On the Dynamics of Accelerated Observers in Special Relativity Theory

P. Martins[1,2*], L. Nero Alves[1,2], J. C. Mendes[2]

[1]Departamento de Electrónica, Telecomunicações e Informática, Universidade de Aveiro, Portugal

[2]Instituto de Telecomunicações, Aveiro, Portugal

*Corresponding author Email: dias.martins@ua.pt

This paper investigates the dynamics of the motion of accelerated observers in Special Relativity (SR) theory. The presented discussion highlights the memristive character of the force dynamics. This description contrasts with the standard phase-space representation used in Physics.

In SR, the analysis of the effects of acceleration is accomplished using a specific set of reference frames: an inertial reference frame considered at rest at the origin of the space-time coordinates, and a co-moving reference frame [1]. In the inertial reference frame, the 4-vector $X^{\mu}=(x, y, z, t)$ represents the space-time position vector of the object. The velocity and acceleration 4-vetors are defined in the usual way, taking the derivative against proper time of the X^{μ}. It is simple to demonstrate that the 4-vector of force, is given by:

$$F^{\mu} = \left(\gamma^4 m\vec{a}, \gamma^4 \frac{m}{c}\vec{a}\cdot\vec{v}\right) \qquad \vec{a} = \frac{d\vec{v}}{dt}$$

where, $\gamma=(1-(v/c)^2)^{-1/2}$, m is the mass of the object, $\vec{v}$ is the 3-vector of velocity, $\vec{a}$ its 3-vector of acceleration and c is the velocity of light. It is apparent from this equation, that all the components of F^{μ} have memristive dynamics, with velocity taking the role of state variable [2]. Figure 1 depicts the characteristic hysteresis loop, present in the spatial components of F^{μ}. The acceleration assumed a sinusoidal time dependence, with maximum acceleration of $10m/s^2$. Figure 1 also shows that the force dynamics forms a closed path without self-crossings in the 3-dimensional (v, a, F) space. Figure 2 shows the phase-space representation of the motion under the framework of the twin paradox [3]. The total trip forms a closed trajectory in phase-space without self-crossings.

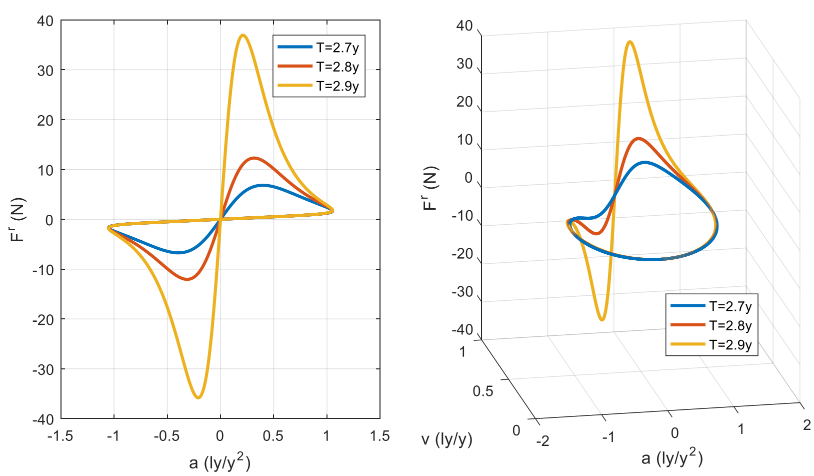

Figure 1: State-space representation: left (a, F), right (v, a, F)

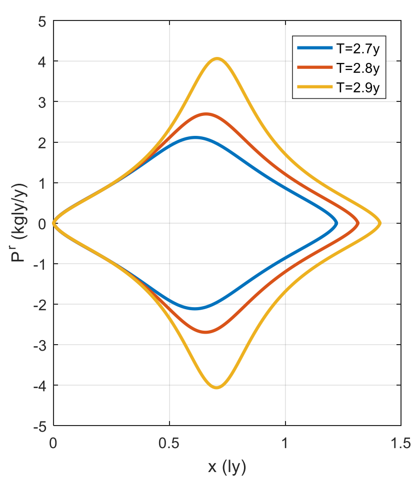

Figure 2: Phase-space representation

References

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