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Generalized maximum entropy in electrical energy price modeling for households and non-households in Portugal

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Abstract

In a high tax burden environment related to electricity prices in Portugal, it is needed to establish fiscal and economic-political measures to lower the electrical price weight imposed on both households and non-households. The most effective way is to first be aware of its determinants. This paper contributes to the existent literature by analyzing how important are fuel costs, emissions, gross domestic product, and renewable energy share in energy consumption in explaining the variations in Portuguese electricity prices for both households and non-households, using generalized maximum entropy estimation. Results suggest that when income increases, electrical energy prices increase 0.0026 Euro/kilowatt-hour for households and 0.0034 Euro/kilowatt-hour for non-households. Fuel costs even lead to higher electricity prices for both, with the highest burden being on the non-household side. Results also point that carbon emissions negatively influence electricity prices and that reductions in electricity bills for non-households may be achieved through the increased share of the industrial sector energy in gross final energy consumption. Thus, increased renewable energy shares do not seem to be the solution to lower electricity prices, despite being necessary to reach sustainability goals.

Keywords: Electrical energy prices; Ill-conditioned problems; Info-metrics; Regression modeling

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1. Introduction

In the Statista Research Department [1] report it is noticed that since the first half of 2010, household electricity prices have assumed an emerging trend. In a plain pandemic period, namely the second half of 2020, where households were obliged to remain at their homes, prices were around 23.66–21.33-euro cents per kWh (decreasing the higher the consumption interval is). Besides paying some of the highest prices in Europe, Portuguese household

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electricity prices are higher than in many countries worldwide [1]. This is mostly associated with the high tax burden, where approximately half of the price of electricity is tax-related [2]. Additionally, higher energy costs place challenges in industry competitiveness once electricity represents a high proportion of firms' total energy costs [3,4]. This article considers annual data for the period 2010–2019 using as determinants of the electrical price for households and the electrical price for non-households, data as the final consumption - industry sector - energy use, the final consumption - energy use, the share of renewable energy in gross final energy consumption, GDP per capita in PPS (purchasing power standards), gas prices for households, gas prices for non-households, carbon dioxide emissions, Brent oil prices and coal prices, following the spirit of the analysis of Moreno et al. [3]. To explore the determinants of electric energy prices for both households and non-households, and since both models are ill-conditioned, and both are based only on ten observations, generalized maximum entropy estimation is computed as a powerful alternative to traditional methodologies that usually provide unstable estimates in these empirical scenarios. Results highlight some important policy directions to be pursued to lower the electricity burden in both groups. The contribution of the paper is twofold. First, it differentiates the modeling of electrical energy prices for households and non-household at once, and not just for the industrial sector as it is typically done [3]. Second, it relies upon data of one of the most electricity expensive countries in Europe, Portugal, and as well in the world [1], and uses the generalized maximum entropy estimation which allows producing accurate estimates in the presence of data scarcity and ill-posed problems, revealing to be a useful and reliable technique. Furthermore, carbon emissions and the share of renewable energy in gross final energy consumption are added to capture the interactions between the sustainability goals and the price burden associated with the electrical energy price in Portugal. Moreno et al. [3] analyze the impact of the price of gas, price of coal, price of petroleum, and gross domestic product in industrial electricity prices in Spain, using generalized maximum entropy estimation, although with slightly different technical premises than the ones adopted in this work. The remaining paper is organized as follows. Section 2 presents a brief literature review, while Section 3 presents the data and methods followed. Section 4 exposes the empirical results, while some conclusions are provided in Section 5.

2. Framework and literature review

High electrical energy prices place challenges to both households and non-households, especially in economies highly dependent on fossil fuels for electricity generation [4,5] as in Portugal. This paper focuses on how important energy fuel cost variables are, and other variables, to explain changes in Portuguese electricity prices for both household and non-household consumers by estimating price elasticities. For that, annual prices had to be collected from different sources and the scarcity of available data lead us to a small data spanning of 10 years (2010–2019). These sample limitations prevent us from using traditional estimation techniques like least squares. However, when dealing with small samples, the generalized maximum entropy method is useful and has gained popularity in the empirical literature [3,6]. Thus, the present article explores its possibilities in the estimation of household and non-household electric energy price models, usually characterized by small datasets.

As highlighted by the Statista Research Department [1], across European Union countries and World countries as well, retail prices for electricity paid by household and non-household consumers comprise both fixed and variable cost components and differ widely. These disparities between household and non-household electricity price formation are also visible in the differences regarding different charged fossil fuel costs (such as natural gas prices) [7]. Furthermore, Haar [8] highlights that within individual countries, electricity prices change under the level of annual consumption or customer categories. Usually, electricity prices are highest for residential and commercial consumers [7]. This is justified by the fact that industrial consumers use more electricity and can receive it at higher voltages, turning electricity supply to these customers more efficient and less expensive [7]. The use of alternative (household versus non-household) electricity prices helps to increase the robustness of the analysis in comparison to previous empirical studies (as that of [3]).

The empirical literature points to a direct and strong effect of fossil fuel prices over electricity prices [3,6,9]. Bernal et al. [9] find that crude oil, natural gas, and coal prices have a significant positive impact on domestic electricity rates for Mexico in the short run. Moreno et al. [3] conclude that imported fuel prices affect Spanish manufacturing, and other industries, competitiveness. Furthermore, the results in the literature confirm a long-run co-integration between the prices of carbon and fossil fuels [10]. Dagoumas and Polemis [11] even confirm that electricity firms almost fully internalize the cost of carbon permits.

More recently, through Granger causality methods, Abbasi et al. [12] find that electricity consumption, prices, and real GDP are cointegrated in Pakistan. Also, for Pakistan, Abbasi et al.'s [13] results indicate the existence of a long-run relationship between electricity consumption, price, and real gross domestic product in the industrial sector. Therefore, we are left to believe that both GDP and share of sector energy in gross final energy consumption will be drivers of electricity prices. Finally, the share of renewable energy in gross final energy consumption will be a determinant of electric energy prices. Following Maciejowska's [14] results, there is a price-dampening merit order effect of renewable energy sources. The authors find that a rise in wind and solar leads to a fall of all quantiles of electricity prices, whereas solar is more successful in reducing the occurrence of positive price spikes. Similar conclusions have been reached by Sirin and Yilmaz [15] and for further inferences of drives of electricity prices, a complete review of algorithms applied to forecast energy prices is provided by Lago et al. [16].

3. Data and methods

The variables considered for the regression modeling are the electrical energy price households (EPh; medium size households; in Euro/kWh) and the electrical energy price non-households (EPnh; medium size consumers; in Euro/kWh) as the dependent variables, and gross domestic product per capita (GDP; in purchasing power standards, EU27, 2020), all three taken from the Eurostat, gas prices households (GPh; medium size households; in Euro/GJ, from the BP Statistical Review of World Energy), gas prices non-households (GPnh; medium size consumers; in Euro/GJ, from the BP Statistical Review of World Energy), oil prices (OP; in Euro/liter, Brent crude petroleum price by barrel obtained from the BP Statistical Review of World Energy), coal prices (CP; in Euro/per tonne, taken from the BP Statistical Review of World Energy), carbon dioxide emissions (CO2; in million tonnes, from the Eurostat data source), share of renewable energy in gross final energy consumption (SRE; in %, taken from Eurostat source) and share of industrial sector energy consumption in gross final energy consumption (SIS; in %; from Eurostat) as the independent variables.

To briefly introduce the generalized maximum entropy (GME) estimator [17,18], consider a linear regression model defined as

$$\boldsymbol{y} = \boldsymbol{X}\boldsymbol{\beta} + \boldsymbol{e}, \tag{1}$$

where, as usually, $\boldsymbol{y}$ denotes a $(N \times 1)$ vector of observations, $\boldsymbol{X}$ is a $(N \times K)$ design matrix of explanatory variables, $\boldsymbol{\beta}$ is a $(K \times 1)$ vector of parameters to be estimated and $\boldsymbol{e}$ is a $(N \times 1)$ vector of errors. Golan et al. [17] introduced a reformulation of the model in (1) as

$$\boldsymbol{y} = \boldsymbol{X}\boldsymbol{Z}\boldsymbol{p} + \boldsymbol{V}\boldsymbol{w}, \tag{2}$$

where $\boldsymbol{\beta} = \boldsymbol{Z}\boldsymbol{p}$ and $\boldsymbol{e} = \boldsymbol{V}\boldsymbol{w}$. In this reformulation, $\boldsymbol{Z}$ is a $(K \times KM)$ matrix of support spaces for the parameters, $\boldsymbol{V}$ is a $(N \times NJ)$ matrix of support spaces for the errors, $\boldsymbol{p}$ and $\boldsymbol{w}$ are respectively a $(KM \times 1)$ and a $(NJ \times 1)$ vectors of probabilities to be estimated. Given the model in (1) and the corresponding reparameterization in (2), the optimization problem in this econometric approach consists in maximizing the entropy function $\boldsymbol{H}(\boldsymbol{p}, \boldsymbol{w})$, subject to the model constraints,

$$\boldsymbol{y} = \boldsymbol{X}\boldsymbol{Z}\boldsymbol{p} + \boldsymbol{V}\boldsymbol{w}, \tag{3}$$

the additivity constraints for $\boldsymbol{p}$,

$$\boldsymbol{1}_K = \left(\boldsymbol{I}_K \otimes \boldsymbol{1}'_M\right)\boldsymbol{p}, \tag{4}$$

and the additivity constraints for $\boldsymbol{w}$,

$$\boldsymbol{1}_N = \left(\boldsymbol{I}_N \otimes \boldsymbol{1}'_J\right)\boldsymbol{w}, \tag{5}$$

where $\otimes$ represents the Kronecker product. The resulting GME estimator is given by

$$\underset{\boldsymbol{p},\boldsymbol{w}}{\operatorname{argmax}} \left\{-\boldsymbol{p}' \ln \boldsymbol{p} - \boldsymbol{w}' \ln \boldsymbol{w}\right\}, \tag{6}$$

and, as there is no closed-form solution to this problem, numerical optimization techniques are needed to obtain the point estimates of the unknown parameters and errors, by $\widehat{\boldsymbol{\beta}} = \boldsymbol{Z}\widehat{\boldsymbol{p}}$ and $\widehat{\boldsymbol{e}} = \boldsymbol{V}\widehat{\boldsymbol{w}}$. All the codes are implemented in

MATLAB by the authors. Additional details on maximum entropy estimation can be found in [17] and [18]. The statistical models are defined as

$$EPh_i = b_1 + b_2 GDP_i + b_3 GPh_i + b_4 OP_i + b_5 CP_i + b_6 CO2_i + b_7 SRE_i + b_8 SIS_i + e_i, \tag{7}$$

and

$$EPnh_i = b_1 + b_2 GDP_i + b_3 GPnh_i + b_4 OP_i + b_5 CP_i + b_6 CO2_i + b_7 SRE_i + b_8 SIS_i + e_i, \tag{8}$$

for $i = 1, 2, \ldots, 10$ where e represents the error component. Regarding the supports for the parameters of both models, instead of using estimates from ordinary least squares as prior information to establish the bounds for supports [3], different supports with wide bounds, namely $[-10, 10]$ and $[-5, 5]$, both with five points, are tested and the stability of the estimates are investigated. Since the impact on the estimates using these two different supports is considered not relevant, only the results using the supports $[-10, 10]$ are presented here, reflecting a lower impact of the supports over the optimization problem. The standard deviation of the noisy observations and the 3-sigma rule is used as a guide to define the supports for the errors (the usual strategy in the literature). Finally, and following Moreno et al. [3], since an increase in economic activity and an increase in primary energy prices lead to an increase in electrical energy prices, the parameters b_2, b_3, b_4 and b_5 are restricted to be non-negative, and the corresponding supports are adapted to $[0, 10]$ accordingly.

4. Results and discussion

The estimated parameters of the models in (7) and (8) are presented in Tables 1 and 2. The median presented in both tables corresponds to the median of the 1000 estimates obtained by bootstrapping residuals. And with this information from resampling, the percentile method is used to compute confidence intervals (other confidence levels are available upon request to the authors).

The relevance of the analyzed variables in explaining the variations in Portuguese electricity prices for both households and non-households was measured from the confidence intervals obtained. From Tables 1 and 2, one can conclude that variables gross domestic product, gas prices (households and non-households), oil prices, coal prices, carbon dioxide emissions (except for the 1% significance level), and the share of industrial sector energy in gross final energy consumption are consistently significant when considering any of the models, at all the confidence levels accounted for. Variable share of renewable energy in gross final energy consumption does not seem to impact the Portuguese electricity prices at either the household or the non-household levels, and, when considering model (7) and the confidence interval at the 99% confidence level, also the carbon dioxide emissions variable has a regression coefficient that cannot be considered significantly different from zero.

Table 1. Results from model in (7): electrical energy price households.

	Estimate	Median	CI 90%	CI 95%	CI 99%
b_1	−0.1077	−0.0961	(−0.1164, −0.0760)	(−0.1209, −0.0714)	(−0.1274, −0.0568)
b_2	0.0026	0.0048	(0.0023, 0.0082)	(0.0020, 0.0090)	(0.0015, 0.0111)
b_3	0.0127	0.0195	(0.0140, 0.0255)	(0.0128, 0.0267)	(0.0114, 0.0296)
b_4	0.58e−09	0.15e−08	(0.48e−11, 0.56e−06)	(0.19e−11, 0.20e−05)	(0.14e−12, 0.10e−02)
b_5	0.85e−03	0.15e−02	(0.44e−03, 0.28e−02)	(0.28e−03, 0.31e−02)	(0.12e−03, 0.56e−02)
b_6	−0.0043	−0.0103	(−0.0186, −0.0030)	(−0.0203, −0.0019)	(−0.0234, 0.0007)
b_7	−0.0004	−0.0039	(−0.0138, 0.0076)	(−0.0152, 0.0099)	(−0.0194, 0.0140)
b_8	−0.0100	−0.0098	(−0.0132, −0.0062)	(−0.0138, −0.0053)	(−0.0150, −0.0031)

Considering the 95% confidence intervals computed, the estimates obtained by the GME procedure let us know that a one-unit increase in purchasing power standards of GDP leads to an expected increase in the electrical energy price in between 0.0020 and 0.0090 Euro/kWh for households and in between 0.0022 and 0.0085 Euro/kWh for the non-households, all other factors being constant. Similarly, a one-unit increase in the costs of gas price, oil price, and coal prices lead to an average increase in the household electrical energy price (in Euro/kWh) in between 0.0128 and 0.0267, 0.19e−11 and 0.20e−05, and 0.00028 and 0.0031, respectively. In the same way, a one-unit increase in the costs of gas price, oil price, and coal price lead to an average increase in the non-household electrical energy price in between 0.0165 and 0.0321, 0.26e−11 and 0.20e−06, and 0.70e−04 and 0.0022 Euro/kWh, respectively, of

Table 2. Results from model in (8): electrical energy price non-households.

	Estimate	Median	CI 90%	CI 95%	CI 99%
b_1	−0.1093	−0.0974	(−0.1122, −0.0837)	(−0.1164, −0.0810)	(−0.1213, −0.0666)
b_2	0.0034	0.0049	(0.0025, 0.0079)	(0.0022, 0.0085)	(0.0016, 0.0098)
b_3	0.0160	0.0239	(0.0179, 0.0306)	(0.0165, 0.0321)	(0.0141, 0.0398)
b_4	0.47e−10	0.48e−09	(0.58e−11, 0.58e−07)	(0.26e−11, 0.20e−06)	(0.45e−12, 0.50e−03)
b_5	0.47e−03	0.92e−03	(0.13e−03, 0.20e−02)	(0.70e−04, 0.22e−02)	(0.76e−05, 0.48e−02)
b_6	−0.0055	−0.0114	(−0.0181, −0.0051)	(−0.0197, −0.0041)	(−0.0214, −0.0019)
b_7	0.0012	0.0031	(−0.0043, 0.0115)	(−0.0053, 0.0128)	(−0.0086, 0.0164)
b_8	−0.0113	−0.0101	(−0.0129, −0.0072)	(−0.0134, −0.0065)	(−0.0141, −0.0040)

course, under *ceteris paribus* interpretation. On the other hand, the carbon dioxide emissions tend to harm electrical energy prices, more specifically, an increase in a million tonnes of carbon dioxide emissions is expected to lead to an average decrease between 0.0203 and 0.0019 Euro/kWh on the household electrical energy price, and an average decrease in between 0.0197 and 0.0041 Euro/kWh on the non-household electrical energy price, maintaining all other factors constant. Finally, the share of industrial sector energy consumption in gross final energy consumption tends to impact negatively the electrical energy prices in Euro/kWh, as an increase of one percent in the share causes an average decrease in between 0.0138 and 0.0053 for household context, and in between 0.0134 and 0.0065 for non-household context, all other factors being constant.

5. Conclusions

The GME methodology allowed us to conclude the impact of the analyzed variables with a degree of stability that very hardly could have been obtained with traditional regression analysis techniques. Given the high standard errors that would be obtained within the classical framework, the much higher amplitudes of the confidence intervals would certainly lead to a higher level of indecision in what concerns the impact of each variable on the Portuguese electricity prices for both households and non-households. Considering the high level of collinearity present, the small number of observations available, and the almost none distributional assumptions made on the data, we were able to conclude on the explanatory power of the aforementioned variables, whereas traditional regression methodologies are not suitable for empirical analysis in the described context. By modeling electricity energy prices for both households and non-households, through this more suitable technique, we conclude that income leads to higher electrical energy prices for both groups, although the higher burden is supported by non-households. As well, reductions in electricity bills for non-households may be achieved through the increased share of the industrial sector energy in gross final energy consumption. Results also point that renewable energy shares do not seem to be the solution to lower electricity prices, meaning that policymakers should redefine policies concerning these. Further recommendations would be to increase the implementation of energy conservation projects, upsurge energy conservation awareness, and resource efficiency for both household and non-household consumers in Portugal. This could be a way to reduce the vulnerability of electricity prices and ensure competitiveness. For that, a clear tax burden alleviation would as well be detrimental, as results point that higher production capacity, and shares of fossil fuels, will only increase electric energy prices.

Declaration of competing interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

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