Accepted Manuscript

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PII: S0020-7403(19)30896-3
DOI: https://doi.org/10.1016/j.ijmecsci.2019.04.042
Reference: MS 4889

To appear in: *International Journal of Mechanical Sciences*

Received date: 12 March 2019
Accepted date: 22 April 2019

Please cite this article as: R.M.F. Paulo , P. Carlone , R.A.F. Valente , F. Teixeira-Dias , F. Rubino , Numerical simulation of the buckling behaviour of stiffened panels: Benchmarks for assessment of distinct modelling strategies, *International Journal of Mechanical Sciences* (2019), doi: https://doi.org/10.1016/j.ijmecsci.2019.04.042

Highlights

- Two benchmarks are presented, for the validation of modelling/simulation aspects of stiffened panels subjected to buckling effects;
- Different numerical strategies are presented to properly model the unstable behavior of the panels, including path-following approaches, geometric and material nonlinear effects;
- The models account for initial geometric imperfections coming from friction stir welding joining operations;
- Modelling and simulation guidelines are presented for subsequent researchers involved in the design process of stiffened panels for aeronautic applications.

Numerical simulation of the buckling behaviour of stiffened panels: Benchmarks for assessment of distinct modelling strategies

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Keywords: *Buckling; Numerical simulation; Riks Method; Stiffened panels; Structural stability; Collapse loads*

Abstract

This work deals with the effective modelling and simulation of the behavior of stiffened panels, when subjected to compressive (buckling) loads. Within the Finite Element Method, two numerical strategies are compared, namely the Riks method and the displacement incremental control method, including damping effects. The capabilities and limitations of both approaches are explored for two distinct benchmarks: a panel with a blade stiffener, and a panel with a T shaped stiffener. In both cases, material (plasticity) and geometrical (large displacements) nonlinearities are considered, together with a modelling strategy based on shell elements. Following previous works of the authors, each panel accounts for initial geometric imperfections coming from friction stir welding joining operations. The paper shows a number of considerations that must be undertaken when choosing between one of the two modelling strategies. Both benchmarks involve a number of challenges from the point of view of modelling unstable structural behaviors, and therefore the proposed benchmarks can represent a valid set of case studies in the understanding of the capabilities of current numerical simulation codes.

1. Introduction

Modelling the behaviour of panels under compression involves dealing with instability effects, namely buckling (local or global) and collapse. Post buckling behaviour is therefore a topic of interest in structural engineering field. It may include sudden changes in the mode-shapes involved, commonly referred as mode-jump, mode-switch or mode-change [1]. From the point of view of modelling and numerical simulation using the Finite Element Method (FEM), such abrupt changes where load / displacement responses can show negative stiffness values (with the structure releasing strain energy to keep equilibrium) typically require specific modelling approaches. To this end, different numerical methodologies can be found in the literature, such as: (i) quasi-static analyses using arc-length control methods; (ii) quasi-static analyses with the aid of artificial damping; (iii) dynamic analyses; and (iv) hybrid methodologies [2].

The typical behaviour of stiffened (reinforced) panels under longitudinal compression, as can be seen from a schematic load vs. displacement curve, can present the effects shown in Figure 1. The segment of the curve on the right (the "snap-back" region) shows a deformation pattern that can appear associated to a mode-change. A quasi-static numerical methodology based on an incremental-iterative strategy will necessarily stop in the neighbourhood of such a turning point, which is associated to a force decrease or a displacement decrease. This holds true either using incremental force control (Figure 2(a)) or incremental displacement control (Figure 2(b)), with a conventional Newton-Raphson approach scheme being employed. Alternatively, these limitations can be solved with an incremental arc-length control methodology, schematically represented in Figure 2(c) together with the incremental values of force and displacement,

In the method shown in Figure 2(c), an additional variable (the arc-length) is introduced into the Newton-Raphson algorithm, making possible to measure the progress of the solution along the static equilibrium path in load/displacement spaces and regardless the response being stable or unstable [2],[3]. Doing so, the search range for an equilibrium equation is highly improved by means of this extra variable Δl (Figure 2(c)), which in turn is responsible for an adaptive evolution of the iterative $\delta u^{(j)}$ and accumulated $\Delta u^{(j)}$ displacements, between increments i and $(i+1)$. The combination of the extra variable with the Newton-Raphson evolution strategy allows the numerical algorithm to correctly follow unstable (snap-back and snap-through) paths.

Within this category, different formulations have been proposed for the selection of the most suitable increment, following the works of Riks' [4] and Crisfield's [5], among others [6],[7]. A so-called "modified" Riks algorithm (as, for instance, implemented in Abaqus commercial Finite Element software [8]) has also been adopted in the literature dealing with the prediction of the behaviour of stiffened panels under compressive loads. Reference works in the field can furthermore include sensitivity analyses [9],[10], as well as optimisation procedures for the cross-section profiles of stiffened panels [11],[12].

Figure 1 here

Fig. 1. Loading path showing an unstable response, both in loading and displacement control (adapted from [13]).

Figure 2 here

Fig. 2. Distinct incremental methodologies - and their limitations - for: (a) load control instability; (b) displacement control instability; and (c) the solution coming from the iterative/incremental character of the arc-length method (adapted from [13]).

Another approach to solve the instability pattern of the numerical solution is to adopt a quasi-static analysis with a simple incremental displacement control, but now accounting for damping effects to stabilize the behaviour of the structure. While the simple use of incremental displacement control would fail in snap-back cases, the use of damping (dissipation of energy) can lead to a numerical solution that follows the vertical dashed line shown on the right side of Figure 1, until the displacement starts to increase again. This methodology has been used, for instance, in numerical analyses of aluminium panels with riveted stiffeners [14], being also used in FEM analyses of the mechanical response of carbon fibre composite stiffened panels [15]. This last work (and Abaqus manual [8]) points out to the critical aspect of choosing a proper adjustment of a damping parameter (ξ), since low values can lead to convergence problems, while high values will lead to inaccurate results.

Therefore, setting the ideal value may require a cumbersome trial and error process before arriving at a converged, to which it is assumed that the dissipated stabilisation energy is small enough to not artificially influence the numerical results.

Regarding the alternative of adopting a methodology based on a dynamic procedure, this can lead to a valid solution in the analysis of instability phenomena. Implicit or explicit methodologies can be used, although implicit dynamic methods can be computationally expensive [3],[8],[16]. Finally, it is worth mentioning the adoption of hybrid procedures, in which two or more of the before mentioned methodologies can be used in sequence [3],[8]. As an example, the Abaqus manual suggests starting the analysis with a static procedure, afterwards switching to an implicit dynamic procedure as soon as the static solution becomes unstable [8].

Following previous publications on the behaviour of reinforced panels when subjected to compressive loads and buckling patterns, in the present paper different strategies for the numerical simulation of those structures are described and compared. Special attention is dedicated to the correct reproduction (and prediction) of unstable paths following from the compressive stress states on such slender structures, as well as the sensitivity of the numerical results regarding different choices on the numerical strategies to be followed. Doing so, a set of benchmark problems are introduced for future assessment of alternative methodologies, with the modelling campaign being carried out for two representative cross sections, displaying different structural behaviours, and using the numerical methodologies available in Abaqus FEM software. Conclusions are then taken about the consequences of different choices for the numerical (input) parameters, together with their influence on the overall quality of the obtained numerical simulation results.

2. Description of the numerical model

The analysis performed and the results presented in this work refer to two panel geometries: (a) a panel with a blade stiffener (named “panel B45”); and (b) a panel with a T shaped stiffener (named “panel T”); both cross-sections being shown in Figure 3. Following the described methodology of using Abaqus FEM code in this research, the panels were discretized using bilinear (four nodes) shell elements with a reduced integration formulation (S4R shell element), accounting for one in-plane integration point together with 5 integration points across the thickness directions [8]. Figure 3 also shows the lines (in red) corresponding to the reference mid-surfaces to be meshed (positioned at half the thickness of the plates), with both panels being modelled with an initial length of 600 mm.

Figure 3 here

Fig. 3. Cross-section of the stiffened panels to be modelled, with the respective dimensions (length of the subsection and thickness values), with the indication of the mid-thickness reference lines in red.

In Figure 4 the boundary conditions are shown for a unit Panel T structure. Following Abaqus nomenclature, in each box, the boundary conditions are expressed in terms of restrictions of displacements along directions (*x, y, z*), and in terms of restrictions of rotations related to each of those directions (*rx, ry, rz,* respectively). The impositions of symmetry boundary conditions along the plane *yz* (left and right of the single panel) are aimed to reproduce the fact that each single reinforced panel (shown in the figure) works in a modular way, together with other panels, and forming a reinforced wall.

Following the work of the authors, geometric distortion values coming from preliminary simulations of friction stir welding joining operations [17] were considered in the present analysis as initial imperfections in the model, triggering the onset of buckling. The idea behind this approach is to provide each individual panel with a realistic geometry (as well as geometric deviations) as coming from preliminary welding processes forming the reinforced walls. The material properties considered in the models correspond to those of an Aluminium Alloy 2024-T3, where an isotropic elasto-plastic model was used in the numerical simulation. For the Panel B45 the same considerations regarding boundary conditions, initial geometric imperfections coming from welding and material properties were followed.

Figure 4 here

Fig. 4. Mechanical boundary conditions for the compressive structural analyses (panel T)

3. Description and comparison of analysis methodologies

In the following, two solving methodologies were tested to predict the behaviour of individual panels under longitudinal compressive loads: (a) an arc-length method (Riks); and (b) a displacement-based incremental control method, including damping effects.

The Riks method (within the categories of arc-length methods) was applied in Abaqus by means of the *RIKS command [8]. Either load-type or displacement-type boundary conditions can be imposed to the moving end (Figure 4), leading to similar numerical results.

In static analyses with a displacement control method, the *STABILIZE command was used to allow for energy dissipation effects and, as a consequence, to help in the stabilisation and convergence of the results. An optimal damping parameter value (ξ) (i.e., the minimum value of the parameter that leads to convergence), was determined by means of a trial and error approach. The collapse loads coming from different damping parameter values are listed in Table 1, with the corresponding load/end shortening curves being shown in Figure 5, for Panel B45 (Figure 5(a)) and for Panel T (Figure 5(b)).

Table 1

Variation of the collapse load magnitude using different damping parameter values.

Table 1 here

Figure 5 here

Fig. 5. Influence of the damping parameter: (a) panel B45 and (b) panel T.

The two panels show distinct buckling and collapse behaviours. While for Panel B45 an abrupt mode-change occurs for a load level lower that the collapse magnitude (Figure 5(a)), in Panel T the mode-change shown is associated with a collapse phenomenon (Figure 5(b)).

Concerning the results for Panel B45, a convergence of results in terms of collapse load and load/end shortening curve was observed using the range [10^-6 $\leq \xi \leq$ 10^-11]. For lower values of the damping parameter the analysis stopped when the mode-change occurred, as represented by the black line in Figure 5(a) (corresponding to a value ξ = 10^-12). For values of the damping parameter ξ > 10^-6 the accuracy of the results is strongly affected, namely the prediction of the mode-change and the collapse load, with the error magnitude increasing with the increase of the damping parameter magnitude (Table 1).

Regarding the analyses for Panel T (Figure 5(b)), it was possible to complete the simulation until reaching the collapse and without any energy dissipation (ξ = 0). However, in this kind of structural analyses it is necessary to verify if a given point really corresponds to a collapse point, rather than a mode-change situation. For that purpose, the analyses results should show an evident continuous decrease in the strength that defines the collapse of the structure. For values of the dissipation parameter $\xi \leq$ 10^-7, the same collapse load magnitude was obtained. Nevertheless, the use of ξ < 10^-7 has led to an early end of the simulation after the collapse, due to convergence problems, being the results not sufficient to clearly show the existence of the collapse. This fact is shown on Figure 5(b) for the curves corresponding to dissipation values of ξ = 10^-8 and ξ = 10^-9. The use of values ξ > 10^-7 has led to errors in the collapse load prediction (Table 1).

After the optimal value of the damping parameter was determined, the two methodologies (arc-length; and displacement control method with damping) can then be compared, leading to similar results in terms of the predictions of mode-changes and collapse load magnitudes. The results obtained for the two benchmarks can be seen in Figure 6 (for Panel B45) and in Figure 7 (for Panel T).

Figure 6 here

Fig. 6. Results for the compression of panel B45, using distinct solving methodologies:

(a) load/end shortening curves; and (b) deformed shapes corresponding to the marks on the curves (displacement values are magnified 15 times along *Ox* and *Oy*).

Figure 7 here

Fig. 7. Results for the compression of panel T, using distinct solving methodologies: (a) load/end shortening curves; and (b) deformed shapes corresponding to the marks on the curves (displacement values are magnified 15 times along *Ox* and *Oy*).

The difference in the collapse load predictions using the two methods is smaller than 0.06%, for both benchmarks. Furthermore, a similarity in the results using these two methods was observed for all the setups that were tested during the preliminary and final analyses of a larger study performed, which included the modelling and simulation of distinct panel geometries and combinations of welding effects.

From the previous pictures it can be seen that, by using the displacement based control with the damping option, the abrupt mode-change phenomenon on Panel B45 was solved in a single increment without a decrease in the end-shortening due to the dissipation of energy. Meanwhile, by using the Riks method the solution required a snap-back to deal with the same effect (zoomed area in Figure 6(a)). In Figure 6(b) it is shown the predicted deformed shapes, before and after the instability, which can be seen to be the same using the two methodologies.

A similar pattern was observed in the behaviour of Panel T, although in this case with a mode-change associated with the collapse, as can be seen in the load/end shortening curve represented in Figure 7(a). As before, the deformed configurations of the reinforced plate are shown in Figure7(b), again with similar results as coming from the two numerical approaches.

It should be noted that the definition of the parameters inherent to these methodologies (namely, the initial increment value, the maximum and minimum increment sizes) can significantly affect convergence and relies mostly on the user experience [8]. The computational costs of the two methodologies depend on the set of parameters used, although in most of the cases similar times can be obtained if the optimal set of increment definitions is chosen.

4. Conclusions

Two numerical methodologies were tested in the reproduction of the buckling behaviour of stiffened panels operations (i.e., the Riks method and the displacement incremental control method with damping). It was shown that both approaches have led to similar results in terms of the predictions of mode-changes shapes and collapse load magnitudes. The optimal set of parameters for the Riks method can be difficult to be determined when aiming to an improved convergence rate until the post-collapse region, strongly relying on user experience. On the other hand, when using the incremental displacement control method with the damping option, the use of a small maximum increment together with the high damping parameter value have shown to led to convergence of the results in a post-buckling region, although the accuracy of the results can be compromised. This last approach requires therefore a study on the damping parameter itself, in order to obtain an appropriate minimum value that would allow to accurately describe the post-buckling region. Being examples that involve a number of challenges from the point of view of the modelling of unstable behaviours using the Finite Element Method, the authors believe that the two proposed benchmarks can represent a valid set of problems of interest in the deep understanding of the capabilities of both user and commercial simulation codes.

Acknowledgments

The authors wish to acknowledge the support given by *Fundação para a Ciência e a Tecnologia, Ministério da Ciência, Tecnologia e Ensino Superior*, Portugal, under grants PTDC/EME-PME/113835/2009 and SFRH/BD/82456/2011. The support from the Center for Mechanical Technology and Automation (TEMA), University of Aveiro, under projects UID/EMS/00481/2013-FCT and CENTRO-01-0145-FEDER-022083, is also acknowledged.

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Tables

Table 1

Variation of the collapse load magnitude using different damping parameter values.

Panel	Damping parameter (ξ)	Collapse load magnitude (MPa)	Collapse load variation (%)
B45	10^{-12}	148.04	-17.87
	10^{-6} to 10^{-11}	180.25	0.00 (ref.)
	10^{-5}	180.31	0.03
	10^{-4}	180.79	0.30
	10^{-3}	185.41	2.86
	10^{-2}	239.65	32.95
T	$\leq 10^{-7}$	256.54	0.00 (ref.)
	10^{-6}	256.61	0.03
	10^{-5}	257.02	0.19
	10^{-4}	259.33	1.10
	10^{-3}	263.58	2.77
	10^{-2}	300.04	17.09

Figures

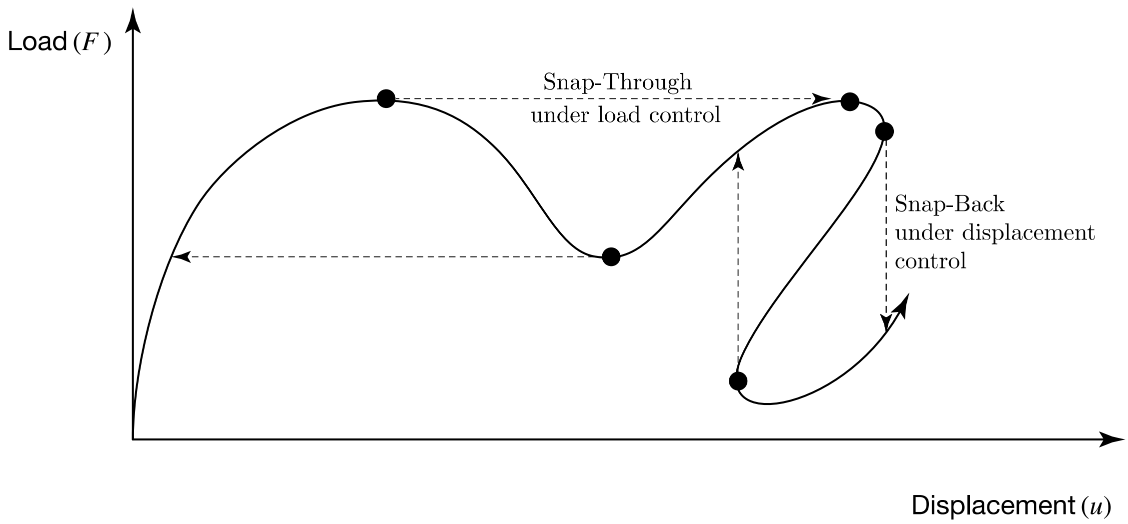


Fig. 1. Loading path showing an unstable response, both in loading and displacement control (adapted from [13]).

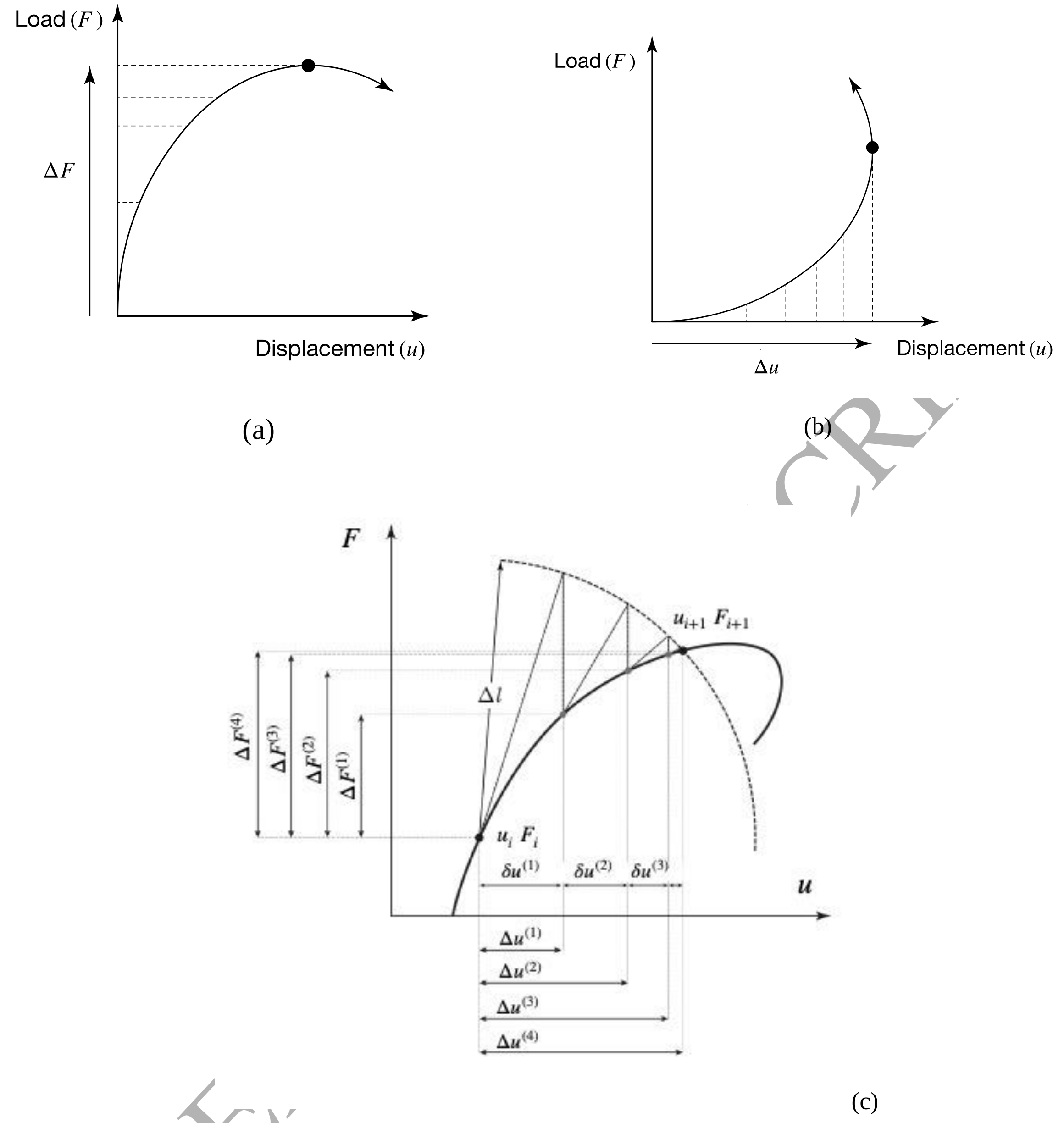


Fig. 2. Distinct incremental methodologies for tracing instability behaviours: (a) a load control method, with its stopping point; (b) displacement control method, with its stopping point; and (c) an arc-length method, showing the capabilities of a variable search length Δl along the iterative process within an increment and avoiding the stopping points (adapted from [13]).

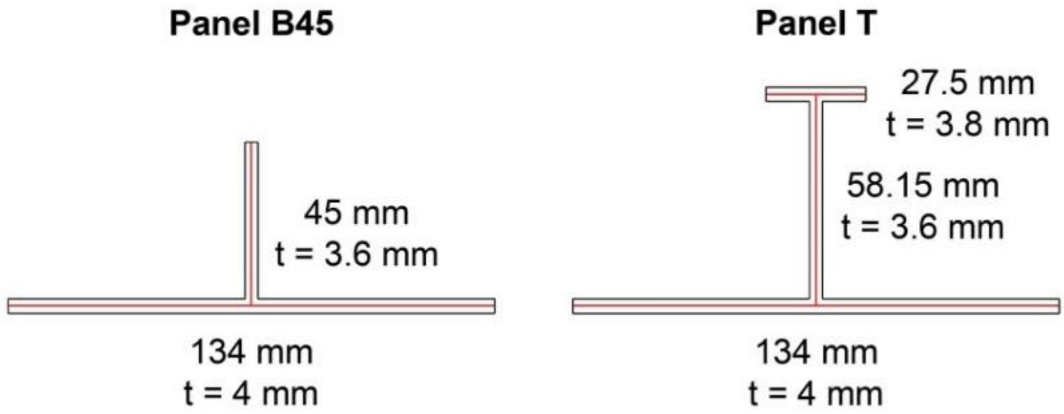


Fig. 3. Cross-section of the stiffened panels to be modelled, with the respective dimensions (length of the subsection, and thickness values), with the indication of the mid-thickness reference lines in red.

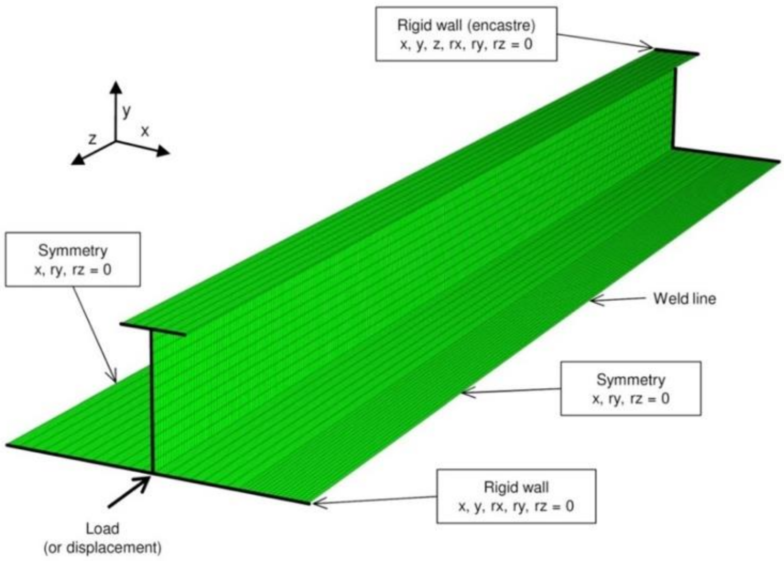
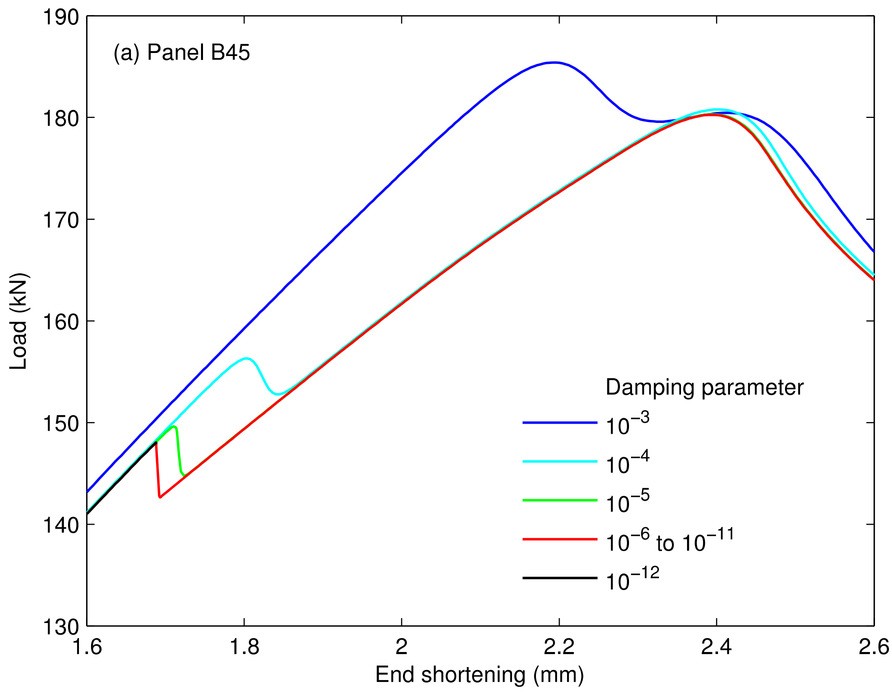
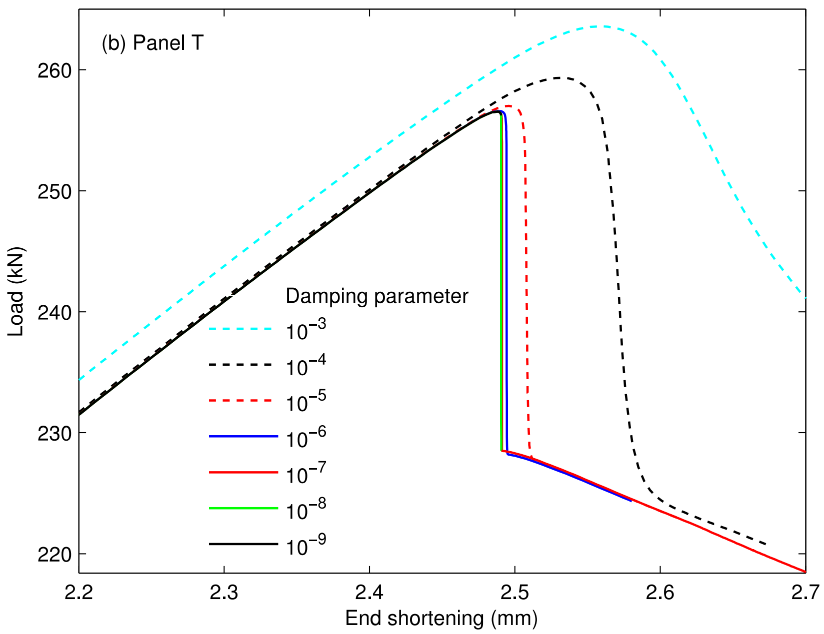


Fig. 4. Mechanical boundary conditions for the compressive structural analyses (panel T).



(a)



(b)

Fig. 5. Influence of the damping parameter for: (a) panel B45; and (b) panel T.

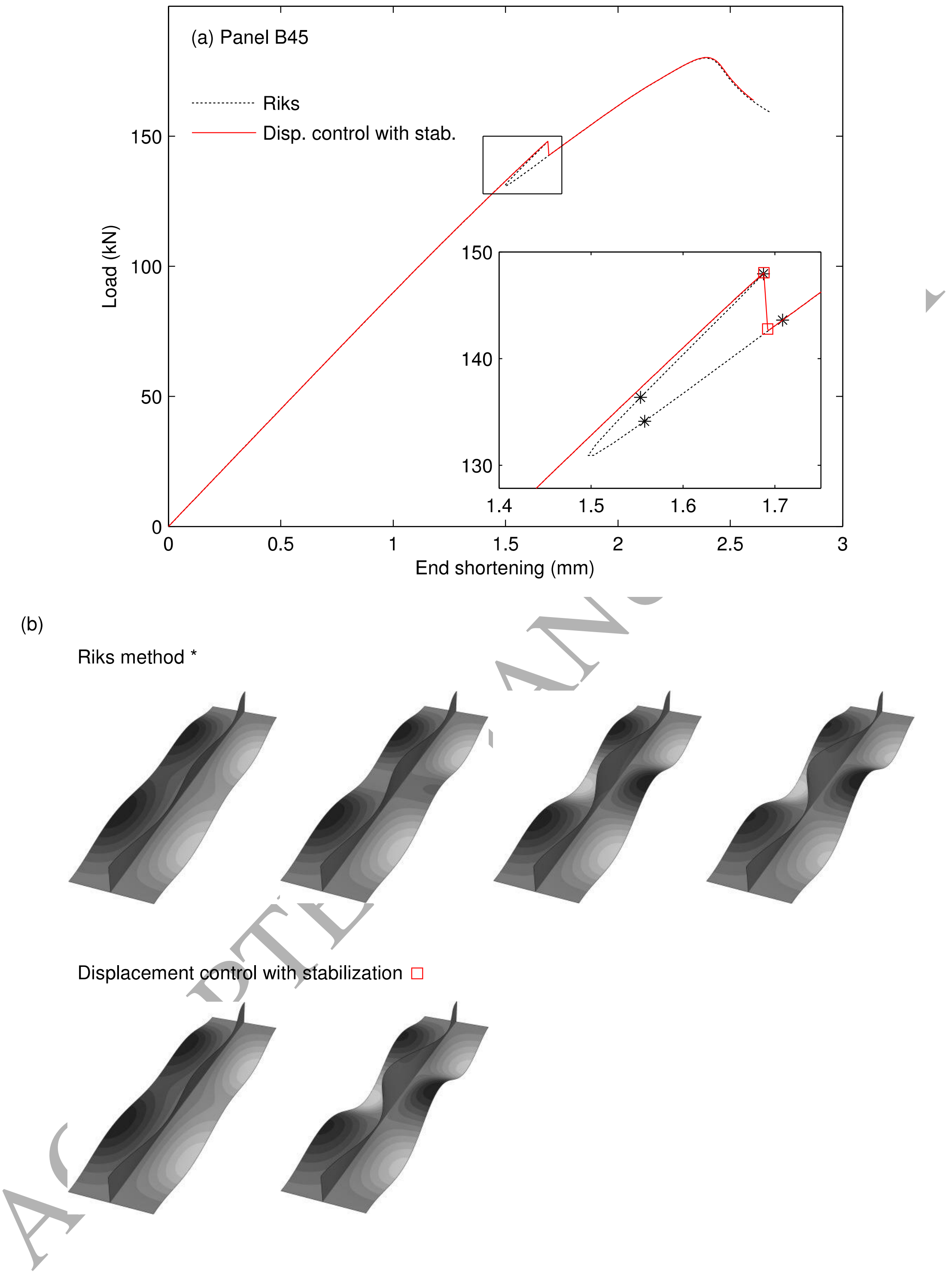
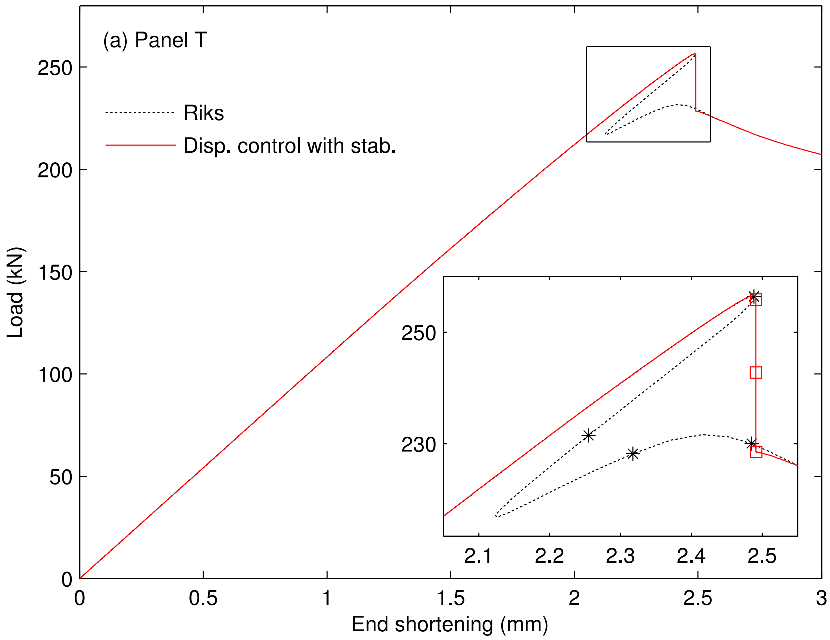


Fig. 6. Results for the compression of panel B45 using distinct solving methodologies: (a) load/end shortening curves; and (b) deformed shapes corresponding to the marks on the curves (displacement values are magnified 15 times along *Ox* and *Oy*).



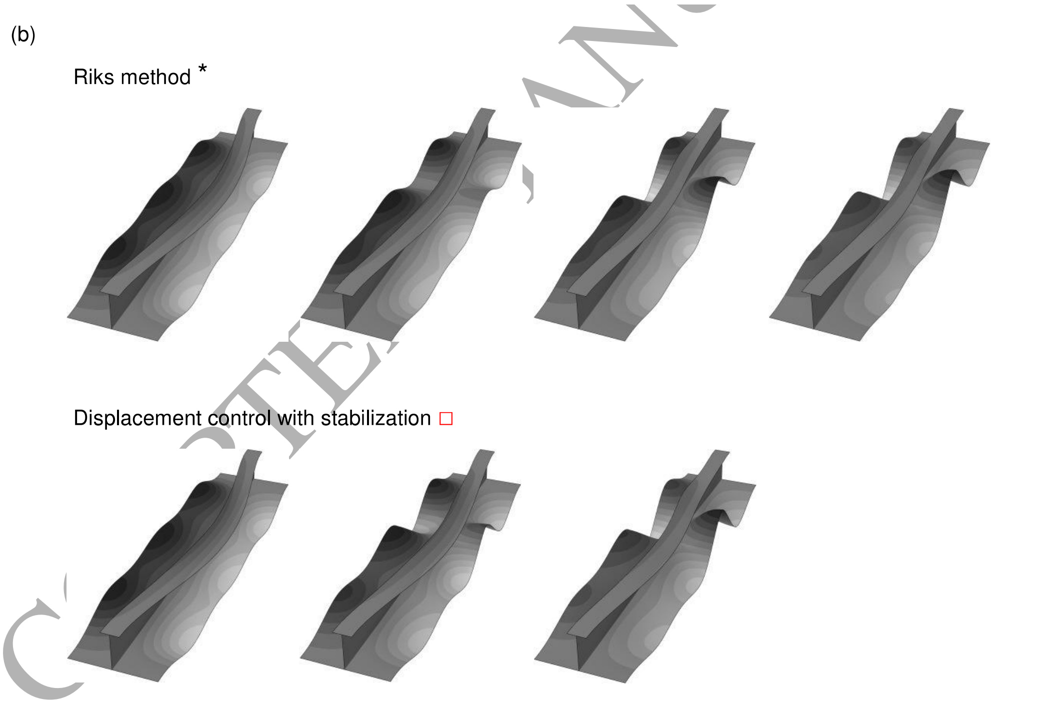


Fig. 7. Results for the compression of panel T, using distinct solving methodologies: (a) load/end shortening curves; and (b) deformed shapes corresponding to the marks on the curves (displacement values are magnified 15 times along *Ox* and *Oy*).