numerical simulations. The indices obtained with the metamodel were compared to those obtained with the traditional approach based on a quasi-Monte Carlo method. The metamodel allowed to reduce the computational cost in about 90% when compared to the traditional approach, without compromising the accuracy of the results.

Introduction

Sheet metal forming processes are among the most common and important metal working operations associated with the automotive, aeronautics and metalworking industries [1]. Numerical simulation is a well-established tool for the design and optimization of these processes [2]. However, the traditional use of the finite element method (FEM) is based on a deterministic approach [3], which does not take into account the various sources of uncertainty that are inevitable in a real industrial environment. These sources of uncertainty have a significant effect on the quality of the final product [4,5], leading to an inefficient production and, eventually, to the expensive redesign of the forming process. For all these reasons and due to the increasing availability of big data coupled with the growth of computer performance, the uncertainty analysis of these processes is a current scientific and industrial interest [1,6-8].

In recent years, distinct methods, such as, Monte Carlo Simulation [7-9], design of experiments [3,8] and metamodels [1,10] have been used to model the influence of uncertainty. Sensitivity analyses are used to quantify the influence of each uncertainty source in the variability of the forming results [3,11-13]. Variance-based sensitivity analysis (Sobol's indices [14]) is one of the most common methods to quantify this influence [6]. However, this analysis usually requires a large computational cost in order to obtain accurate sensitivity results [6]. This drawback contributes to the computational inefficiency of the uncertainty analysis, delaying its full and suitable employment in industry.

This work presents a numerical study on the influence of the material and process uncertainty in the results of a square cup forming process. The square cup test was chosen for two main reasons: (i) it is a commonly used benchmark test to represent sheet metal forming processes [1,6,15-18]; (ii) it is a relatively fast process to simulate, which is suitable for performing a large number of numerical simulations. A polynomial chaos expansion (PCE) metamodel is used for reducing the computational cost of the Sobol's indices assessment [19]. The PCE metamodel establishes mathematical relationships between the square cup forming results and the uncertainty sources associated with the elastoplastic material properties of the blank (Hooke's law parameters, hardening law parameters, anisotropy coefficients) and process conditions (blank thickness, friction coefficient and the blank holder force). Then, Sobol's indices are estimated with the trained PCE metamodel, without resorting to additional numerical simulations. The indices obtained with the PCE metamodel were compared to those obtained with the traditional approach based on a quasi-Monte Carlo (q-MC) method.

Stochastic Model

Numerical Model. The square cup forming process was modelled with the same numerical model used in a previous work [6], as shown in Fig. 1. The geometry of the tools was adapted from the NUMISHEET' 93 benchmark [18]. The square blank has an initial thickness t_0, and a side length of 75 mm. The numerical simulation of the forming process consists of three phases: (i) First, the blank holder moves downwards, pressing the blank against the die, until a prearranged black holder force (BHF) is reached; (ii) Then, the punch moves 40 mm downwards, drawing the blank into the die, with a constant BHF; (iii) The final step consists in removing the tools (black holder, punch and die), resulting in the springback of the square cup. The numerical simulations were performed with the software DD3IMP (Deep Drawing 3D Implicit Code) [20]. Only a quarter of the model is simulated due to symmetries in the material, geometry and boundary conditions, and to reduce the computational cost. The blank is discretized with 1800 (8-node hexahedral solid) elements, with 2 elements in thickness and 30x30 elements in the sheet plane. The contact between the blank and the tools is described by the Coulomb's law with a constant friction coefficient, μ_0. In average, the duration of each simulation is approximately 4 minutes and 34 seconds in a computer equipped with an Intel® Core™ i7-8700K Hexa-Core processor (4.7 GHz).

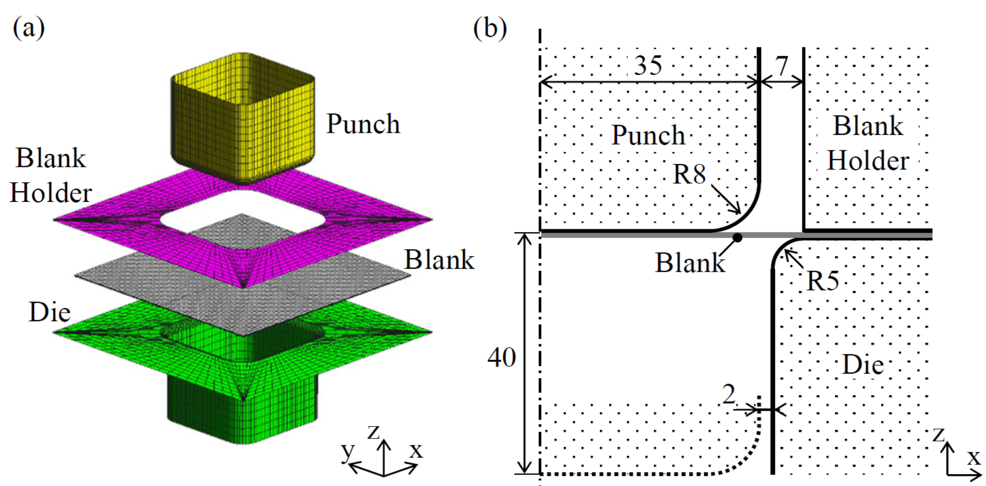

Fig. 1. Square cup forming process: (a) Numerical model [6]; (b) Dimensions of the tools in mm [6].

The mechanical behaviour of the metal sheet is modelled by: (i) the generalized Hooke's law, where E and ν are the Young's modulus and the Poisson's ratio, respectively; (ii) the Swift hardening law; (ii) and the Hill'48 yield criterion. The yield criterion is defined by:

$$F(\sigma_{yy}-\sigma_{zz})^2+G(\sigma_{zz}-\sigma_{xx})^2+H(\sigma_{xx}-\sigma_{yy})^2+2L\tau_{yz}^2+2M\tau_{xz}^2+2N\tau_{xy}^2=Y^2, \quad (1)$$

where σ_{xx}, σ_{yy}, σ_{zz}, τ_{xy}, τ_{yz} and τ_{xz} are the components of the Cauchy stress tensor; Y is the yield stress; F, G, H, L, M and N are anisotropy parameters. The parameters follow the condition $G+H=1$ and $L=M=1.5$ (von Mises). F, G, H and N are obtained from the anisotropy coefficients r_0, r_{45} and r_{90}, by:

$$F={r_0}/{r_{90}(r_0+1)};\ G={1}/{r_0+1};\ H={r_0}/{r_0+1}$$
$$N={(r_0+r_{90})(2r_{45}+1)}/{2r_{90}(r_0+1)} \quad (2)$$

The Swift hardening law is given by:

$$Y=C(\varepsilon_0+\bar{\varepsilon}^p)^n \quad (3)$$

where $\bar{\varepsilon}^p$ is the equivalent plastic strain; n, C and ε_0 are hardening parameters. The initial yield stress is $Y_0=C(\varepsilon_0)^n$.

Input and Output Parameters. The sensitivity analysis is focused on 11 input parameters, 8 associated with the material behaviour (E, ν, n, C, Y_0, r_0, r_{45} and r_{90}) and 3 associated with the blank thickness, t_0, friction coefficient, μ_0, and blank holder force, BHF. The uncertainty of the input parameters is assumed to follow a normal distribution characterized by a mean, μ, and a standard deviation, σ, whose values are given in Table 1 [6].

Table 1. Mean and standard deviation of the normal distribution associated to the uncertainty of each input parameter [6].

	E [GPa]	ν	n	C [MPa]	Y_0 [MPa]	r_0	r_{45}	r_{90}	t_0 [mm]	μ_0	BHF [N]
μ	206.00	0.300	0.259	565.32	157.12	1.790	1.510	2.270	0.780	0.1440	2450.0
σ	3.85	0.015	0.018	26.85	7.16	0.051	0.037	0.121	0.013	0.0288	122.5

The uncertainty influence was analysed for 4 output parameters associated with the forming results, namely, the punch force (PF), the equivalent plastic strain ($\bar{\varepsilon}^p$), the thickness change (TC) and the geometry change (GC). The PF and the $\bar{\varepsilon}^p$ values are directly obtained from the numerical simulation, while the TC and the GC are defined by [6]:

$$TC\ [\%]=100\times(t_0-t_f)/t_0 \quad (4)$$

$$GC\ [mm]=\sqrt{(\bar{x}_f-x_f)^2+(\bar{y}_f-y_f)^2+(\bar{z}_f-z_f)^2} \quad (5)$$

where t_0 and t_f are the initial and final sheet thickness, respectively, in a given region of the square cup; (x_f,y_f,z_f) and $(\bar{x}_f,\bar{y}_f,\bar{z}_f)$ are, respectively, the final spatial position of a given node for the numerical simulation with and without uncertainty (i.e., using the mean values of Table 1). The GC quantifies the positional difference of a given node between the deterministic and the stochastic simulation. In this work, only the maximum values of the four outputs were analysed.

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Sensitivity Analysis

Sobol's Indices. Sobol's Indices are a sensitivity measure of the influence of the input parameters on the output parameters [14]. Two distinct sensitivity indices can be used to quantify this influence, the 1st order indices, S_i, and the total sensitivity indices, S_i^T, which can be defined as follows [14]:

$$S_i = {V[E(\mathrm{y}|x_i)]}/{V(\mathrm{y})} \tag{6}$$

$$S_i^T = 1 - \left({V[E(\mathrm{y}|x_{\sim i})]}/{V(\mathrm{y})}\right) \tag{7}$$

where $V(\mathrm{y})$ is the unconditional variance of the result y; $V[E(\mathrm{y}|x_i)]$ is the conditional variance of the expected value of y when all input parameters, but x_i, are fixed; and $V[(\mathrm{y}|x_{\sim i})]$ is the conditional variance of the expected value of y when only the input parameter x_i is fixed. The 1st order indices, S_i, quantify the individual influence of each input parameter, x_i, on the result y; while the total sensitivity indices, S_i^T, quantify not only the individual influence of each input parameter, x_i, on the result y, but also the influence of the interactions between the input parameter x_i and the remaining, on the result y.

The indices S_i and S_i^T were already computed and published for the above model of the square cup forming process [6]. These indices were computed with the traditional method proposed in [21], and using the estimators proposed by [22], which allow to significantly improve the stabilization of the indices for a lower number of numerical simulations. A base sample of 3000 simulations was generated with a Sobol's sequence [23], in order to also achieve a faster stabilization. For a base sample of 3000 simulations, a total of 39000 simulations were needed to evaluate the sensitivity indices for the 11 input parameters, accordingly to the traditional procedure [21]. The chosen size of the base sample guarantees the stabilization of the sensitivity indices [6].

Polynomial Chaos Expansion. A Polynomial Chaos Expansion (PCE) metamodel is used to reduce the computational cost of the Sobol's indices evaluation [21]. The PCE metamodel allows to estimate the outputs (i.e., forming results), $\mathrm{y}^{PCE}(\mathbf{x})$, as a function of the input parameters (i.e., uncertainty sources), $\mathbf{x}$, by using an orthogonal polynomial basis, $\Psi_{\boldsymbol{\alpha}}$. The output value predicted by the PCE metamodel is given by [19]:

$$\mathrm{y}^{PCE}(\mathbf{x}) = \sum_{\boldsymbol{\alpha} \in \mathbf{A}} \beta_{\boldsymbol{\alpha}} \Psi_{\boldsymbol{\alpha}}(\mathbf{x}) \tag{8}$$

where $\beta_{\boldsymbol{\alpha}}$ are expansion coefficients and $\mathbf{A}$ is a set of pre-selected multi-index $\boldsymbol{\alpha} = [\alpha_1, \alpha_2, \dots, \alpha_k]$ (k is the number of input parameters). The elements α_i indicates the degree of the polynomial associated with the input parameter x_i. Hermite polynomials are used to build the polynomial basis, $\Psi_{\boldsymbol{\alpha}}$, since the input variables follow a gaussian distribution (see Table 1) [19]. The set $\boldsymbol{\beta}$ of expansion coefficients $\beta_{\boldsymbol{\alpha}}$ is determined with the ordinary least squares method [24]:

$$\boldsymbol{\beta} = (\boldsymbol{\Psi}(\mathbf{x})\boldsymbol{\Psi}(\mathbf{x})^{\mathrm{T}})^{-1}\boldsymbol{\Psi}(\mathbf{x})\mathbf{y}^*(\mathbf{x}) \tag{9}$$

where, $\mathbf{y}^*(\mathbf{x})$ is a set of q output results obtained with the q training simulations of the numerical model; and $\boldsymbol{\Psi}(\mathbf{x})$ is a $q \times q$ matrix of Hermitian polynomials of degree m. More details about the construction of $\boldsymbol{\Psi}(\mathbf{x})$ can be found in [19]. To avoid a high computational cost, only polynomials up to degree $m \leq 3$ and low order iterations between input variables are considered, following a hyperbolic truncation scheme [25].

Due to the orthogonality property of the polynomial basis, it is possible to directly evaluated the 1st order Sobol's indices by [26]:

$$S_i^{PCE} = \sum_{\boldsymbol{\alpha} \in \mathbf{A}^*}(\beta_{\boldsymbol{\alpha}}^2) \Big/ \sum_{\boldsymbol{\alpha} \in \mathbf{A}}(\beta_{\boldsymbol{\alpha}}^2) \tag{10}$$

where $\mathbf{A^*}$ is a subset of $\mathbf{A}$ in which the multi-index $\boldsymbol{\alpha}$ is only associated to the input variable x_i (i.e., no other input variable is associated to the multi-index). The total Sobol's indices can be evaluated by [26]:

$$S_i^{T\,PCE} = \sum_{\boldsymbol{\alpha} \in \mathbf{A}^{\mathbf{T}}}(\beta_{\boldsymbol{\alpha}}^2) \Big/ \sum_{\boldsymbol{\alpha} \in \mathbf{A}}(\beta_{\boldsymbol{\alpha}}^2) \tag{11}$$

where $\mathbf{A^T}$ is a subset of $\mathbf{A}$ in which the multi-index $\boldsymbol{\alpha}$ is associated to the input variable x_i, even if $\boldsymbol{\alpha}$ is simultaneously associated with other input variables. Based on the above equations it is evident that the Sobol's indices are instantaneously calculated after the evaluation of the expansion coefficients $\beta_{\boldsymbol{\alpha}}$, i.e., after the metamodel training.

The metamodel was trained with the same 3000 base simulations, previously used to compute the indices S_i and S_i^T with the traditional approach. Four metamodels were trained each one for a given output PF, $\bar{\varepsilon}^p$, TC and GC. The metamodel was tested for other 1000 simulations by comparing the predicted PCE output, $\mathrm{y}^{PCE}(\mathbf{x}^*)$, with the one assessed with the testing simulations $\mathrm{y}(\mathbf{x}^*)$. The performance of each metamodel was evaluated with the root-mean-square error, $\sqrt{\epsilon}$, and the coefficient of determination, R^2, given by [26]:

$$\sqrt{\epsilon} = \sqrt{1/_{q^*} \sum_{i=1}^{q^*} \left(\mathrm{y}(\mathbf{x}_i^*) - y^{PCE}(\mathbf{x}_i^*)\right)^2} \tag{12}$$

$$R^2 = 1 - \epsilon/_{V(\mathbf{y}(\mathbf{x}^*))} \tag{13}$$

where q^* is the number of testing simulations, $\mathrm{y}(\mathbf{x}_i^*)$ and $y^{PCE}(\mathbf{x}_i^*)$ are the simulation and predicted output for the set of input parameters, $\mathbf{x}_i^*$, of the i^{th} testing simulation. $V(\mathbf{y}(\mathbf{x}^*))$ is the variance of the outputs evaluated for the q^* testing simulations. The root-mean-square error, $\sqrt{\epsilon}$, and the coefficient of determination, R^2, of the metamodels trained for each output, are indicated in Table 2.

The PCE metamodels for the outputs PF, $\bar{\varepsilon}^p$ and TC achieved the best performances, with R^2 values close to 1. On the other hand, the PCE metamodel for the output GC had the poorest performance, with a R^2 value of 0.8834. Fig. 2 compares the simulated outputs of the testing dataset, $\mathrm{y}(\mathbf{x}_i^*)$, with those predicted by the PCE, $y^{PCE}(\mathbf{x}_i^*)$. It can be observed that the PCE metamodels were able to accurately predict the simulation outputs, with the exception of GC.

Table 2. Root-mean-square error and coefficient of determination of the metamodels trained for the outputs PF, $\bar{\varepsilon}^p$, TC and GC.

	PF	$\bar{\varepsilon}^p$	TC	GC
$\sqrt{\epsilon}$	0.2207[kN]	0.0036	0.0841%	0.0412[mm]
R^2	0.9956	0.9734	0.9845	0.8834

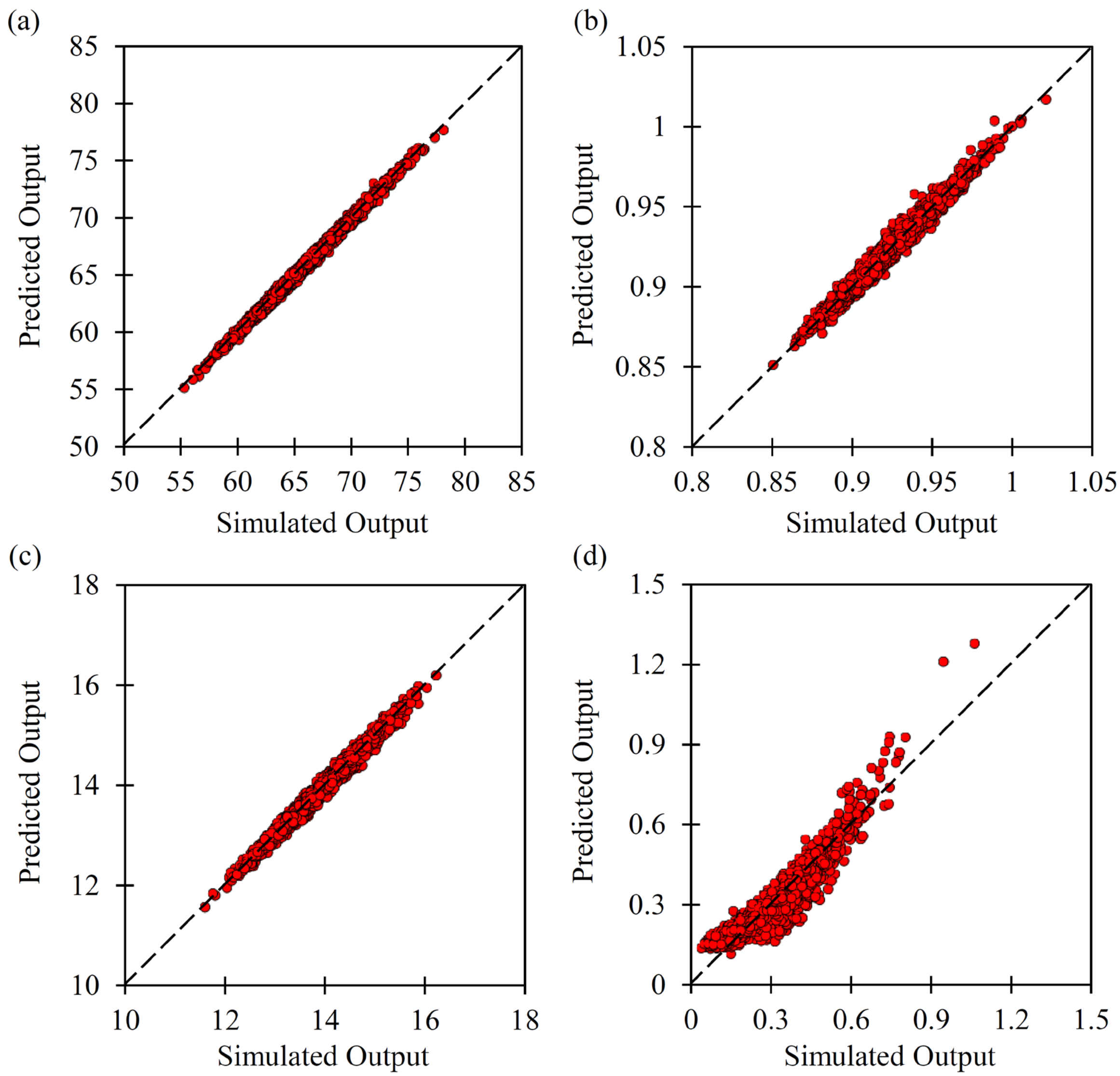

Fig. 2. Predicted (PCE metamodel) and simulated outputs: a) PF [kN]; b) $\bar{\varepsilon}^p$; c) TC [%]; and d) GC [mm]. The dashed line represents the optimal metamodel response, in which predicted outputs are equal to simulated outputs.

Sensitivity Results

In this section, the Sobol's sensitivity indices evaluated with the traditional approach are compared with those evaluated using the PCE metamodel. In this context, Fig. 3 and Fig. 4 show the 1st order and the total Sobol's sensitive indices, respectively, for the outputs PF, $\bar{\varepsilon}^p$, TC and GC.

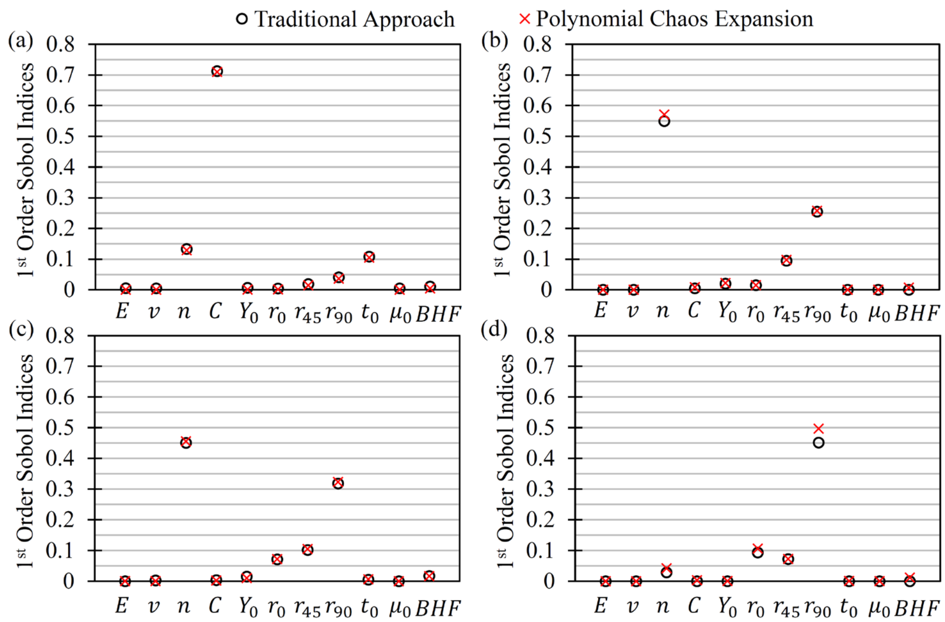

Fig. 3. 1st order Sobol's indices, computed with the traditional approach and with the PCE metamodel, for the outputs: a) PF; b) $\bar{\varepsilon}^{p}$; c) TC; and d) GC.

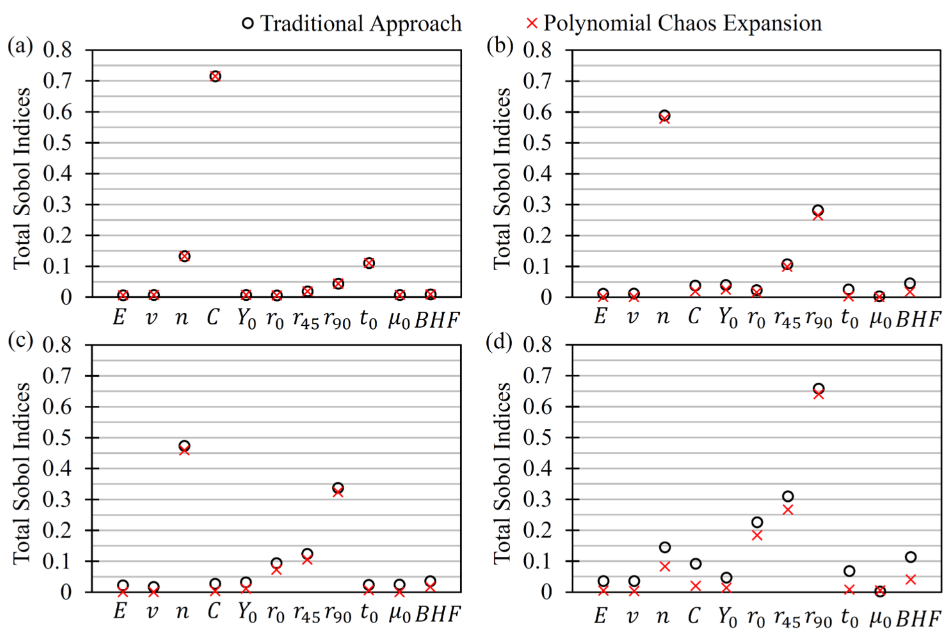

Fig. 4. Total Sobol's indices, computed with the traditional approach and with the PCE metamodel, for the outputs: a) PF; b) $\bar{\varepsilon}^{p}$; c) TC; and d) GC.

Based on Fig. 3 and Fig. 4, it can be observed that the PCE metamodels are able to accurately predict both sensitivity indices. The only significant difference between the indices computed with the traditional and PCE metamodel is observed for the output GC, where the maximum absolute difference between the sensitivity indices of both methodologies is 0.073. This occurs due to the lower accuracy of the metamodel to predict the GC output, as shown in Fig. 3 (d). Nevertheless, even in this case the accuracy of the sensitivity indices computed by the PCE metamodels is suitable to quantify and rank the influence of the input parameters. It is noteworthy that, that the computation of the Sobol indices with the traditional approach required 39000 simulations, while the computation with the PCE metamodel only required 4000 simulations (3000 for training and 1000 for testing the metamodel).

In summary, the PCE metamodel allowed to evaluate the Sobol's indices with accuracy and computational efficiency, requiring about 90% less numerical simulations when compared to the traditional approach.

Summary

In this work, a polynomial chaos expansion (PCE) metamodel is used to compute sensitivity indices, with the goal of reducing the computational cost associated with the traditional approach. The sensitivity indices were assessed with both methodologies for 4 outputs/results: the punch force (PF), the equivalent plastic strain ($\bar{\varepsilon}^p$), the thickness change (TC) and the geometry change (GC) of the square cup forming process. In this study was assumed uncertainty in 11 input parameters, namely, elasticity parameters, anisotropy coefficients, hardening parameters, blank thickness, friction coefficient and blank holder force.

The PCE metamodel allowed to establish mathematical relationships between the square cup forming results and the sources of uncertainty. The predictive performance of the PCE metamodels was tested, and it was concluded that the metamodels were able to accurately predict the simulation outputs. Then, Sobol's indices were estimated with the trained PCE metamodels and the traditional approach based on a quasi-Monte Carlo (q-MC) method. Both methodologies obtained similar 1st order and total Sobol's indices for the PF, the $\bar{\varepsilon}^p$ and the TC. Small differences in the Sobol's indices were observed for the GC output, but without compromising the sensitivity results.

In summary, the PCE metamodel allowed to reduce the computational cost in 90%, when compared to the traditional approach, without compromising the results accurately. In future works, other metamodel techniques will be tested to further improve the prediction accuracy, particularly, in the case of the geometry change. Furthermore, it is also intended to optimise the number of base simulations required to train and test the metamodel to further reduce the computational cost.

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