HISTÓRIAS COM CIÊNCIA NA BIBLIOTECA ESCOLAR [(HI)STORIES WITH SCIENCE IN THE SCHOOL LIBRARY]

A project to bring topics of History of Science to secondary schools in Aveiro (Portugal)

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O binómio de Newton é tão belo como a Vénus de Milo.
O que há é pouca gente para dar por isso.

Newton's Binomial theorem is as beautiful as the Venus de Milo.
The problem is that very few people are able to realize it.

Álvaro de Campos [heteronymous of Fernando Pessoa]

ABSTRACT

In this paper, we intend to present the project *Histórias com Ciência na Biblioteca Escolar* [(Hi)stories with Science in the School Library], a project which brings History of Science topics to high schools in the city of Aveiro (Portugal). This project is a joint project of the University of Aveiro and the *Rede de Bibliotecas Escolares* [School Libraries Network, a program of the Ministry of Education]. The goal of this project is to join the University and High Schools in promoting scientific dissemination, namely the research carried out in the University, as well as extending the scientific culture to a pre-university audience. This project is coordinated by Professor António Andrade (Department of Languages and Cultures) and consists of a cycle of nine conferences in high school libraries by several investigators of the University. The themes of these conferences are very diverse and include areas such as astronomy, medicine, botany, literature, physics and mathematics. These conferences are always centered on episodes of the history of science, for instance, botany in the work of the epic Portuguese poet Camões, the importance of amateur astronomers in the past, the history of syphilis, the importance of the phonograph in Portugal, among others. In this paper, we will present the three mathematical conferences in detail.

1 Introduction

The History of Mathematics Group of the University of Aveiro participates in the project *Histórias com Ciência na Biblioteca Escolar* [(Hi)stories with Science in the School Library] with three conferences: Portuguese Arithmetic Books in the Portuguese Discoveries (Teresa Costa Clain), Real Problems – Historical Mathematical Solutions (Hélder Pinto) and Amateur Mathematicians – passions with limits? Simple problems, big challenges… (Helmuth Malonek).

In Clain's conference, the practical arithmetic treatises written in Portugal during the 16th century are presented. According to the traditional model, these treatises are mathematics texts with a practical vocation and with the objective of answering to the needs of professional training in the commercial world. Commercial arithmetic also became a source and a vector for the dissemination of an important set of problems that would mark the history of knowledge for centuries. In this session, the *Praticad'*

Arismetica (1540) by Ruy Mendes is briefly presented and some problems proposed by Bento Fernandes are analyzed, which illustrate the ludic side of mathematical knowledge at the time.

Pinto's conference shows several historical examples of real problems that were solved using mathematics, for instance, how to measure the distance to a ship in the sea (Thales), how to determine the size of the earth (Eratosthenes), how to measure the height of a mountain (China), how to measure the sun´s altitude (Portuguese instrument) and how to improve calculation with rudimentary calculators. In this conference, as an introduction, several examples that everyone who knows some mathematics are presented, such as percentages (e.g.: taxes and store promotions), measuring areas, "reading" schedules and tables, and so on (for instance, as an example to students, even when using a clock, several mathematical notions are being used: why is 17:15 the same as "a quarter past 5 pm"?).

Malonek's conference was not presented in the ESU-8 meeting because it is the most recent one in this project, only being implemented this year. However, in this paper the situation of the project is updated and this conference is also presented.

Finally, note that this project is not only about the contents presented in the conferences, the major goal is to enhance the scientific culture of high school students. This is a small step to increase, in the future, the audience that can understand the importance of science and mathematics through the history of mankind. Mathematics and science are very useful and beautiful; the major problem is that very few people realize this.

2 The project *Histórias com Ciênciana Biblioteca Escolar*

The University of Aveiro and the Network of School Libraries, a program of the Ministry of Education, met to jointly develop a project of scientific dissemination among high school students, centered on the School Library.

Figure 2.1: The project *logo*.

This project, entitled *[Hi]stories with Science in the School Library*, is made up of nine conferences, stimulated by a group of teachers and researchers with research developed in their areas of knowledge and composed by the following sessions:

- Plants in the Lyric and Epic of Camões. (Jorge Paiva / Functional Ecology Center of the University of Coimbra);

- Amateur astronomers - passions without limits? (Vitor Bonifácio / UA-DF / CIDTFF);
- The unicorn and the bezoar: between myth and reality. (António Andrade / UA-DLC / CLLC);
- The Phonograph, which presented on Sunday constipated and hoarse, presented on Monday clear and clear as never before - The phonograph and its presence in the teaching and popularization of science (19th century). (Isabel Malaquias / UA-DF / CIDTFF);
- Portuguese books of arithmetic in the Discoveries. (Teresa Clain, History of Mathematics Group, CIDMA – UA; D. Maria II High School);
- Real Problems - Historical Mathematical Solutions. (Hélder Pinto, History of Mathematics Group, CIDMA – UA; ESE – Instituto Piaget);
- Changes in history seen from a Chemistry perspective: some examples of molecules that have changed the world. (João Oliveira / UA-DQ / CESAM);
- Madness, medicine and literature (from the Archipathology of Filipe Montalto). (Joana Mestre Costa, UA-ISCA / CLLC);
- Amateur Mathematicians - passions with limits? Simple problems, big challenges… (Helmuth Malonek, History of Mathematics Group, CIDMA - UA).

The coordination of the conference cycle links schools and researchers. In this project, the School Library represents an aggregator of diverse knowledge and resources that could be implicated in the change of educational practices, by supporting learning methods and the curriculum.

The project *[Hi]stories with Science in the School Library* began with high schools in the city center of Aveiro and quickly spread to other schools in the region of Aveiro (and beyond). In these sessions, we try to show that Science is a "passion", the result of human curiosity in an attempt to discover the world and everything that is part of it, including Mankind. Without knowledge of the origins, we hardly understand the present, hence the importance of this cycle of lectures.

In the following text, the contributions of the History of Mathematics as part of the history of science will be presented, namely the sessions facilitated by the History of Mathematics Group (GHM) of CIDMA of the Department of Mathematics of the University of Aveiro.

2.1 Portuguese Arithmetic Books in the Portuguese Discoveries (Teresa Costa Clain)

In the following subsection, the practical arithmetic treatises written in Portugal in the 16th century will be presented: *Tratado da Pratica d'arismetica* (Nicolas, 1519), *Pratica d'Arismetica* (Mendes, 1540) and *Tratado da Arte de Arismetica* (Fernandes, 1555). The main goal of this presentation is: to show that mathematical themes are present in the arithmetic treatises, as well as to disclose the authors' performance in the face of challenges of the surrounding commercial world and their contributions to the mathematical knowledge of the time, giving them a place in the historiography of Mathematics in Portugal. For this purpose, some axes on which the session will be developed are presented, that is, an approach around a socio-economic framework addressing the following issues: What was the economic, social and geographical context? Who were the authors? What do they tell us about themselves and their motivations?

At this point, the Portuguese empire and commercial expansion is explained. The large deals associated with expansion were not "going the right way". The arithmetic treatises are testimony of a preparation for the daily life of the national merchant in order to open up to the international markets. Territorial expansion was also synonymous with commercial expansion, and new social and natural realities. The arithmetic treatises addressed some business issues and dealt with problems related to the *New World*. One example was the spices business, which often had a loss of merchandise due to poor travelling conditions. Further, the "weight" of the fourth and twentieth tax on goods was added on top of this, as Gaspar Nicolas claims. Two important institutions of the Portuguese commercial network are also referred to: the House of India and Flanders Factory.

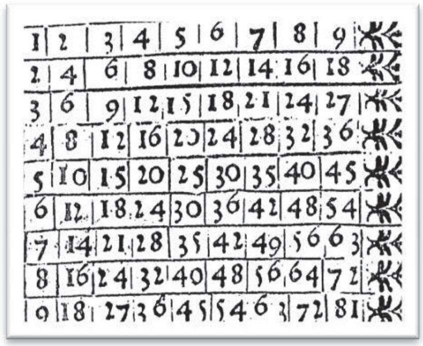
Biographical data on the authors is scarce; however, their motivations are explicit. A reading of the three treatises leads us to believe that the three authors presented similar themes through different methodologies to serve the same objectives. Gaspar Nicolas frequently mentions the questions that were asked in the House of India when he arrived in the city of Lisbon. Ruy Mendes, dealing with the same subjects, does so in a "more academic" manner, by organizing the subjects, considering his taste in problems with numbers and through the method in which he introduces the themes. Bento Fernandes transmitted through his work his experience as a merchant. He wrote a treatise for merchants and gave special emphasis to commercial rules, focusing on the importance of training merchants.

A global approach to these works will be: What is the organization of the treatises? What are the topics covered? New commercial routes, with an enormous amount and variety of products in circulation, as well as the emergence of increasingly complex commercial techniques required that the actors act wisely. Merchants needed to record, calculate earnings and predict risks. For these purposes, arithmetic knowledge was necessary.

In the session, the structure of the work by Ruy Mendesis presented. This is the only one of the three authors that separates the knowledge of basic rules from their applications. In this treatise, there are "raw" mathematics and commercial mathematics.

An approach around the mathematical language of the works is used: What are the concepts covered? What is the scientific vocabulary used? What arithmetic was it? Also, notice that the use and vulgarization of the Indo-Arab numbers transform the notion of arithmetic and allowed for the dissemination of written calculation.

To begin, basic calculations and basic operations are presented, such as the Lattice method of multiplication according to the Table of Multiplication.



1	2	3	4	5	6	7	8	9
2	4	6	8	10	12	14	16	18
3	6	9	12	15	18	21	24	27
4	8	12	16	20	24	28	32	36
5	10	15	20	25	30	35	40	45
6	12	18	24	30	36	42	48	54
7	14	21	28	35	42	49	56	63
8	16	24	32	40	48	56	64	72
9	18	27	36	45	54	63	72	81

Figure 2.2: Multiplication Table by Ruy Mendes (f. 13)

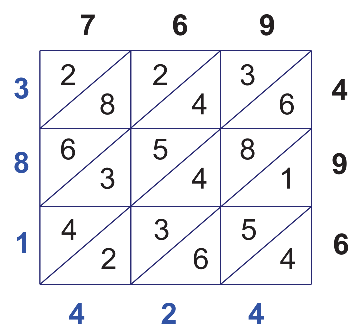


Figure 2.3: Lattice method of multiplication by Gaspar Nicolas (769 x 496 = 381424)

The entire division by *galera* method (Galley division) is also introduced.

	0	2					
	3	5	6				
0	4	7	8				
1	2	3	2	9			
9	**8**	**7**	**6**	**5**	(2	2	8
4	**3**	**2**	2	2			
	4	3	3				
		4					

Figure 2.4: An example of Galley division.

An example is presented by choosing **98765** as the dividend and **432** as the divider. The Galley division algorithm is considered difficult; however, this activity is proposed to the students who attended the session. The algorithm is outlined in the following intermediate steps by interpreting figure 2.4:

$987 - 2\times 432 = 123$

$1236 - 2\times 432 = 1236 - 2\times(400+30+2) = 1236-(800+60+4) = (1236-800)-(60+4) =$

$376 - 4 = 372$

$3725 - 8\times 432 = 3725 - 8\times(400+30+2) = (3725-3200)-(240+16) =$

$(525-240)+16 = 285 - 16 = 269$

The students know the theme of "sequences" and in this context, a problem of "walks" is proposed, where several steps need to be fulfilled:

-Understanding the problem in the original text;

-Studying and comprehending the author's solution;

-Solving the same problem using knowledge acquired in school.

Here, the problem at hand is outlined: consider two men walking, which can be described by two arithmetic progressions. One of the men walks 9 miles each day and the other walks 1 mile more than the previous day. The author wants to know when they meet for the first time. The author's solution: duplicate 9 and take 1 away, we have 17.

In current language, the equation can be solved with a second degree $9n = \frac{n(n+1)}{2}$. To "avoid" the resolution of the two-degree equation, Nicolas proposed the equivalent expression $9 = \frac{n+1}{2}$, where $n = 9 \times 2 - 1 = 17$(Nicolas, 1963, f. 52 v).

Classical themes, such as progressions, and square and cubic roots, though disconnected from the mercantile world, have taken a prominent place in the Portuguese treatises. These themes are associated with calculating practices. Moreover, they present rough sketches of processes similar to those associated with mathematical thinking.

In the lecture, local characteristics are linked to the treatises: How have the themes presented been adapted to Portuguese commerce? What are the specific rules in Portuguese commerce?

The main goal of the treatises was essentially a commercial application of the developed methodologies. Among the classic rules considered, such as companies and leagues, there are particular Portuguese rules of commerce, such as the fourth and twentieth tax rule, and the rule of the account of Flanders. The fourth and twentieth tax rule established a model for calculating the payable tax for goods from the East by applying the rule of threes (80---23---a (to the tax) and 80---57---b (to the merchant)). This tax was an important source of revenue for the kingdom and corresponded to 28.75% of earnings. The account of Flanders was a conversion rule. Through these models, a mathematical understanding of the issues can be reached, as referred in Almeida (Almeida, 1994).

Nicolas, Mendes and Fernandes present the problems of the fourth and twentieth rule by addressing three steps to calculate the tax, see (Clain, 2016, p.352): a quarter of the merchandise, the twentieth of three-quarters of the remainder and finally what the merchant will keep after the taxes have been paid. The presence of this trichotomy in the statements for the practice of the fourth and twentieth rule can be observed.

On the rule of companies, the students are presented with problems similar to(Mendes, 1540, f. 71):

> Three merchants, Pedro, Luys, and Andre created a company in which Pedro invested 56 *cruzados*, Luys 78 and Andre 85. They earned a hundred *tostões*, how should they distribute the gains between the three of them?

With the students, the relation of the gains is established through the model

$$\frac{g_1}{i_1} = \frac{g_2}{i_2} = \frac{g_3}{i_3} = \frac{100}{219}$$

and we have, for example, Pedro's gains in this company

$$\frac{g_1}{56} = \frac{100}{219} \Leftrightarrow g_1 = \frac{100 \times 56}{219}$$

In the scope of proportional division, some problems of a playful nature, such as the classic inheritance problem, is presented:

> A man was about to die and his wife was pregnant, and he made a will in this way: he said that he left 600 *cruzados* in money, which he ordered to be distributed in this way: in case the woman had a male child, her son had 2/3 of the 600 *cruzados* and his wife 1/3. If his wife had a daughter, the daughter received 1/3 and the wife received 2/3. After the man died, the wife had a daughter and a son. How are the 600 *cruzados* to be distributed among the family? (Fernandes, 1555, f. 101v)

In the session, the problem is solved in current mathematical language, through a system of three equations and three unknowns (x – son; y – mother and z – daughter):

$$\begin{cases} x + y + z = 600 \\ x = 2y \\ y = 2z \end{cases}$$

These treatises have a large number of problems. The mathematical contents are found in the solutions presented by the authors. Through some stories told in the library and the examples presented, the students perceive that they are currently studying subjects that are found in the 16th century treatises, although the mathematical language is different. Symbolic language was not part of the 16th century, but mathematical thinking was. We leave the testimony of the students of the LH2 11th grade class:

> The class considered that the lecture was very enriching, constructive and appropriate to our course since it displayed mathematics in the historical context, more specifically, in the Discoveries. It allowed us to acquire a perception of how calculations were performed at that time and how much it has since evolved. The fact that we put into practice some of the processes used at the time of the Discoveries, made the speech more dynamic, interesting and useful. Given the vastness of the theme, we felt it was a short time since there was much more to be addressed. In short, the class considered that this kind of activity should be implemented in the classes more often.

2.2 Real Problems – Historical Mathematical Solutions (Hélder Pinto)

In this session, Pinto begins to sensitize the students to everyday life mathematics. The speaker intends to express the following message: mathematics are more present in our life than we imagine and, sometimes, those mathematics are very simple. The topics covered are diverse and offer different perspectives. Some examples are described:

- Hours and clocks: the sexagesimal system that everyone knows, fractions $\frac{1}{2}$, $\frac{1}{4}$ (half an hour, a quarter), the multiplication table of the number 5 (that "appear" in the minutes clock pointer) and Modular arithmetic (12h / 24h) are presented. In the conference, the high number of divisors of 60 (1, 2, 3, 4, 5, 6, 10, 12, 15, 20, 30 and 60) are referred to, which constitutes a possible reason for the appearance of this numbering system. This session is an occasion to review some aspects of number theory that are outside the curriculum of high school education in Portugal.
- Percentages (bank loans, sales, taxes, etc.): in Portugal, the Languages and Literature students take the Applied Mathematics to Social Sciences course, where financial models, among others, are studied. In this course, students are faced with financial calculations necessary in the everyday life of a taxpayer (state taxes on supermarket purchases, property tax, etc.). The students attending the Science and Technology course are not familiar with these topics since they arenot in the course´s curriculum. However, it is important for students to have an education in finance. A special focus on the use of percentages is addressed to calculate bank loans, sales and taxes. This knowledge fosters a practice for citizenship, avoiding unrealistic spending situations, and promoting consistent financial education.

After the first part of the conference, we explain that the utility of mathematics can be possibly found in many topics in the history of mathematics. Specifically, remark that the geometry of measurement is present in the mathematics curricula in Portugal and students are familiar with the basics of trigonometry. So, to introduce topics of the history of Mathematics, it is pertinent, above all, to show that mathematics has always been present, over time, in solving everyday life problems, such as construction, navigation, among others (see, for instance, Swetz (1994) and Katz (2000) for activities and to understand the importance of using History in teaching). In this context, several themes are presented:

- Measure the distance to a ship at sea (*Thales of Miletus*): in this part, different possibilities on how this measurement could have been done at the time are shown using only similarity or equality of triangles.

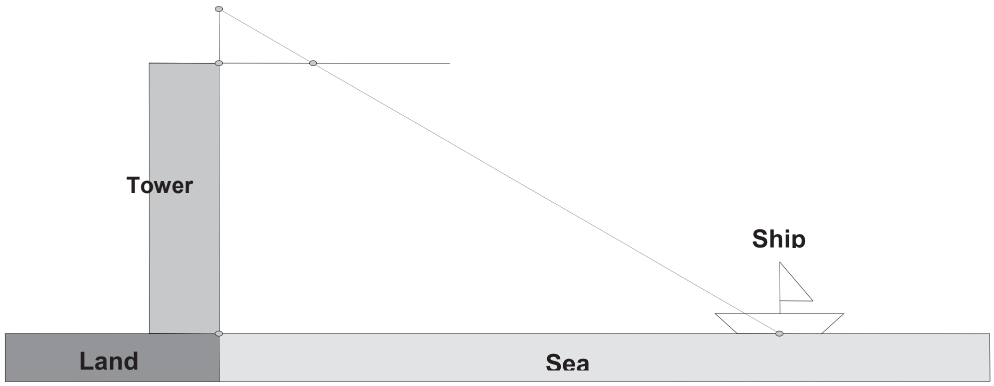


Figure 2.5: One possibility using similarity of triangles.

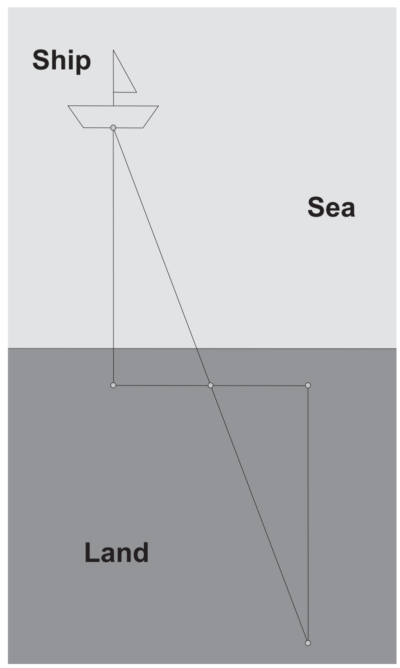


Figure 2.6: Another possibility using equality of triangles

- The height of a pyramid (*Thales of Miletus*): the situation with proportions is presented but it is also pointed out that the easier solution is to wait for the time of the day when all objects have shadows “perfectly equal” to themselves…

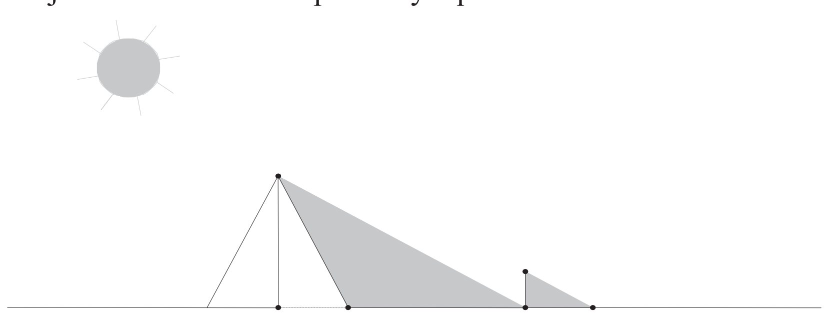


Figure 2.7: A representation of a pyramid and a vertical stick with their shadows

- Determination of the earth´s meridian length (*Eratosthenes)*: how amazing that it was possible to do this measurement with absolutely no modern technology just using mathematics!

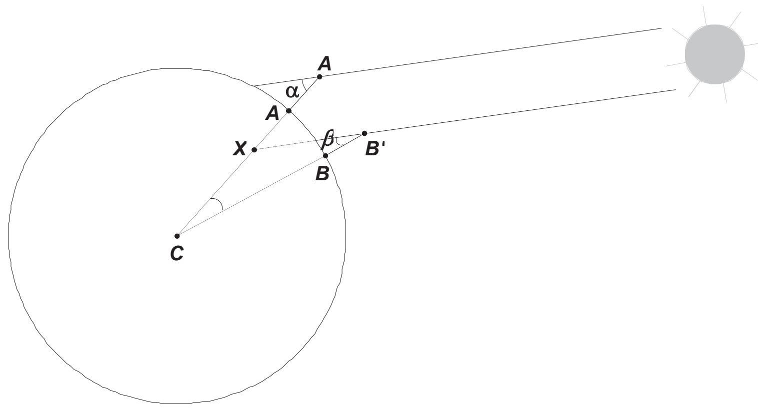


Figure 2.8: Picture shown to students to explain the Eratosthenes method

In another part of the conference, Napier rods and Genaille-Lucas rulers are presented in detail. We explain how these ancient instruments of calculation work (suitable only for multiplications) and highlight that the second instrument is an ingenuous improvement of the previous one (the second rulers are easier and quicker to work with). The aim in this part is to show that mathematics, like other sciences, is the consequence of the effort of several people and has evolved overtime. For a description of these instruments and how to use them in the classroom, see Pinto (2009) and (2010); for more information about these calculation methods see Seaquist, Seshaiyer & Crowley (2005).

Figure 2.9: Examples of these rulers used in classroom

Finally, the shadow instrument of Pedro Nunes is shown in this conference. Pedro Nunes (1502-1578) was a mathematician, cosmographer and professor at the University (the only one in Portugal in that time). Nunes lived in the period of the Portuguese Discoveries and had many contributions to the introduction of rigor in geometry and mathematics in 16th century Portuguese culture.

In his works, Pedro Nunes suggested several instruments that he imagined were useful for astronomical navigation used during the Discoveries. At stake was the measurement of the height of the Sun and other stars, namely the Polar Star. Using this measure, pilots could calculate latitude, which was very important in astronomical navigation; as stated in the conference, it was like the “GPS” of thetime. Among his creations, there are three that have survived and heavily contributed to the progress of scientific instrumentation: the *Nónio*, the Nautical Ring and the Instrument of Shadows, of which several examples are known. Among these, the instrument of the shadows is highlighted, which was a modest instrument, similar to a solar clock, but with a very ingenious innovation that made it possible to directly measure height through the shadows projected by the Sun.

The Instrument of Shadows was constructed like this: the base was a plate, usually square, where a circle was inscribed and a tangent to that circle was drawn. Mounted on the circle was a board similar to a style or gnomon, as in solar clocks. This plate had the shape of a rectangular isosceles triangle, with the legs´ length equal to the radius of the circle. The triangle had a hut resting on the radius of the circle that touched the tangent line, as shown in figure 2.10. A diameter parallel to the tangent was marked in the circle and the circumference from 0° to 90° was drawn in the directions from the diameter to the point of the tangent. To measure the height of the sun, one started by placing the base of the instrument horizontally. This base was then rotated until the edge of the shadow of the triangle coincided with the tangent line. The height of the star was directly “read” in the graduated circle. However, remark that this instrument was not used in real life because it was very difficult to have a completely horizontal surface athigh sea…

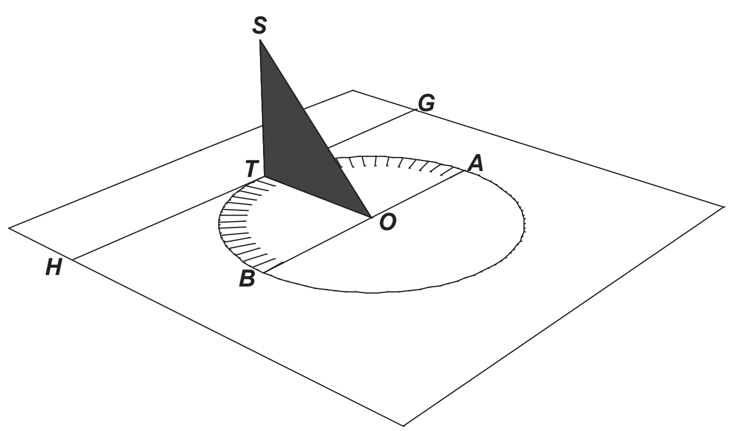


Figure 2.10: Representation of the Instrument of Shadows

The geometric principle of this instrument is very easy to understand: the sun’s rays make an angle with the horizontal plane that corresponds to the angular height of the star. When placing the plate to match the shadow with the tangent line, this angle is transferred to the horizontal plate, where it is directly “read”. The geometric demonstration is very basic and only use two mathematical results that students already know (see figure 2.11): the angles SS’T and TS’O are equal because the triangles [SS’T] and [TS’O] are equal (they have one common side as [TS’], [ST] and [TO] have the same length because of the instrument’s construction and both triangles are rectangle; the LAL criteria is used); the angles TS’O and XOA are equal because they are alternate angles between parallel lines. For more detailed information about this instrument, see Crato (2003).

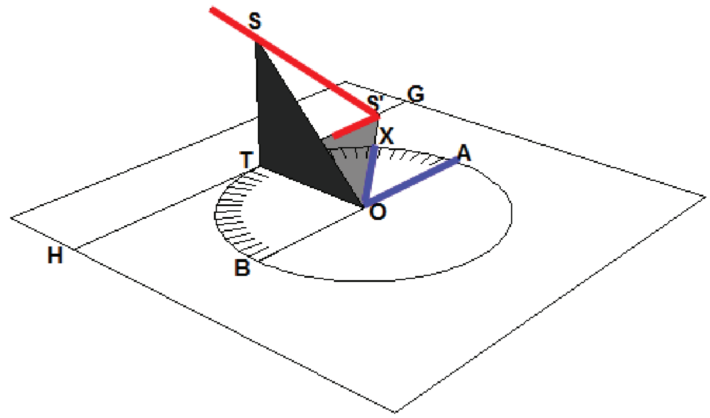


Figure 2.11: The correct position of the shadow in the instrument

The Instrument of Shadows is easily introduced in the classroom because it is very easy to construct replicas using only paper and cardboard (figure 2.12). For detailed explanations about how to construct these replicas and how to create activities with this instrument in the classroom, see Pinto (2009) and (2010).

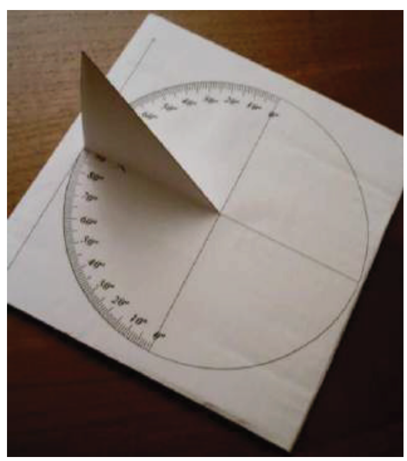


Figure 2.12: A replica of the instrument made with paper and cardboard

Pinto's lecture aims to motivate the learner to the practices based on the construction and use of simple instruments. In addition, Socio-Economic Sciences and Humanities course students are familiar with the theme of the Discoveries through the History of Portugal classes. Pedro Nunes is a heavily mentioned figure in the literature and his important contributions to the scientific knowledge are presented.

The students of the SE1 10th grade class left their opinion about this session:

> Professor Hélder Pinto showed that Mathematics is present in everything, from a simple clock to the most advanced spacecraft. In times when knowledge and technology were more rudimentary, mathematics was used to solve real problems, which today are considered substantially simpler. The class was surprised by the fact that certain measures of interest, such as the earth's radius,

> were very similar to the values obtained using more modern and rigorous processes. The lecture was very enriching and allowed us to realize that since the earliest times Man has always tried to perceive the world around him and to interact with it. We would like to participate in more activities of this nature, since it gave us a different perception of mathematical knowledge.

2.3 Amateur Mathematicians - passions with limits? Simple problems, big challenges… (Helmuth Malonek)

In this lecture, Malonek always starts by talking about his taste for mathematics, emphasizing the difference between amateur mathematicians and professional mathematicians. He introduces and explores several examples of problems developed by amateur mathematicians. Among others, the "friendly numbers"and the famous Goldbach conjecture, which is one of the oldest unresolved problems in mathematics, which says "any even number greater than 2 can be represented by the sum of two prime numbers". It is known that computer-based techniques have already confirmed this conjecture; however, the actual mathematical demonstration has not yet been presented. During the lecture, Malonek interacts with the students by telling some stories about his experience as a mathematician, raising their interest. Furthermore, the school library, where the lecture occurs, fosters a taste for reading. Malonek presents the book of Apostolos Doxiadis *The Uncle Petros and the Goldbach Conjecture*, a bestseller of the year 2001 (Doxiadis, 2001). During this session some chapters of this book are addressed, the main topics of the book are highlighted, i.e., the passion for studies, the discovery of mathematics, learning persistence, and the importance of a pedagogical practice based on trial and error. At the end of the conference, the students were invited to give their opinion on the session (The session was held in the library of the High School of Caldas das Taipas with the students of the SE2 10th grade class). We quote a student:

> It was a very enjoyable session, where we had the privilege of being able to listen to an experienced and well-known mathematician and above all, to learn. We all became aware of the reason for this theme and also had the opportunity to learn a little more about various mathematicians who, while amateurs, made a valuable contribution to the development of mathematics.

3 Final Remarks

The project is now in a consolidation process. The number of conferences available to schools is increasing and each speaker goes, on average, to three or four schools each year. The majority of the students that attend to conferences are, typically, 16-18 years old but, in some cases, the attendants were younger (13-15 years old). Overall, this is a very low cost project (each school only provides, sometimes, transportation to the lecturers) and each school chooses the best way of implementing this project: in some schools, the conferences were given along the school year (usually on the same week day, typically two each month); in others, the conferences were given in special weeks, for instance, during "science weeks", integrated in other initiatives of the schools. In the figure below, we show that the three mathematical conferences were given on the same dayin the High School of Caldas de Taipas, integrated in the Math day of the *Semana de Ciência e Tecnologia* [Science and Technology Week].

Figure 3.1: Photo in upper left corner: from left to right, Hélder Pinto, Teresa Clain, Helmuth Malonek and Fernanda Carvalho (library coordinator of the highschool, Escola Secundária de Caldas das Taipas). Photo in the lower right corner: from left to right, Teresa Clain, Gorete Branco (coordinator of the math department of the highschool, Escola Secundária de Caldas das Taipas), Helmuth Malonek and Hélder Pinto. The lectures are announced on posters. In this case, by Americo Costa of the same highschool (the first three images of the bottom line).

With these mathematical conferences, we intend to change the students' perspective about mathematics and humanize the discipline. In the end, we hope that the history of mathematics can help students understand the following:

> Mathematics contains some very subtle devices that serve not only to satisfy those who are intrigued by mathematical problems but also to help with all practical and mechanical endeavors and to lessen men's labors. (Descartes, 1637, p.7).

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REFERENCES

Almeida, A. (1994). *Aritmética como descrição do real (1519-1679)*. Lisboa: Imprensa Nacional, Casa da Moeda.

Clain, T. C. (2016). Les règles d'un quarte et un vingtième et des comptes de Flandre comme modélisation du réel. In L. Radford, F. Furinghetti & T. Hausberg (Eds.), *HPM 2016: Proceedings of the 2016 ICME Satellite Meeting* (pp. 351-358). Montpellier: IREM de Montpellier.

Crato, N. (2003). O Instrumento de Sombras. In *Centro Virtual Camões – Ciência em Portugal, Personagens e*

Episódios. Retrieved from http://cvc.instituto-camoes.pt/ciencia/e32.html, accessed November 8, 2018).

Descartes, R. (1637). *Discours de la méthode pour bien conduire sa raison, et chercher la vérité dans les sciences*. Leyden.

Doxiadis, A. (2001). *Uncle Petros and Goldbach's Conjecture*. New York: Bloomsbury.

Fernandes, B. (1555). *Tratado da Arte de Arismetica*. Porto: Germão Galharde.

Katz, V. (Ed.) (2000). *Using History to Teach Mathematics: An International Perspective*. MAA Notes, vol. 51.Washington, DC: The Mathematical Association of America.

Mendes, R. (1540). *Pratica darismetica nouamente agora composta pelo licenciado Ruy Mendez: na qual se descraram por boa ordem e craro estilo as quatorze especias darte darismetica .&. as sete dellas por numeros inteyros e as outras sete por numeros quebrados: e assi mesmo trinta e cinco regras da dita arte muito sortil e breue e craramente decraradas*. Lisboa: Germão Galharde.

Nicolas, G. (1519 [1963]). *Tratado da Pratica d'Arismetica* [*facsmile*]. Portugal: Livraria Civilização.

Pinto, H. (2009). *História da Matemática na Sala de Aula*. Lisboa: Associação Ludus.

Pinto, H. (2010). The History of Mathematics in the classroom – some activities. In É. Barbin, M. Kronfellner, & C. Tzanakis (Eds.), *History and Epistemology in Mathematics Education: Proceedings of the Sixth European Summer University – ESU 6* (pp. 245-257). Wien: Holzhausen Verlag.

Seaquist, C., Seshaiyer, P., Crowley, D. (2005). Calculation Across Cultures and History. *Texas College Mathematics Journal*, *1*, 15-31.

Swetz, F. (1994). *Learning Activities from the History of Mathematics*. Portland: J. Weston Walch Publisher.