



Technical note

A new approach for the prediction of speed-adjusted pump efficiency curves

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ABSTRACT

Researchers and engineers commonly rely on modelling and simulation computer programs to improve the management, design and operational efficiencies in water supply systems. The reliability of some efficiency measures, however, is known to be highly dependent on the modelling accuracy of the selected modelling/simulation tool. Although a number of papers have been recently published related to the use of variable-speed pumps in water networks, there are no relevant studies on the analysis and improvement of the methods used to model the behaviour of these pumps. Consequently, the present work focuses on the methods used to model efficiency curves of variable-speed pumps. A new approach for the prediction of the speed-adjusted curves is proposed and preliminary results using two distinct pumps are presented and discussed. The proposed method is capable of satisfactorily predicting the behaviour of the pumps, demonstrating its potential to be effectively used in modelling tools.

Keywords: Hydraulic efficiency; hydraulic and pneumatic machinery; hydraulic structure design & management; modelling; pump design curves; water supply systems

1 Introduction

The interest in introducing variable-speed pumps in water supply systems (WSS) has grown steadily in recent years, mostly due to efficiency and sustainability related concerns (Coelho & Andrade-Campos, 2014a). Therefore, the number of papers regarding the efficient use of variable-speed pumps has also increased in recent years (e.g. Bene & Hős, 2012; Hashemi, Tabesh, & Ataekia, 2014; Moreira & Ramos, 2013). However, only a minor number of these papers focus on the analysis and quantification of the real effects of the use of variable-frequency drives (VFD) to improve the operational efficiency of water pumps (e.g. Marchi, Simpson, & Ertugrul, 2012). Nevertheless, when using simulation tools, such as the simulation software EPANET,¹ the information concerning the hydraulic and efficiency curves of the pumps when operating at variable speeds is needed. Very often, this information is obtained automatically by the hydraulic simulators through assumptions and approximations. One direct consequence of this may be that,

when performing cost analyses prior to the installation of VFD, the results may be strongly affected by those assumptions and approximations.

This Technical Note explores the use of existing methods to model the behaviour of variable-speed pumps (VSP), including the approximated numerical method followed by EPANET (Rossman, 2000), the hydraulic simulator most frequently used by the scientific community. With this in mind, a new approach for the modification of the efficiency of pumps with speed variation is proposed and compared with existing models. To verify and validate the proposed method, a quantitative analysis of the energy savings is performed using three different water distribution networks.

2 Modelling variable-speed pumps

To model the behaviour of pumps operating at variable speed resulting from the use of VFD it is necessary to adjust

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the pump's characteristic curve, the power and, consequently, the efficiency curves for each speed. Affinity laws (AL) are commonly used to predict these curves (e.g. Quintela, 1981; Rossman, 2000; Walski, Zimmerman, Dudinyak, & Dileepkumar, 2003). These laws state that pump flow, head and power can be described, respectively, by the following linear, quadratic and cubic functions of the pump speed ratio:

$$\frac{Q_1}{Q_2} = \frac{N_1}{N_2}, \quad \frac{H_1}{H_2} = \left(\frac{N_1}{N_2}\right)^2, \quad \frac{P_1}{P_2} = \left(\frac{N_1}{N_2}\right)^3 \quad (1)$$

where N_1 and N_2 are two pump speeds ($N_i = 2\pi\omega/60$ is the rotational speed, in rpm, and ω is the angular velocity, in rad s^{-1}). The above equations assume that, at the best efficiency point (BEP), the pump efficiency is independent of the pump speed. Thus, the efficiency curve is only moved to the left when the pump speed is reduced or, consequently, moved to the right when increased (Marchi & Simpson, 2013). This is the main drawback of using AL since the real behaviour of pumps shows a decrease of the BEP with the speed reduction.

The effects of varying speed on the flow, head, power and efficiency of centrifugal pumps are shown in Fig. 1a (adapted

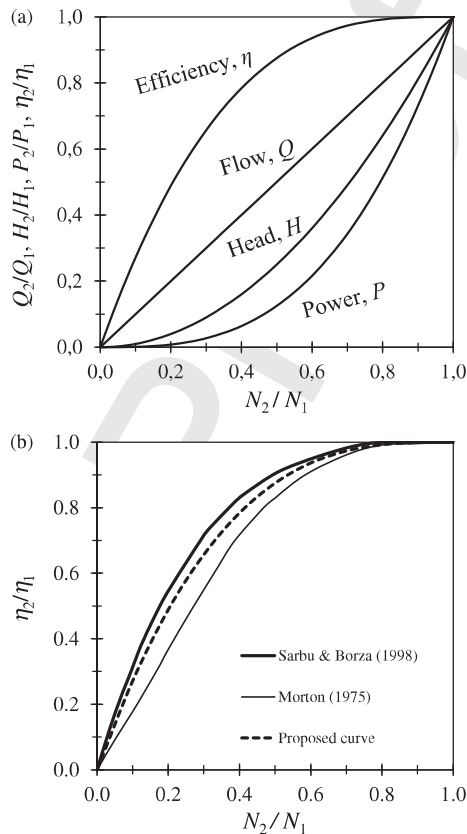


Figure 1 (a) Design flow ($Q = Q_2/Q_1$), head ($H = H_2/H_1$), power ($P = P_2/P_1$) and efficiency ($\eta = \eta_2/\eta_1$) of a centrifugal pump as a function of the speed ratio (approximation) and (b) comparison between the proposed efficiency variation curve with the exact solutions by Morton (1975) and Sárbu and Borza (1998)

from Morton, 1975; Sárbu & Borza, 1998). As can be seen, the flow, head and power curves change with the AL. However, this efficiency curve is in fact an approximation of the curves proposed by those authors. An exact representation of these curves is shown in Fig. 1b.

It is not possible to accurately analyse the relationship between the efficiency of the pump and the speed given that Morton (1975) only provided a graphical representation. Nonetheless, it should be stated that such a representation should only be considered accurate for pump speeds greater than 50% of the full-speed (Morton, 1975). Below that speed threshold, the reduction of efficiency often becomes more evident for smaller pumps and less evident for larger ones. Taking this into account, Sárbu and Borza (1998) proposed the following equation describing the effect of the variation of speed on the efficiency of the pumps:

$$\eta_2 = 1 - (1 - \eta_1) \left(\frac{N_1}{N_2}\right)^{0.1} \quad (2)$$

where η_1 and η_2 are the original and the speed-adjusted (i.e. the efficiency of the pump at speed N_2) efficiencies, respectively. This equation, as stated by Simpson and Marchi (2013), is an approximation to the equation proposed by Gulich (2003) and initially introduced by Osterwalder and Hippe (1984), for scaling efficiency curves in hydraulically smooth surfaces, which is dependent on the Reynolds number.

Interestingly, Eq. (2) can be used to predict distinct efficiency curves for different original efficiencies η_1 . In fact, its mathematical formulation does not describe the graphical representation presented by its authors (Fig. 1b). The curves obtained by replacing the original efficiency by different values in Eq. (2) are the represented in Fig. 2. As stated by Sárbu and Borza (1998), this formulation can lead to negative efficiency values, which is a disadvantage. Additionally, the formulation describes a sharper decrease of the efficiency with the reduction of the speed for lower η_1 efficiencies (Fig. 2).

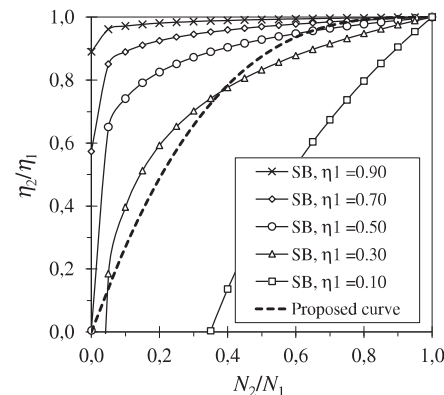


Figure 2 Variation of the efficiency with the speed ratio for different original efficiencies, η_1 , using the relation proposed by Sárbu and Borza (1998) (SB) and comparison with the method proposed in this work

221 Walski et al. (2003) also performed experiments using a
 small pump (nominal flow of 0.631 s^{-1} and head of 0.6 m)
 for speed reductions of up to approximately 30% of the full-
 speed. They analysed changes in the efficiency curve when
 using variable-speed drives (VSD) and concluded that changes
 226 similar to the AL were observed. However, for lower efficien-
 cies, a deviation relative to the AL curve was expected to be
 observed. Nevertheless, as stated by Walski et al. (2003), the
 proposed results are not representative since the experiment was
 performed with only one small size pump.

231 2.1 Prediction of the speed-adjusted efficiency

A method to predict the speed-adjusted efficiency is proposed
 in this paper, based on the graphical representation of the pump
 power function in Fig. 1a. This approach will lead to an equation
 236 describing a graphical representation of the speed-adjusted effi-
 ciency similar to the curves proposed by Morton (1975) and
 Sárbu and Borza (1998) (Fig. 1b).

The cubic dependency of the pump power ($P = P_2/P_1$) with
 241 speed ratio ($N = N_2/N_1$) can be seen in Fig. 3, quadrant I (in
 grey, thick solid line). Extending this power function into quad-
 rant III, the shape of the curve becomes what should be expected
 for the variation of the efficiency with the speed ($\eta = \eta_2/\eta_1$).
 Thus, starting from the power function $P(N)$, displacing it as
 246 in step A ($N \rightarrow N + 1$) and again as in step B ($P \rightarrow P + 1$),
 leads to a function that describes the variation of the efficiency
 of the pump with the speed (dotted line). Mathematically, this
 procedure can be described as:

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$$\eta(N) = P(N - 1) + 1 \quad (3)$$

Considering the affinity law for power (Eq. (1c)) and
 knowing that $P = P_2/P_1$, $N = N_2/N_1$ and $\eta = \eta_2/\eta_1$, Eq. (3)

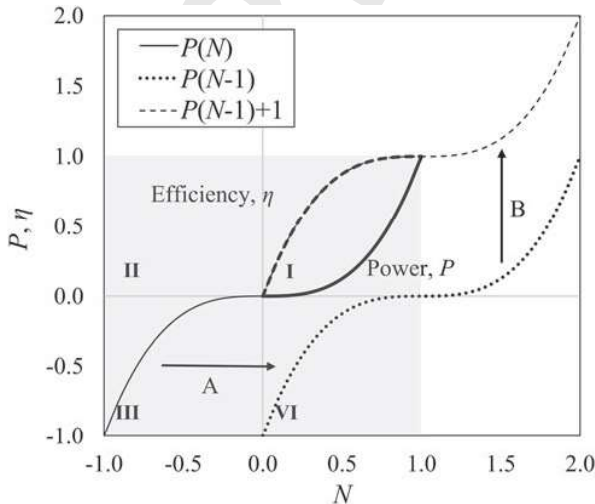


Figure 3 Proposed method for the prediction of the speed-adjusted efficiency. $P(N - 1)$ is a horizontal translation of $P(N)$ and $P(N - 1) + 1$ is a vertical translation of $P(N - 1)$

becomes

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$$\frac{\eta_2}{\eta_1} = \left(\frac{N_2}{N_1} - 1 \right)^3 + 1 \quad (4)$$

which is a new formulation for the efficiency of the speed-adjusted pump.

The efficiency method proposed by the authors, described above, approximates quite well the results and observations presented by Morton (1975) and Sárbu and Borza (1998), as can be seen in Fig. 1b. On the one hand, this new method never leads to negative efficiency values, which is a clear improvement on the SB formulation. On the other hand, however, regardless of the original efficiency considered, the dependency of the efficiency on the speed is unchanged, that is, the speed-adjusted curve is always the same independently of the original efficiency (see Fig. 2).

3 Results and discussion

The method proposed in this paper (see Section 2.1 above) for the prediction of speed-adjusted efficiencies was validated with real data and compared with (i) the AL; (ii) the formulation proposed by Sárbu and Borza (1998) (SB); and (iii) the method implemented in EPANET 2.0 (EPA), which considers a constant efficiency curve. All methods were also tested with a set of benchmark networks of pumps in order to verify the effect in the computation of energy savings from the use of VSD.

3.1 Performance assessment

The experimental data in Table 1 describes the BEP of a large and a small pump, and is used to compare to predictions obtained with each method.

For the large pump, where the speed is reduced from 1525 to 1182 rpm, the SB approach provides the best results. However, for the smaller pump, where changes in efficiency are often more significant and difficult to predict, and considering a reduction in speed from 3600 to 2000 rpm, the proposed method provides the best prediction of the efficiency.

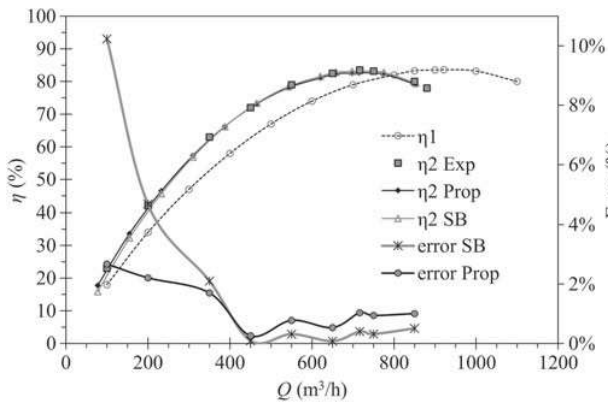
This comparison is based only on the BEP values and is thus not reliable for pumps operating at different speeds and flow rates. In such cases more efficiency points should be evaluated. A comparison between the different methods, considering several points of the available curve for the large pump was also performed and is shown in Fig. 4.

Although the proposed method gives the largest absolute difference in the BEP prediction, the efficiency curve fits well the experimental observations, with an average error of 1.2%, as can be seen in Fig. 4. However, the curve obtained with the hydraulic simulator EPANET (η_1 in Fig. 4), with a BEP difference of only 0.12%, cannot accurately predict the experimental values for η_2 . This also shows that the BEP value cannot be used to evaluate hydraulic efficiency curves. The SB formula

331 Q1 Table 1 Experimental and predicted values for the best efficiency point (BEP) of two real pumps, considering distinct prediction methods: (i) 386
the affinity laws (AL); (ii) the formula proposed by Sárbu and Borza (1998) (SB); (iii) the method used by EPANET 2.0 (EPA); and (iv) the
method proposed in this work (Prop.)

Pump type	Power (kW)	Experimental			Prediction			Difference (%)			
		BEP at N_1 (η_1)	BEP at N_2 (η_2)	BEP AL (η_2)	BEP SB (η_2)	BEP EPA (η_2)	BEP Prop. (η_2)	AL	SB	EPA	Prop.
Large ^a	556	83.60	83.50	83.60	83.18	83.60	82.65	0.12	-0.39	0.12	-1.02
Small ^b	5.5	56.00	52.00	56.00	53.34	56.00	51.08	7.69	2.57	7.69	-1.76

341 ^aSulzer pump HPL 54-30-20; data reported by Ulanicki, Kahler, and Coulbeck (2008). $N_1 = 1525$ rpm, $N_2 = 1182$ rpm. ^bPump 50-32-160 396
HT, from TKL catalogue (1989); data reported by Simpson and Marchi (2013). $N_1 = 3600$ rpm, $N_2 = 2000$ rpm.



356 Q4 Figure 4 Comparison between the experimental efficiency curve of the large pump (Ulanicki et al., 2008) (Exp) and the predicted curves 411
357 Q7 using the prediction methods proposed by Sárbu and Borza (1998) (SB) and the method proposed method in this work (Prop)

361 also fits the experimental results well, with an average error of 2.1%. However, the main differences between the SB and the 416
proposed method occur most evidently at the lower efficiency points, where the pump often does not operate.

366 It can be concluded that the proposed method has an accuracy similar to the SB formula considering the previous preliminary 421
371 results obtained with the presented amount of data existing in the literature. Although both approaches are significantly more 426
376 accurate than the one used by EPANET, real data provided by pump manufacturers is necessary in order to obtain more 431
conclusive results regarding the best possible alternatives for the replacement of low-accuracy methods commonly used in hydraulic simulators. This lack of information has also been mentioned by researchers dealing with efficiency studies concerning the use of pumps as turbines (PAT) for energy recovery in WSS (Carravetta, Fecarotta, Martino, & Antipodi, 2014).

3.2 Impact on the computation of energy savings

381 For the analysis of different networks under different operational conditions, the optimization methodology presented by Coelho 436
and Andrade-Campos (2013) was used to obtain the best operational conditions using the VSD (i.e. the best pump controls 437
in order to minimize the associated energy costs). This was

done by assuming that the pumps are initially operating at a nominal constant speed n , and then using a VSD to allow for a range of pump speeds. This methodology uses EPANET for the simulation of the networks and the computation of energy costs. However, an adjustment to the power computation was performed in order to implement the discussed and proposed methods for the prediction of the efficiency. All simulations presented in this paper consider one day, divided in 24-h steps.

Single-pump water supply system

The proposed method is tested and validated with the network shown in Fig. 5a. The system's head curve, the efficiency and head curves of the pump are shown in Fig. 5b. The operating

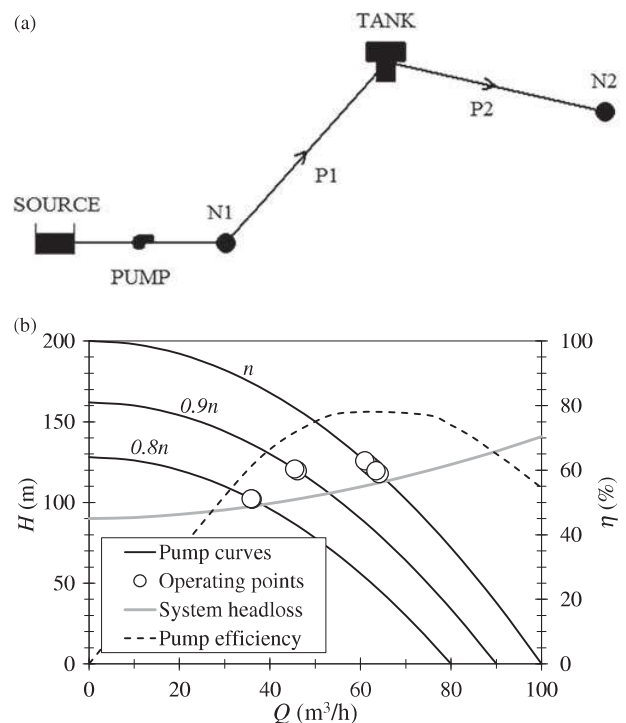
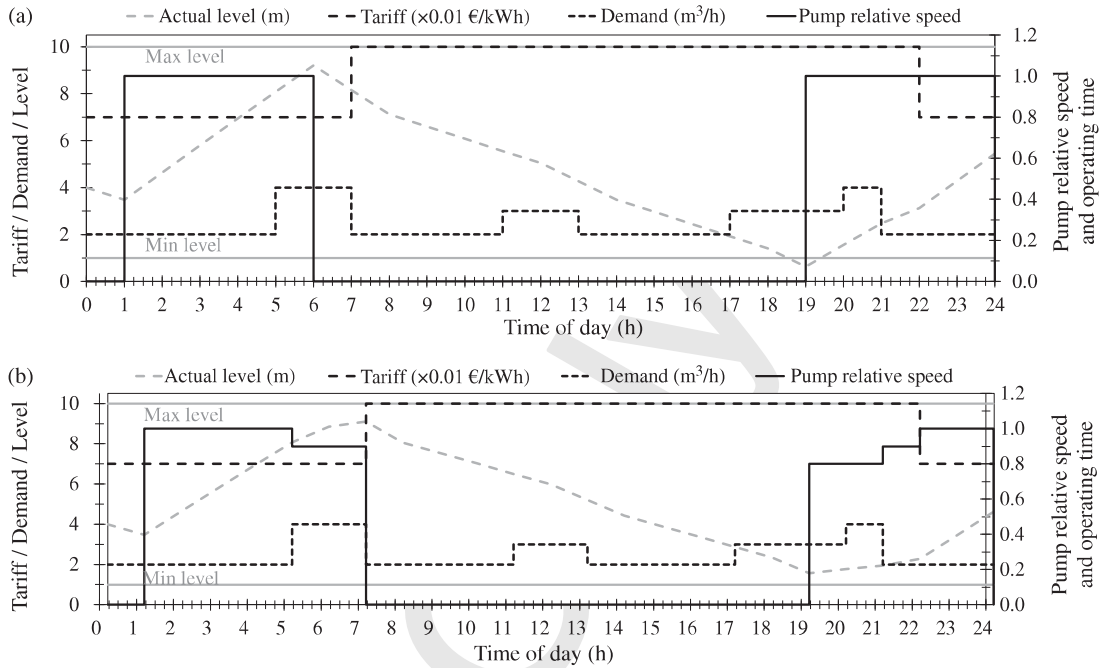


Figure 5 (a) EPANET model of the single-pump network and (b) system headloss, pump nominal efficiency (the curve used by EPANET for all speeds n , $0.9n$ and $0.8n$) and pump characteristic curves (at nominal speeds n , $0.9n$ and $0.8n$)

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Figure 6 Operational conditions of the single-pump network for the 24 h simulation with the pump operating at (a) a nominal constant speed and (b) at variable speed

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points shown correspond to the best operational condition found using the pump at variable speeds. In this network, both pipes P1 and P2 have lengths of 2000 m, diameters of 200 mm and Hazen–Williams roughness coefficients of 50. The minor head-loss coefficient was set to zero in order to ignore such losses and a variable water demand was associated to node N2 (Fig. 6). The resulting operational conditions of the network, including the water levels in the tank, considering the pump at fixed and variable speed, are shown in Fig. 6a and b, respectively.

The energetic performance and cost efficiency of the single-pump network under distinct operational conditions (fixed and

variable speed) are listed in Table 2. The savings obtained from the use of variable speed instead of the pump’s nominal constant speed are also provided.

As expected, the obtained results confirm that EPANET does not accurately compute the efficiency of the pump. This simulator clearly underestimates the savings obtained when using a variable-speed pump (4% of daily energy savings against approximately 7% of savings computed by the remaining methods). It should be said, however, that for different set-ups (with, for example, a different pump or system head curve) the results obtained with EPANET can overestimate the real savings.

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Table 2 Results obtained from the simulation of the single-pump network considering distinct methods to predict the efficiency of the pump: (i) the affinity laws (AL); (ii) the formula proposed by Sárbu and Borza (1998) (SB); (iii) the method used by EPANET 2.0 (EPA); and (iv) the method proposed in this work

Type of control	Efficiency curve	Avg eff. (%)	Avg power (kW)	Daily cost (€)	Avg energy (kWh m ⁻³)	Daily energy (kWh)
Fixed speed	Nominal	77.82	26.61	21.01	0.424	266.14
Variable speed	AL	75.86	22.56	18.82	0.419	248.14
	SB	75.66	22.60	18.87	0.420	248.62
	EPA	73.25	23.21	19.47	0.436	255.34
	Proposed	75.73	22.58	18.85	0.420	248.43

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Savings from pump speed variation (%)

Method	Avg power	Daily cost	Avg energy	Daily energy
AL	15.24	10.40	1.23	6.76
SB	15.07	10.19	0.97	6.58
EPA	12.78	7.34	-2.66	4.06
Proposed	15.14	10.27	1.06	6.65

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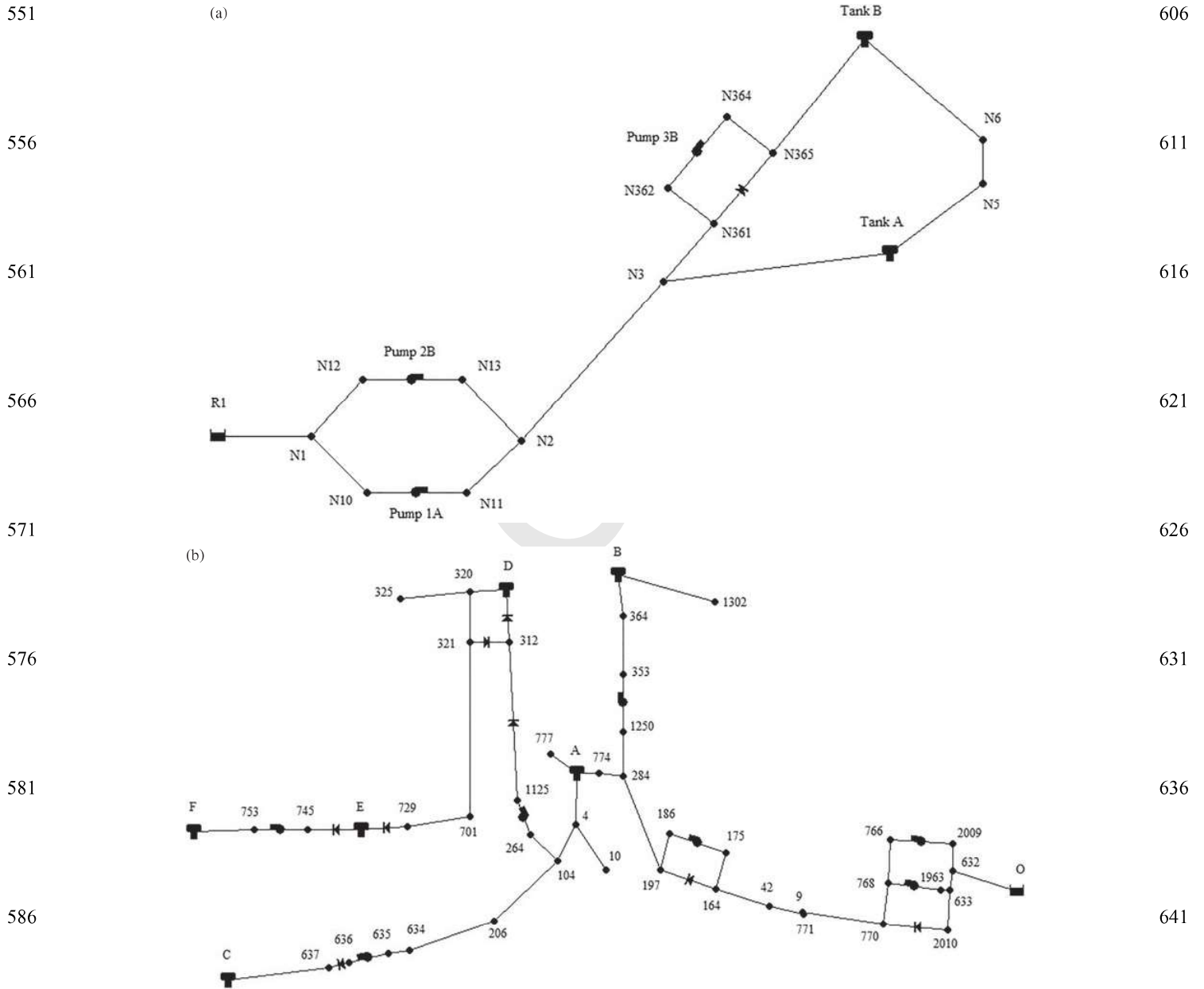


Figure 7 EPANET model of (a) the van Zyl test network (van Zyl et al., 2004) and (b) the Richmond simplified network (University of Exeter, 2014)

The proposed method produced results that are similar to those obtained using the AL and SB methods. It should also be noted that the operational conditions coincide with the region where this method approximates the other methods, i.e. the pump speed is always above 80% of the nominal speed and the original pump efficiency (η_1) is higher than 0.7.

Van Zyl and Richmond water supply systems

The same approach described in the previous section was also applied on the two benchmark networks shown in Fig. 7a and b, here designated by van Zyl and Richmond networks, respectively. The Richmond network, part of the Yorkshire water supply area in the UK, is a benchmark of

the Centre for Water Systems Resources of the University of Exeter and can be retrieved from their website (University of Exeter, 2014). Details concerning the van Zyl benchmark can be found in van Zyl, Savic, and Walters (2004) or Coelho and Andrade-Campos (2014b). The computed daily energy costs for both benchmarks at two distinct operational conditions, i.e. with pumps running at fixed and at variable speed, considering the four methods previously described for the speed-adjusted efficiency computation, are listed in Table 3.

In the described benchmarks, the daily energy cost computed by EPANET 2.0 also overestimates the values obtained with different speed-adjusted efficiency curves. The savings from

661 Table 3 Results for the two benchmark networks considering distinct pump efficiency prediction methods: (i) the affinity laws (AL); (ii) the approach proposed by Sárbu and Borza (1998) (SB); (iii) the method used by EPANET 2.0 (EPA); and (iv) the method proposed in this work

666	Network	Type of control	Eff. curves	Daily cost (£)	Savings (%)
	van Zyl	Fixed speed	Nominal	345.24	—
			Variable speed	AL	222.69
			SB	222.23	35.63
			EPA	231.16	33.04
			Proposed	221.27	35.91
671	Richmond	Fixed speed	Nominal	15,632.72	—
			Variable speed	AL	12,308.10
			SB	12,310.10	21.25
			EPA	12,403.06	20.66
			Proposed	12,320.30	21.19

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the use of VSD are, however, underestimated by the hydraulic simulator.

681 4 Concluding remarks

It is crucial to use accurate models to assess the effectiveness of implementing efficiency measures in water supply systems. The analytical formulation proposed in this paper, which can be used to predict pump efficiency curves with a reduction of speed, proved to be an appropriate choice when considering variable-speed pumps. This new method is an alternative to existing approaches. However, more experimental data, including efficiency curves of pumps of distinct types and sizes, operating at a wider range of speeds, are necessary in order to obtain more reliable and definitive conclusions on the efficiency of the proposed method.

Using different efficiency prediction methods can have a considerable effect in lowering the computational costs associated to the use of variable-speed drives in water supply systems.

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Note

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1. EPANET is the hydraulic simulator developed by the U.S. Environmental Protection Agency (EPA). For more details, see Rossman (2000).

711 Notation

AL = affinity laws
BEP = best efficiency point

EPA = EPANET 2.0 716

 H = head (m) N = relative speed or speed ratio of the pump (—) P = power of the pump (W)

Prop = proposed method

 Q = flow ($\text{m}^3 \text{ s}^{-1}$) 721

SB = Sárbu and Borza (1998)

 η = efficiency of the pump (%) ω = angular velocity of the pump (rad s^{-1})

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