AN EXPERIENCE WITH DESMOS IN THE STUDY OF THE QUADRATIC FUNCTION

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Abstract

In this paper we present a didactic experience in the subject of Mathematics carried out in a distance learning context, on the topic Quadratic Function, using the digital and free platform Desmos. The use of this tool was determinant for the teaching and learning of quadratic function since its teaching took place in distance education, due to the pandemic situation.

In a pandemic context, the use of tools to gauge student learning was a necessity, but practices such as the one described in this paper should be incorporated into a normal classroom environment, promoting discovery through graphical and algebraic manipulation.

Keywords: Mathematics, quadratic function, Desmos, feedback.

1 INTRODUCTION

At a time when students have multiple demands in terms of technology and the resources and tools available are more and more varied, the school must respond with more attractive activities. It is, therefore, essential to provide students with digital tools accessible through the devices they are most familiar with, mobile phones and computers. By using these tools, we are creating a stimulating environment capable of engaging the students, providing them with the means to build their knowledge.

Currently, teachers have at their disposal a wide range of digital tools that promote active learning in the teaching of Mathematics and contribute to pedagogical differentiation, such as Desmos. Desmos is a very appealing free digital platform with an intuitive interface and is a great way to engage students and develop investigation skills [1].

This tool can be accessed through a mobile phone, tablet or computer as long as they have an internet connection, with the exception of applications for mobile devices that do not need this connection. “Desmos is a dynamic mathematics software/application that offers a virtual environment to learn and teach mathematics” [2]. On this platform, sets of interactive activities and tasks can be created that facilitate learning in Mathematics, in order to make them more meaningful and with a high motivational potential.

Ten activities on the quadratic function were constructed, involving various parameters so that the students, observing the changes on the graphs, were able to understand the concepts underlying this subject, namely vertical and horizontal translations of the graph, and the consequent changes on the range, zeros, monotony, extrema, axis of symmetry and coordinates of the vertex of the parabola representing the graph of the quadratic function.

This didactic experience was applied to two classes of 10th grade in Portuguese Educational System, involving forty-seven students, 76.7% of whom highlighted the fact that it facilitated learning in the study of functions, followed by 74.4% who considered the environment dynamic and attractive as well as the feedback given in the answers to the exercises promoted the understanding of the subject.

The use of digital tools should always be complemented with the teacher’s human component. The profile of each student is unique and specific, and the pace of learning differs from student to student. A tool like Desmos, which students are enthusiastic about using, allows monitoring of individual performance and timely and appropriate intervention by the teacher.

ISBN: 978-84-09-42484-9

2 STUDY FRAMEWORK

The 2020/2021 academic year was marked by several changes that affected the daily life of the school community due to the pandemic caused by Covid-19. These changes had direct and indirect effects on the behavior, motivation and performance of students and teachers.

In Portugal Educational system an academic year consists of three distinct periods: the first from the beginning of October to Christmas, the second from January to Easter and the last from April (after Easter holidays) to June.

In the second period, classes began with the implementation of distance learning. Through the Google Meet platform, classes were taught in two modalities: synchronous and asynchronous. In synchronous classes, the contents were exposed by the teacher in a similar way to what was done in the classroom in face-to-face teaching. In asynchronous classes, students had to autonomously develop tasks and activities guided by the teacher. This type of teaching required greater autonomy on the part of the students and an increased effort on the part of the teacher to keep students focused on the class and on the activities developed.

In the third period, only the first two weeks were in the distance learning modality. With the improvement of the indicators for evaluating the state of the pandemic, the Government authorized face-to-face teaching in the third phase of the deflation in April. However, the constraints of the first period in the classroom remained with some rules, such as the mandatory use of masks and social distance. In general, students were satisfied with the return to face-to-face classes. The activity was developed in the first two weeks of the third period in the distance learning format. The evaluation of the activity and the collection of feedback from the students took place in the face-to-face classes of the third period.

For the study of the quadratic function, a sequence of didactic activities was built using the Desmos tool, two questionnaires were prepared, one using the Quizizz tool and the other using Google Forms.

In this way, students participated in the construction and analysis of activities, drawing conclusions about the properties of functions and their characteristics, giving them a more active role.

2.1 Characterization of the population

This experience was carried out at Alcaides de Faria Secondary School, a school attended by 1622 students, which is located in the urban area of the municipality of Barcelos, in the northern region of Portugal.

The participants were two classes of the 10th grade: class D from Scientific-Humanistic course in Sciences and Technologies, involving 25 students, and class F of the Scientific-Humanistic Course of Socioeconomic Sciences, with 22 students. Of the 47 students in the study, 24 were male and 23 were female, thus being a very balanced group in terms of gender.

Regarding the age of the participants, twelve were 14 years old, thirty-four were 15 years old and one was 16 years old, which is in line with the usual age of attendance of the 10th grade.

3 METHODOLOGY

The use of this tool was crucial for the teaching and learning of the quadratic function since its teaching took place in distance learning and the students did not yet have a graphing calculator or any simulator on their computers.

The study of the quadratic function began with a brief framework, where its use in different areas was mentioned with the presentation of some examples. An approach was made to their relationship with geometry, namely in the study of conics, followed by their definition. At the 10th grade level, the study of this function is done considering the families of functions $f(x) = ax^2$, $f(x) = a(x-h)^2$, $f(x) = ax^2 + k$ and $f(x) = a(x-h)^2 + k$, where a, h and k are real parameters and $a \neq 0$.

The aim of the activities implemented was to analyze the influence of the parameters a, h and k on the graph of the function and consequently on its characteristics: range, vertex of the parabola representing the graph of the function, concavity and intervals of monotony (according to the guidelines of the Ministry of Education in Portugal in [3], [4], [5], [6]).

The accomplishment of these online activities was always accompanied with pen and paper registers on a guiding worksheet provided by the teacher, to help consolidate the knowledge on this specific topic. Some examples of the activities developed were translated to English and can be found in https://teacher.desmos.com/activitybuilder/custom/626c165413591d39fac313f1?lang=pt-BR.

The next section is dedicated to the description of some of the activities developed along this study.

3.1 Activities and their implementation

Two classes were created in Desmos platform, 10D and 10F, in order to assign the tasks to the students, and collect data separately.

Although DESMOS allows each student to work at his/her own pace, progressing to the next activity/task after completing the previous one, the option made was only to proceed to the next activity after the completion of the former activity by all students.

While carrying out the activity, the teacher clarified the doubts that arose, explaining them to the all class, so that all the students could benefit from the explanation. Students' resolutions were also monitored by the teacher in order to detect difficulties or misunderstandings, acting promptly and, if necessary, remembering some concepts.

After completing the study, analyzing the influence of the parameters in the graph and characteristics of the quadratic function, thirteen tasks (Fig. 1) were assigned to the students, having different approaches to the topic, to evaluate the learning results of the activities carried out.

Figure 1. Tasks assigned to the students.

One of these tasks was a problem involving the application of the quadratic function to a real situation.

The immediate feedback after answering the exercise (with the exception of those exercises in which it was necessary to write a justification or conclusion), was an asset in enhancing learning.

In the sequence of the exercises, the increase in the degree of difficulty was considered, thus allowing the students to become more motivated as they carried out each of the activities.

Contrary to what was done with the activities, here each student followed his own pace. The increasing degree of difficulty of the proposed tasks allowed the most interested students to be challenged to evolve, avoiding disinterest, and the less interested to gain confidence as they proceeded.

Through the platform's control panel, the teacher monitored the students' performance, allowing her to detect if they needed help with the tool or if they had not yet understood the concepts involved in carrying out the task.

In addition, this panel also made it possible to verify whether or not students fulfilled the proposed tasks.

Some representative examples of the activities and exercises created using Desmos to study the quadratic function are described in the next subsections.

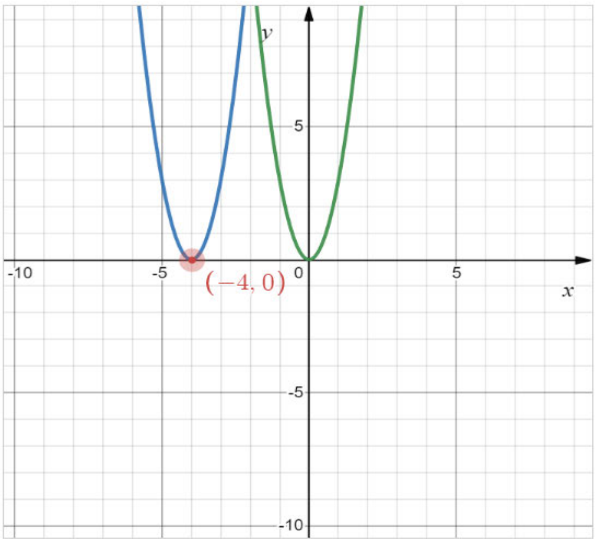
3.1.1 Activity 1

The activity in Fig. 2 allows students to represent the horizontal translation of the parabola with equation $f(x) = ax^2$, moving the point marked in red horizontally and according to the values of h suggested in the left column. They were asked to write the analytic expression of each of the functions and the coordinates of the vector associated with each translation.

The aim of this activity was that students could understand that the graph of functions of the type $f(x) = a(x-h)^2$, with a and h nonzero real parameters, is obtained from the graph of functions of type $f(x) = ax^2$, with $a \neq 0$ through a horizontal translation associated with the vector u of coordinates (h,0).

Task 1

Functions of type $f(x) = a(x-h)^2$, $a \neq 0$ and $h \in IR$



The function represented in green is $y = 3x^2$

Values of h	f(x)=3(x-h)^2	vector associated with the horizontal translation
-4	$3(x+4)^2$	$(-4,0)$
-1		
2		
3		

Conclusion

The graph of the function $f(x) = 3(x-h)^2$ is a horizontal translation of the graph of $y = 3x^2$, associated to the vector $(h,0)$.

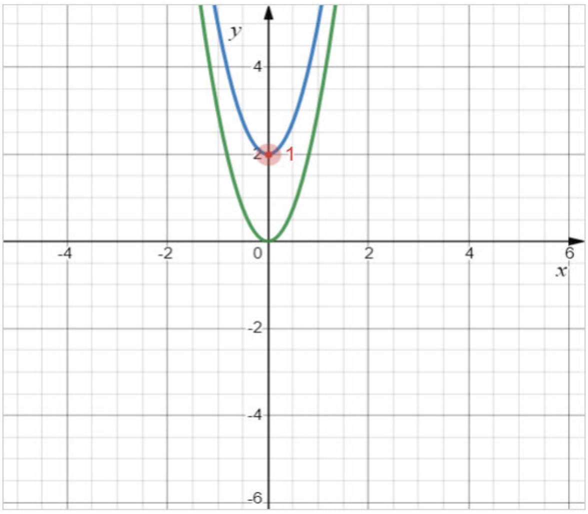
Figure 2. Horizontal translation.

3.1.2 Activity 2

Similarly to activity 1, Fig. 3 shows the vertical translation of the parabola $3x^2$. By moving the point marked in red vertically, and according to the k values suggested in the left column, it is possible to visualize the different parabolas and thus, fill in the two columns relating the graph of these functions with the graph of the functions of the type $f(x) = ax^2$ com $a \neq 0$.

Task 2

Functions of type $f(x) = ax^2 + k$, $a \neq 0$ and $k \in IR$



The function represented in green is $y = 3x^2$

Values of k	f(x)=3x^2+k	vector associated with the vertical translation
-4	$f(x) = 3x^2 - 4$	$(0,-4)$
-1	$f(x) = 3x^2 - 1$	$(0,-1)$
2	$f(x) = 3x^2 + 2$	$(0,2)$
3		

Conclusion

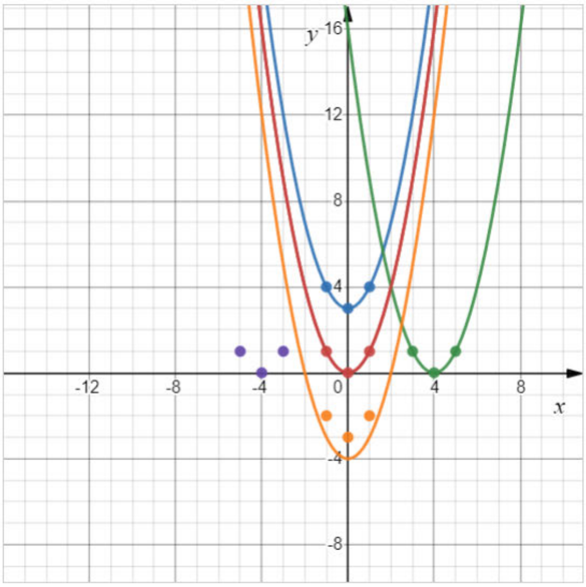
The graph of $f(x) = 3x^2 + k$ is a vertical translation of the graph of $y = 3x^2$ associated to the vector $(0,k)$.

Figure 3. Vertical translation.

3.1.3 *Learning assessment task 1*

After exploring the translations in Activities 1 and 2, the students had to solve the exercise in Fig. 4. The purpose of this exercise was to identify the vertex of each of the parabolas and, from the given graphic representation of the function $f(x) = x^2$ (in red), write the analytical expression for each of the four other functions (blue, green, orange and purple), given the colored points belonging to the graphs.

Task 4



Knowing that the function $f(x) = x^2$ is represented by the red colored parabola complete the table

Parabola	Equation
Red	x^2
Blue	$x^2 + 3$
Green	$(x-4)^2$
Orange	$x^2 - 4$
Purple	

The answer is correct if the graph associated with each color passes through the points of that color.

Reset

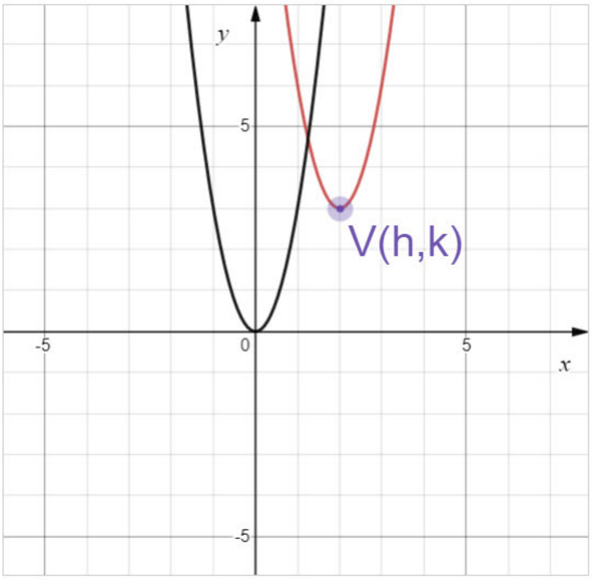
Figure 4. Parabola equation.

3.1.4 *Activity 3*

After understanding the horizontal and vertical translations of the graph of $f(x) = ax^2$, the students were introduced to the more general case $f(x) = a(x-h)^2 + k$, with $a \neq 0$, h and k are real numbers.

Moving the vertex of the parabola $V(h, k)$ (see Fig. 5), the students should infer that the graph is a translation of the parabola $f(x) = ax^2$ associated with the vector of coordinates (h, k).

Study of the functions of type $f(x) = a(x-h)^2 + k$, $a \neq 0$ and $k \in IR$



The graphs of f_1 and f_2 are translations of the parabola $y = 3x^2$ (in black). Knowing this fact, complete the table.

Vertex V(h,k)	Function f(x)=a(x-h)^2 +k
$V(2,3)$	$f_1(x) = 3(x-2)^2 + 3$
$V(-1,-3)$	$f_2(x) =$

Conclusions:

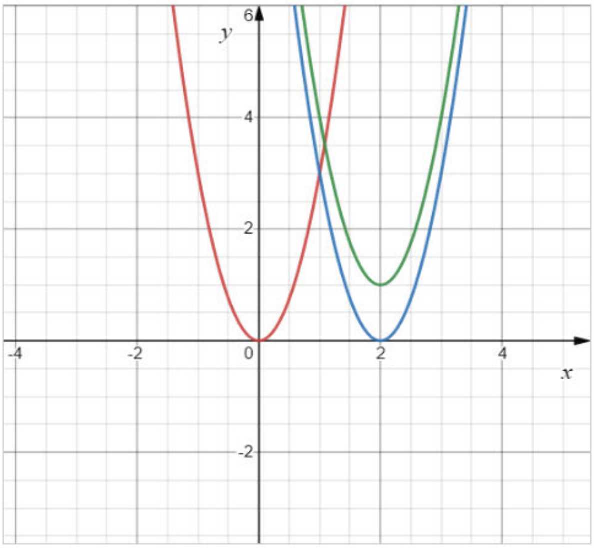
The graph of f_1 is a translation of $y = 3x^2$ associated with the vector $(2,3)$.

Figure 5. Vertex translation.

3.1.5 Activity 4

Besides the graphical approach of the quadratic function, the analytical aspects cannot be neglected. Understanding the influence of the parameters in the range, concavity, intervals of monotony, zeros, etc, was the main purpose of this activity, which is illustrated in Fig. 6.

Study of functions of type $f(x) = a(x-h)^2 + k$, $a \neq 0$ and $k,h \in IR$



	functions
red	$f(x) = 3x^2$
blue	$f_1(x) = 3(x-2)^2$
green	$f_2(x) = 3(x-2)^2 + 1$

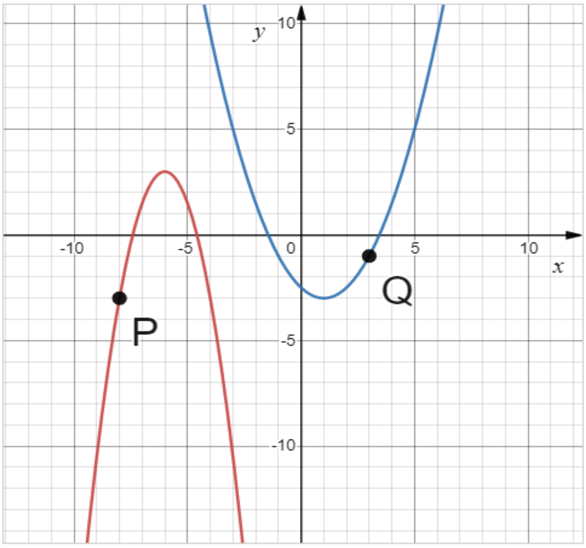
Conclusions

	a>0	a<0
Concavity	*upwards*	*downwards*
Domain		
Range		
Zeros		
Signal		
Monotony		
Vertex		
Parity		
Axis of Symmetry		

Figure 6. Quadratics functions study.

3.1.6 *Learning assessment task 2*

This was an exercise (Fig. 7) which assembled several of the concepts regarding the quadratic function $f(x) = a(x-h)^2 + k$. To answer correctly, the students must understand the relation of the vertex coordinates with the analytical expression of f, but must also use the coordinates of Q (or P) to find a.



The blue parabola represents the graph of f and the red one the graph of g. Knowing that **Q** belongs to the graph of f and **P** to the graph of g, complete the table.

Function	Analytical expression of the function
f	
g	$-\frac{3}{2}(x+6)^2+3$

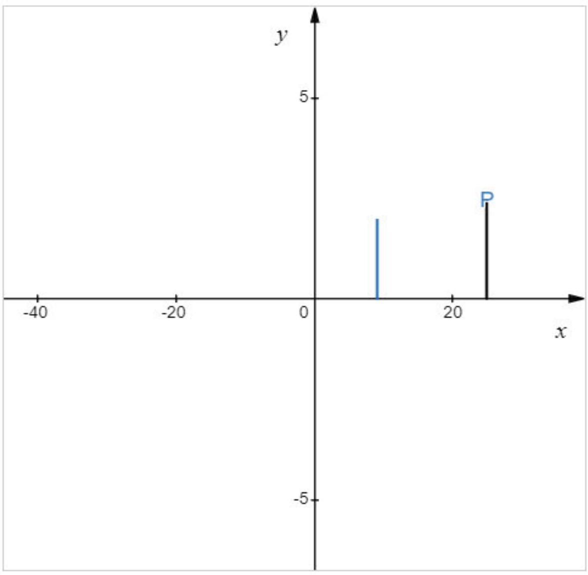
Explain your answer

Enviar

Figure 7. Parabola equation from the graph.

3.1.7 *Learning assessment problem*

Usually students prefer exercises rather than problems, as in the latter they have to understand the formulation in order to apply the mathematical concepts. The example in Fig. 8 is related to football, which is part of our daily life.



In a football game, a free kick is to be taken, 25 meters from the goal line. The barrier is at the mandatory distance of 9.15 meters from the ball. The plane of the ball's trajectory is perpendicular to the goal line. The ball, either hits the barrier or passes over it. If it goes over the barrier, only one of the following three situations can occur:

- the ball goes over the goal bar;
- the ball hits the goal bar;
- the ball enters the goal.

The tallest player in the barrier is 1.95 meters tall.

The goal bar is 2.44 meters off the ground.

Assume that, after being kicked, the ball describes an arc, in such a way that its height, relative to the ground, measured in meters, is given by ,
$0.32x - Bx^2$, where x is the distance, in meters, from the projection of the ball on the ground to the place where it is shot .

What is the value of the parameter B in order to be goal? Justify your answer.

Figure 8. Free kick.

4 RESULTS

In order to understand the advantages of using Desmos from the students' perspective, a survey with 5 questions was carried out on Google forms. Summarizing the 44 responses given to the survey, we have the following conclusions:

- 95,4% of the students found the use of Desmos easy or very easy;
- Regarding the main advantages of using Desmos, 74.4% highlighted the dynamic and attractive environment as well as the feedback given in the tasks, 58.1% the interactivity, 76.7% the fact that it facilitates learning in the study of functions and 44.2% mentioned motivation.
- 90.7% of the respondents agreed with the statement “the Desmos tool facilitated the learning of the quadratic function”.
- 74.4% of the students stated that this tool increased their enthusiasm for the use of technologies, which shows a great aptitude for dealing with new mathematics learning tools. On the other hand, the percentage of 74.4% of answers in "got to know a different/interesting way of learning Mathematics" shows the students' curiosity in learning through diversified tools. However, we should not disregard that the other four opinions (overcoming difficulties, increasing autonomy, increasing interaction with the teacher and giving a more active role to the student) are between 37.2% and 41.9%, revealing that these respondents have the perception that active and student-centered learning is more effective.
- Concerning the effectiveness of Desmos use in the activities, 81.8% of the students considered that it has been excellent or very good. It is noteworthy that no student considered it insufficient.

The mathematics syllabus is quite ambitious and demanding regarding the study of real functions of one real variable, and, as it is a structuring subject in the students' academic training, it is a domain for which a large number of lessons are allocated.

Digital resources like Desmos, allow a different and motivating approach to the study of several topics, namely functions, where students have more learning difficulties and grades are usually lower than in other topics of the curriculum.

This academic year, in spite of all the constraints that arose in the context of the pandemic scenario, the grades were higher than in the past academic years, due, in part, to the fact that there was a greater involvement of students through the use of Desmos.

5 CONCLUSIONS

In the context of the COVID19 pandemic, the alternative used to teach the quadratic function proved to be an asset. In the classes where this curricular topic was taught, the students were very enthusiastic, which was reflected in their learning outcomes.

The activities designed using Desmos, as well as the tasks assigned to consolidate knowledge, improved the students’ grades on this topic when compared to other subjects taught in the same academic year. In spite of being the first contact students had with this tool, they managed to use it and considered its effectiveness very good or excellent. Although certain activities do not format well on mobile screens, choosing the appropriate activities or tasks, the use of cell phones can turn the classroom into a more interactive and fruitful learning space, encouraging student collaboration and discussion.

For Beckmann, [7], “secondary schools are responsible for providing an elementary digital education” and “In order for digital education to become a reality, educational training for teachers must be modernized as well. Digital education requires well trained teachers who are able to use digital media to convey the relevant information to pupils, trainees and college students.” The use of Desmos (or other similar tools) in Mathematics classes should be encouraged.

Given the positive impact that Desmos use had on students in 2020/21, the authors developed some activities to teach other topics, namely on complex numbers for the 12th grade of secondary school which can be found (in Portuguese) in the following links:

https://teacher.desmos.com/activitybuilder/custom/6263cd25a75d8c0d03312e25?lang=pt-BR

https://teacher.desmos.com/activitybuilder/custom/622788ed11f2c30f7fe5c1a7?lang=pt-BR

ACKNOWLEDGEMENTS

The second and the third authors were supported by Portuguese funds through the CIDMA - Center for Research and Development in Mathematics and Applications, and the Portuguese Foundation for Science and Technology (FCT-Fundação para a Ciência e a Tecnologia), within project UIDB/04106/2020.

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