Multifield, Multifrequency Bosonic Stars and a Stabilization Mechanism

Nicolas Sanchis-Gual^{1,2}, Fabrizio Di Giovanni³, Carlos Herdeiro¹, Eugen Radu¹ and José A. Font^{3,4}

¹*Departamento de Matemática da Universidade de Aveiro and
Centre for Research and Development in Mathematics and Applications (CIDMA),
Campus de Santiago, 3810-183 Aveiro, Portugal*

²*Centro de Astrofísica e Gravitação—CENTRA, Departamento de Física, Instituto Superior Técnico—IST,
Universidade de Lisboa—UL, Avenida Rovisco Pais 1, 1049-001 Lisboa, Portugal*

³*Departamento de Astronomía y Astrofísica, Universitat de València, Dr. Moliner 50, 46100 Burjassot (València), Spain*

⁴*Observatori Astronòmic, Universitat de València, C/ Catedrático José Beltrán 2, 46980 Paterna (València), Spain*

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Scalar bosonic stars (BSs) stand out as a multipurpose model of exotic compact objects. We enlarge the landscape of such (asymptotically flat, stationary, everywhere regular) objects by considering multiple fields (possibly) with different frequencies. This allows for new morphologies *and* a stabilization mechanism for different sorts of unstable BSs. First, any odd number of complex fields, yields a continuous family of BSs departing from the spherical, equal frequency, ℓ -BSs. As the simplest illustration, we construct the $\ell = 1$ BSs family, that includes several single-frequency solutions, including even parity (such as spinning BSs and a toroidal, static BS) and odd parity (a dipole BS) limits. Second, these limiting solutions are dynamically unstable, but can be stabilized by a hybrid- ℓ construction: adding a sufficiently large fundamental $\ell = 0$ BS of another field, with a different frequency. Evidence for this dynamical robustness is obtained by nonlinear numerical simulations of the corresponding Einstein-(complex, massive) Klein-Gordon system, both in formation and evolution scenarios, and a suggestive correlation between stability and energy distribution is observed. Similarities and differences with vector BSs are anticipated.

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Introduction.—Recent observations of dark compact objects, via gravitational waves [1–3], very large baseline interferometry imaging of M87* [4] or orbital motions near Sagittarius A* [5] support the black hole hypothesis. Yet, the issue of degeneracy remains a central question. This has been sharpened by recent illustrations, in both the gravitational and electromagnetic channels [6,7], using dynamically robust bosonic stars (BSs) to imitate the observed data.

In spite of many proposed black hole mimicker models [8], imposing an established formation mechanism and dynamical stability, within a sound effective field theory, restricts considerably the choices. The fundamental spherical (scalar [9,10] or vector [11]) BSs, occurring in Einstein’s gravity minimally coupled to a single complex, free bosonic field, fulfill these criteria [12], having become prolific testing grounds for strong-gravity phenomenology. The purpose of this Letter is to enlarge the landscape of dynamically robust BSs, by considering multifield, multifrequency solutions, which will open new avenues of research, both theoretical and phenomenological, for these remarkable gravitational solitons.

Single and multifield BSs.—Single field BSs appear in different varieties [13] besides the aforementioned fundamental spherical (*monopole*) solutions [14], including

spinning BSs [11,15–17] and multipolar (static) BSs [18]. Concerning the former, only the vector case is dynamically robust [19]; concerning the latter, the simplest illustration is the *dipole* BS, shown to be unstable below.

Single-field BSs provide building blocks for multifield BSs, despite the nonlinearity of the model. Appropriate superpositions, moreover, change dynamical properties. An *excited* monopole scalar BS, which is unstable against decaying to a fundamental BS [20], is stabilized by adding a sufficiently large fundamental monopole BS of a second field [21] (see also [22–24]). In the same spirit, for the nonrelativistic BSs of the Schrödinger-Poisson system, a dipole configuration is stabilized by adding a sufficiently large fundamental monopole [25] (see also [26]). These examples turn out to be illustrations of a stabilization *mechanism*, as we shall discuss.

A particular type of multifield BSs, composed of an odd number ($2\ell + 1$, $\ell \in \mathbb{N}_0$) of (equal frequency) complex scalar fields was unveiled in [27] and dubbed ℓ -BSs. These are spherical and can be seen as a superposition of all m multipoles, with the same amplitude, for a given ℓ . ℓ -BSs were shown to be stable in spherical symmetry [28]; but nonspherical perturbations suggest new equilibrium configurations exist with different frequencies for different fields [29]. This will be confirmed herein: ℓ -BS are just the

symmetry-enhanced points of larger continuous families of multifield, multifrequency BSs [30].

The model.—Einstein's gravity minimally coupled to a set of N free, complex, massive scalar fields $\Phi^{(j)}$ is $S = (16\pi G)^{-1} \int d^4x \sqrt{-g} [R - \mathcal{L}]$, where G is Newton's constant, R the Ricci scalar and the matter Lagrangian is

$$\mathcal{L} = \sum_{j=1}^N (g^{\alpha\beta} \Phi_{,\alpha}^{(j)*} \Phi_{,\beta}^{(j)} + \mu^2 \Phi^{(j)*} \Phi^{(j)}); \quad (1)$$

μ is the (common) mass of all fields $\Phi^{(j)}$ and the asterisk (*) denotes complex conjugation.

All BSs studied herein are described by the metric ansatz $ds^2 = -e^{2F_0(r,\theta)} dt^2 + e^{2F_1(r,\theta)} (dr^2 + r^2 d\theta^2) + e^{2F_2(r,\theta)} r^2 \sin^2\theta (d\varphi - W(r,\theta) dt)^2$, in terms of four unknown metric functions of the coordinates (r, θ) ; the two Killing coordinates (t, φ) represent the time and azimuthal directions. The N scalar fields $\Phi^{(j)}$ are

$$\Phi^{(j)} = \phi_j(r, \theta) e^{-i(w_j t - m_j \varphi)}, \quad (2)$$

where $w_j \in \mathbb{R}^+$ are the fields' frequencies and $m_j \in \mathbb{Z}$ the azimuthal harmonic indices. The fields' amplitudes ϕ_j are real functions. This ansatz illustrates symmetry non-inheritance [34]: each $\Phi^{(j)}$ depends on the Killing coordinates but its energy-momentum tensor (EMT) does not [35].

Constructing the enlarged ℓ -BSs family.—Taking an odd number of fields, $N = 2\ell + 1$, for a fixed $\ell \in \mathbb{N}_0$, a spherical ansatz ($W = 0$, $F_1 = F_2$, with no angular dependence), equal frequencies ($w_j = w$), and equal radial amplitudes such that $\phi_j(r, \theta) e^{im_j \varphi} = f(r) Y_\ell^{-\ell-1+j}(\theta, \varphi)$, where Y_ℓ^m are the standard spherical harmonics, one obtains ℓ -BSs [27].

Taking still $N = 2\ell + 1$ but keeping the most general ansatz discussed above new possibilities emerge. We take $m_j = -\ell - 1 + j$, as for ℓ -BSs. For concreteness we focus on the simplest nontrivial $\ell = 1$ case. Then, the problem reduces to solving a set of seven partial differential equations (PDEs), for $F_{0,1,2}$, W , and $\phi_{1,2,3}$. This number reduces for particular cases [36]. These PDEs are solved with boundary conditions: (i) at $r = 0$, $\partial_r F_{0,1,2} = 0$, $\partial_r W = 0$, $\partial_r \phi_2 = \phi_{1,3} = 0$; (ii) at infinity all functions vanish, $F_{0,1,2} = W = \phi_i = 0$; (iii) at $\theta = 0, \pi$, $\partial_\theta F_{0,1,2} = 0$, $\partial_\theta \phi_2 = \phi_{1,3} = 0$; (iv) the geometry is invariant under a reflection along the equatorial plane $\theta = \pi/2$, and, as for ℓ -BSs, ϕ_2 and $\phi_{1,3}$ are parity odd and even functions, respectively. Thus, at $\theta = \pi/2$, $\partial_\theta F_{0,1,2} = \partial_\theta W = \phi_2 = \partial_\theta \phi_{1,3} = 0$. All configurations reported here are *fundamental*, with $n = 0$, where n is the number of nodes along the equatorial plane of $\phi_{1,3}(r, \pi/2)$ [37]. The solutions are constructed

numerically by employing the same approach as for the case of single-field BSs—see, e.g., the description in [38].

The single-frequency, multifield limits.—There are special limits where all fields have the same frequency ($w_j = w$). First, there are two types of single-field configurations: (i) dipole BSs (DBS₀), which are odd parity, obtained by taking only the $m = 0$ mode, $\phi_2 \neq 0$ [39] (see also [40]). Their angular momentum density vanishes ($W = 0$) and so does their total angular momentum, $J = 0$; (ii) spinning BSs (SBS _{± 1}) [11,15–17], which are even parity and have $J \neq 0$, obtained by taking only either $\phi_1 \neq 0$ (SBS₋₁) or $\phi_3 \neq 0$ (SBS₊₁).

Second, combinations of single-field configurations lead to two types of two-field configurations: (iii) spinning dipolar BSs (DBS₀ + SBS _{± 1}), in which case only either $\phi_1 \neq \phi_2 \neq 0$ (SBS₋₁ + DBS₀) or $\phi_2 \neq \phi_3 \neq 0$ (DBS₀ + SBS₊₁). These are novel solutions with $J \neq 0$, carried by the even-parity scalar field; (iv) toroidal static BSs (SBS₋₁ + SBS₊₁), for which $\phi_1 = \phi_3 \equiv \phi \neq 0$. Each field $\Phi^{(1)}, \Phi^{(3)}$ carries a *local* angular momentum density, with the corresponding EMT component $T_\varphi^{(1)} = -T_\varphi^{(3)} = 2e^{-2F_0} w \phi^2$, such that their sum is zero, $T_\varphi^t = 0$, and the spacetime is locally and globally static, with $J = 0$.

Finally, (v) ℓ -BSs (SBS₋₁ + DBS₀ + SBS₊₁), which are static, spherical, and have $\phi_i = \phi(r)(\sin\theta/\sqrt{2}, \cos\theta, \sin\theta/\sqrt{2})$. Figure 1 illustrates the single-frequency limits of the enlarged $\ell = 1$ BSs family as 3D plots.

For each of the five types of solutions described above, there is a one-dimensional family of BSs with $w_{\min} < w < \mu$, where $w_{\min}$ is family dependent. In an ADM mass, M vs frequency w diagram, they describe a spiral-type curve (costumary for BSs)—Fig. 2 [41]. As the frequency is decreased from the maximal value, μ , the ADM mass increases up to a maximum value $M^{(\max)}$ which

FIG. 1. $\ell = 1$ BSs family.

FIG. 2. ADM mass vs frequency for some of the single-frequency limits of the $\ell = 1$ BSs family. (Inset) Fraction of the total mass in $SBS_{-1} + SBS_{+1}$ and in DBS_0 along the static sequence 1 + 2 in Fig. 1 for $w_2 = 0.94\mu$ (and $w_1/\mu = w_3/\mu$ in the x axis).

is family dependent. The energy density distribution of the solutions is illustrated by the morphologies in Fig. 1.

The multifrequency, multifield interpolations.—Relaxing the equal frequency requirement a larger solution space emerges (blue triangle in Fig. 1). There are multifrequency BSs interpolating between the single-frequency ones, which are particular points in a manifold of solutions [42]. As an illustration consider the interpolation $SBS_{-1} + SBS_{+1} \leftrightarrow DBS_0$, which goes through an ℓ -BS. Fix, e.g., $w_2 = 0.94\mu$ for the ℓ -BS along the sequence—Fig. 2 (inset). On the one hand, decreasing $w_1 = w_3$ (w_2 fixed), the toroidal static BS ($SBS_{-1} + SBS_{+1}$) is approached for $w_1 = w_3 \simeq 0.933\mu$ —sequence 1 in Fig. 1. On the other hand, increasing $w_1 = w_3$ (w_2 fixed), the dipole DBS_0 is obtained for $w_1 = w_3 \simeq 0.955\mu$ —sequence 2. These are static BSs sequences; thus $W = 0$.

Similar interpolations occur between configurations with and without angular momentum, as in the transition $SBS_{-1} + SBS_{+1} \leftrightarrow SBS_{+1}$ —sequence 3 in Fig. 1. Starting from a static, toroidal BS with $w_1 = w_3 = 0.8\mu$, varying w_3 (w_1 fixed), the amplitude of $\Phi^{(1)}$ vanishes for a critical value of $w_3 \simeq 0.829\mu$, yielding the single-field SBS_{+1} . All intermediate solutions with $w_1 \neq w_3$ possess a nonvanishing angular momentum. In all sequences, a similar picture holds considering other frequencies.

The manifold of solutions of the $\ell = 1$ BSs family is as follows. Starting from an ℓ -BS with a fixed frequency $w_1 = w_2 = w_3$, the line of static ($J = 0$) BSs is obtained keeping $w_1 = w_3$ and varying the ratio $y \equiv w_1/w_2$ ($= w_3/w_2$). Then $y \in [y_{\min}, y_{\max}]$. y varies the parity of the BSs; the boundary values are the parity even and odd solutions, respectively. Then, for each fixed y one can vary $x \equiv w_3/w_1$, with $x \in [x_{\min}, x_{\max}]$, where the limits are y dependent. x varies J ; for $x > 1$ ($x < 1$), J is positive

FIG. 3. Time evolutions of the energy density for five static BSs in the $\ell = 1$ family. Each column shows six sequential snapshots of the xz ($y = 0$) plane (time running from top to bottom). From left to right the values of $w_1 = w_3$ of the five models are: 0.955 (DBS_0), 0.945, 0.940 (ℓ -BS), 0.935, 0.933 ($SBS_{-1} + SBS_{+1}$).

(negative) [43]. Finally, varying the frequency of the starting ℓ -BS yields a 3D manifold of solutions. Thus, we expect a $(2\ell + 1)D$ manifold of multifrequency, multifield BSs for a model with $(2\ell + 1)$ complex scalar fields, including ℓ -BSs as symmetry-enhanced solutions.

Dynamical (in)stability.—We assess the dynamical stability of representative solutions in the $\ell = 1$ BS family by resorting to fully nonlinear dynamical evolutions of the corresponding Einstein–(multi-)Klein-Gordon system. The infrastructure used in the numerical evolutions is the same as in [19].

Figure 3 exhibits the results for a sequence of static solutions (i.e., along sequences 1 and 2 in Fig. 1, including the dipole, the ℓ -BS and the toroidal static BS). We find that all solutions (except the ℓ -BS) are dynamically unstable, decaying to a multifield BS in which all fields have $\ell = m = 0$. Including J does not improve dynamical stability. The $SBS_{\pm 1}$ are unstable against a nonaxisymmetric instability [19] and all hybrid cases we have studied (such as $SBS_{\pm 1} + DBS_0$) also decay to the fundamental $\ell = 0$ BSs.

Hybrid ℓ -BSs and a stabilization mechanism.—Instead of focusing on a single ℓ -BSs family we now allow superpositions of such families with different ℓ 's. The most elementary example is to add $\ell = 0$ BSs to the $\ell = 1$ family. Thus we add a fourth complex scalar field $\Phi^{(0)}$,

obeying (1) and (2) with $j = 0$ (hence, now $j = 0, 1, 2, 3$) and $m_j = 0$. Its boundary conditions are $\partial_r \phi_0|_{r=0} = \phi_0|_{r=\infty} = \partial_\theta \phi_0|_{\theta=0,\pi} = 0$, besides being parity even. Keeping only $\Phi^{(0)}$, the basic solution is the single-field, fundamental, monopole BS (MBS_0). We now show that adding MBS_0 can quench the instabilities observed in the $\ell = 1$ family.

To be concrete we consider the following superpositions: (A) $\text{MBS}_0 + \text{SBS}_{+1}$ and (B) $\text{MBS}_0 + \text{DBS}_0$. As an illustration of (A), fixing $w_3/\mu = 0.98$, there is a continuous sequence of solutions reducing to the MBS_0 (SBS_{+1}) for $w_0/\mu = 0.964$ (0.975). We refer to the intermediate configurations as “saturns.” Their dynamical evolutions—Fig. 4—exhibit a simple pattern: sufficiently close to the MBS_0 (SBS_{+1}) limit, saturns are stable (unstable). Here, stability means no sign of instabilities for long evolutions ($t \simeq 24\,000$) [44]. Attempting to interpret the transition between the two regimes, we observe a correlation between instability and the r coordinate of the maximum of the

FIG. 4. Time evolution (top to bottom) of two saturns: (left and middle left) $\omega_1/\mu = 0.967$; (middle right and right) $\omega_1/\mu = 0.974$. For each case the energy density and the real part of $\Phi^{(3)}$ on the xy plane are shown. The first (second) saturn is close to MBS_0 (SBS_{+1}) and is stable (unstable).

energy density—Fig. 5: when the latter is at the origin ($r = 0$) no instability is observed.

As an example of (B), fixing $w_0/\mu = 0.97$, there is a continuous sequence of solutions reducing to the MBS_0 (DBS_0) for $w_2/\mu = 0.983$ (0.973). We refer to the intermediate configurations as “pods.” Evolving this sequence of pods reveals analogous patterns: (a) sufficiently close to the MBS_0 (DBS_0) limit, pods are stable (unstable)—see Sec. I of Supplemental Material [45] for snapshots of the evolutions; (b) when the energy density maximum, which, in general, has two symmetric points located on the z axis (i.e., $\theta = 0, \pi$) is at the origin, no instability is observed—Fig. 5 (inset).

Generality and remarks.—An analogous family of vector $\ell = 1$ BSs should exist. Preliminary results show an important difference: the whole sequence 3 (see Fig. 1) is stable in the vector case, including the $\text{SBS}_{-1} + \text{SBS}_{+1}$ static configuration, which is now spheroidal rather than toroidal (see Sec. II of Supplemental Material [45] for details). This is a consequence of the stability of vector $\text{SBS}_{\pm 1}$ [19]. On the other hand, we have evidence that the vector DBS_0 is unstable, as in the scalar case (see Sec. III of Supplemental Material [45]).

A byproduct of our construction is the realization that all single-frequency BSs arising in (combinations of) models of type (1) are continuously connected within a multidimensional solution manifold, interpolated by multifrequency solutions. For instance, spherical (MBS_0) and spinning ($\text{SBS}_{\pm 1}$) BSs, typically described as disconnected, are connected (via saturns).

Adding the fundamental MBS_0 , which is the ground state of the whole family, stabilizes different types of

FIG. 5. Instability timescale (black dots and lines) and maximum of the total energy density (red lines) for the sequence of saturns (main panel) and pods (inset) described in the text. The blue points and line are thresholds when no instability is seen. The z coordinate in the inset is r for $\theta = 0, \pi$. $T_{\text{instability}}$ refers to the time when we observe that the solution begins to clearly deviate from axisymmetry for saturns and from equatorial symmetry for pods.

unstable BSs, such as excited monopole BSs [21], spinning BSs, and dipole BSs. Such stable configurations can actually form from the (incomplete) gravitational collapse of dilute distributions of the corresponding fields and multipoles—see Sec. III of Supplemental Material [45]. It would be interesting to probe the generality of this cooperative stabilization mechanism, and if other (higher ℓ , say) multipolar BSs can be stabilized similarly [50].

Another mechanism for mitigating instabilities is adding self-interactions [52], which was suggested to quench the instability of spinning BSs, without requiring the energy density to be maximized at the origin [53]. It would be interesting to construct the corresponding $\ell = 1$ BSs family in models with self-interactions and investigate whether other members of the family can be stabilized by self-interactions.

Finally, among the possible generalizations, one could consider different masses for the fields in (1). At least for slightly different masses multifield, multifrequency solutions still exist, albeit with a modified solution space, for instance without an exactly spherically symmetric composite state. Different generalizations could be obtained by introducing a target space metric (see, e.g., [54,55]) in the multifield model.

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