Pavimentar a esfera

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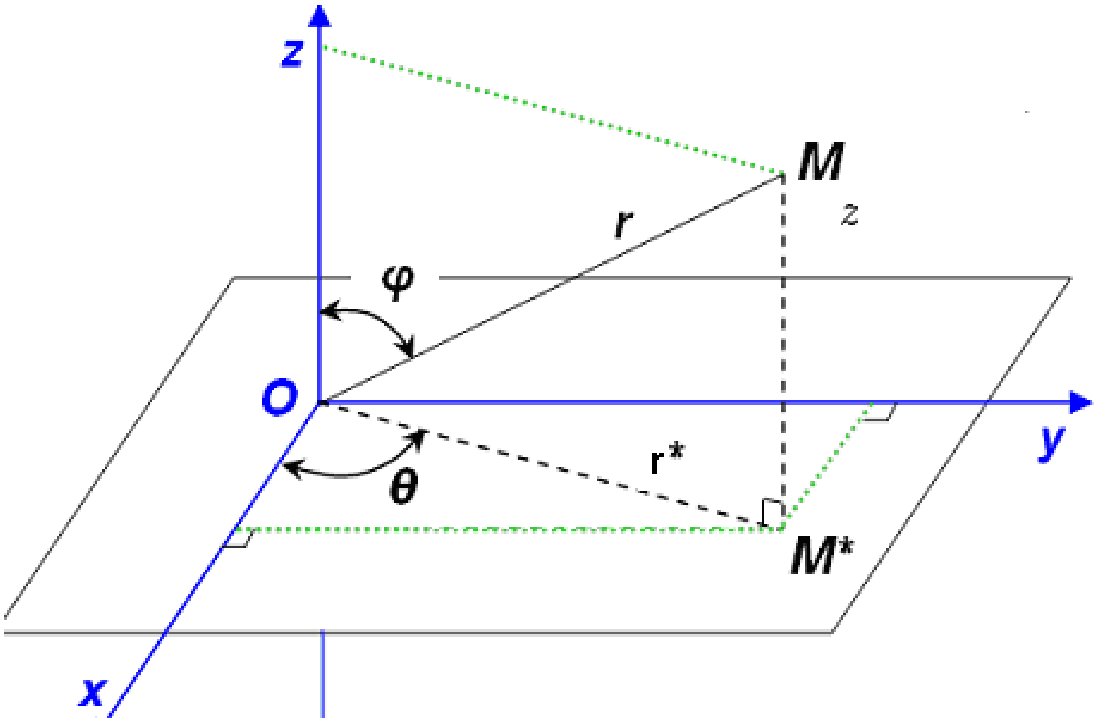
Lisboa 2015

1. Coordenadas Esféricas

$$(r, \theta, \varphi)$$



$$M = (x, y, z)$$

$$M^* = (x, y, 0)$$

$z = r \sin(90 - \phi) = r \cos\phi$
$r^* = r \cos(90 - \phi) = r \sin\phi$

$$x = r^* \cos\theta = r \sin\phi \cos\theta$$
$$y = r^* \sin\theta = r \sin\phi \sin\theta$$
$$z = r \cos\phi$$

$$r \in [0, \infty), \theta \in [0, 2\pi], \phi \in [0, \pi]$$

Coordenadas Esféricas ⟷ Coordenadas Cartesianas

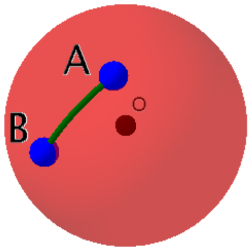
$\boldsymbol{M} = (r, \theta, \varphi)$ → $\boldsymbol{M} = (x, y, z)$

$$x = \mathbf{r}^{*}\cos\theta = r\ \sin\phi\ \cos\theta$$
$$y = \mathbf{r}^{*}\sin\theta = r\ \sin\phi\ \sin\theta$$
$$z = r\ \cos\phi$$

$\boldsymbol{M} = (x, y, z)$ → $\boldsymbol{M} = (r, \theta, \varphi)$

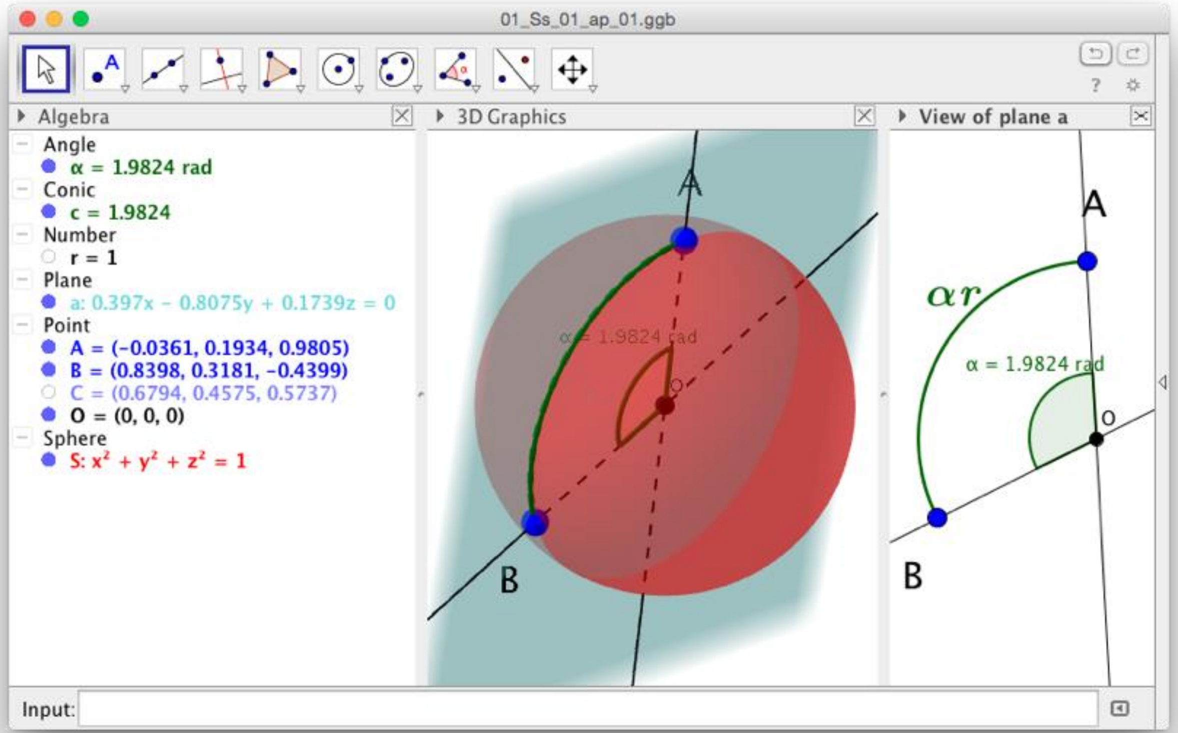
$$r = \sqrt{x^2 + y^2 + z^2}$$
$$\phi = \arctan\frac{y}{x}$$
$$\theta = \arctan\frac{\sqrt{x^2 + y^2}}{z}$$

2. A ferramenta segmento esférico



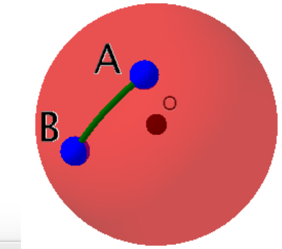
01_Ss_01_t.ggb
01_Ss_01_it.ggt

Questão: Dados dois pontos A e B sobre a esfera construir o arco ou segmento esférico que une A a B.



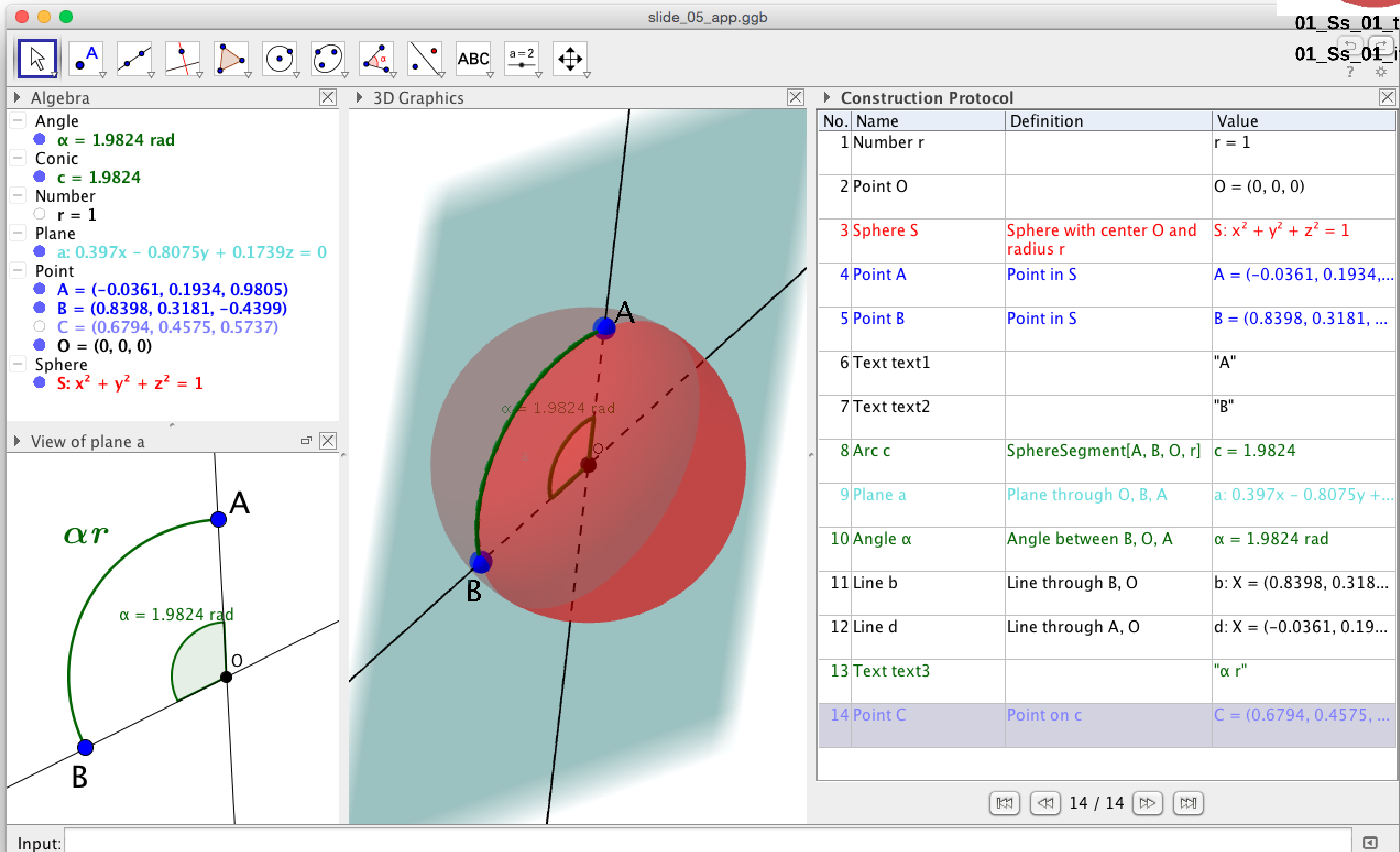
slide_04_app.ggb

2. A ferramenta segmento esférico



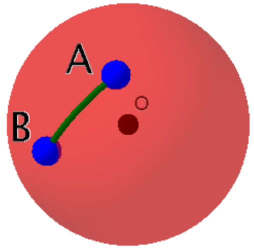
01_Ss_01_t.ggb

01_Ss_01_it.ggt

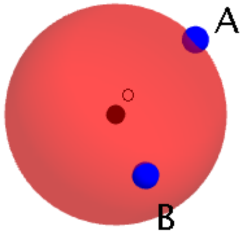


slide_05_app.ggb

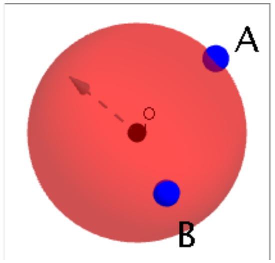
2. A ferramenta segmento esférico



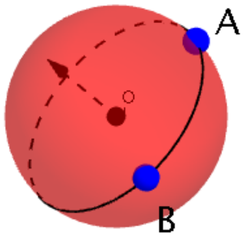
01_Ss_01_t.ggb
01_Ss_01_it.ggt



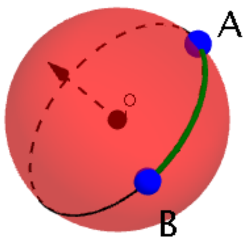
r=1O=(0,0,0)S=Sphere[O, r]
A=PointIn[S]
B=PointIn[S]



PerpendicularVector[Plane[A, O, B]]



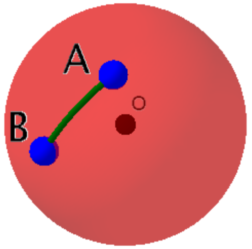
Circle[O, B, PerpendicularVector[Plane[A, O, B]]]



Arc[Circle[O, B, Vector[PerpendicularVector[Plane[A, O, B]]]], B, A]

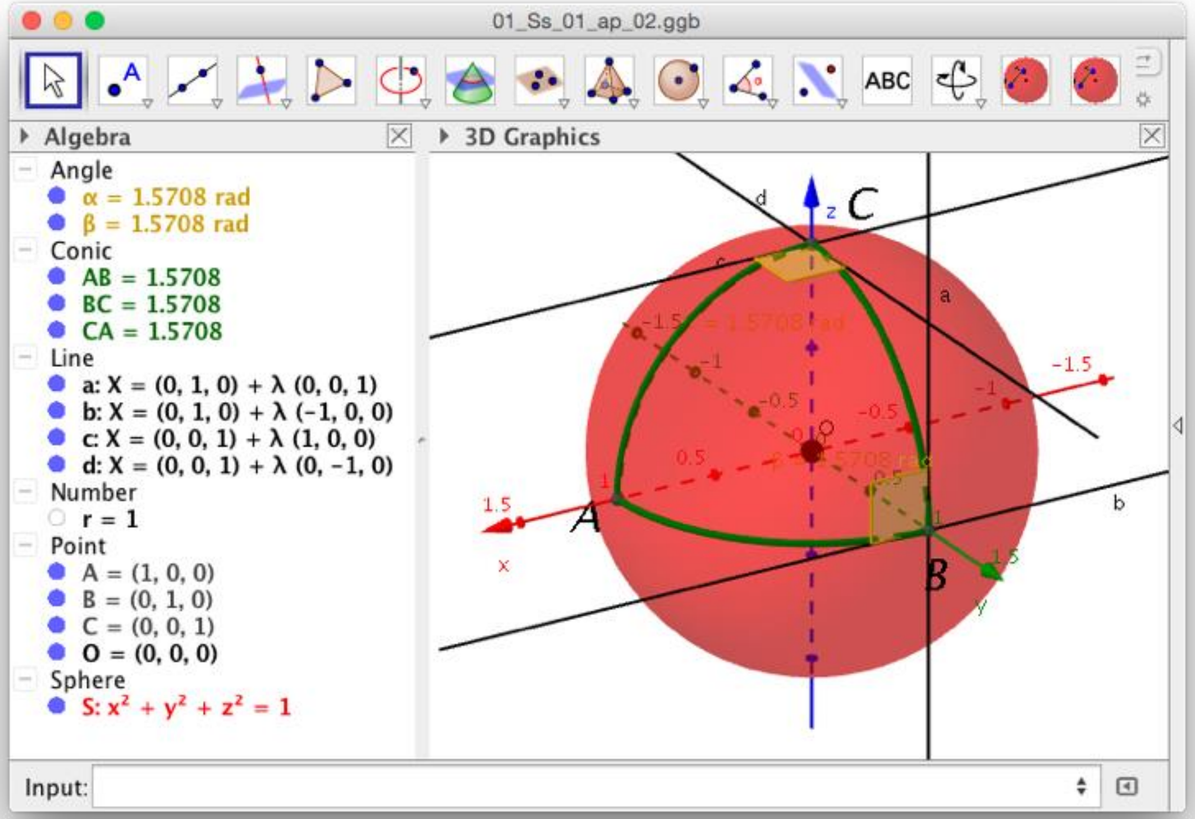
SphereSegment[A, B, O, r] ou SegmentoEsférico[A,B,O,r]

3. Triângulos esféricos, lados



01_Ss_01_t.ggb
01_Ss_01_it.ggt

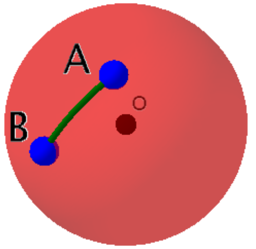
Triângulo esférico ABC, onde A, B e C correspondem à intersecção, no primeiro octante, da esfera de centro em O=(0,0,0) com os eixos coordenados.



slide_07_app.ggb

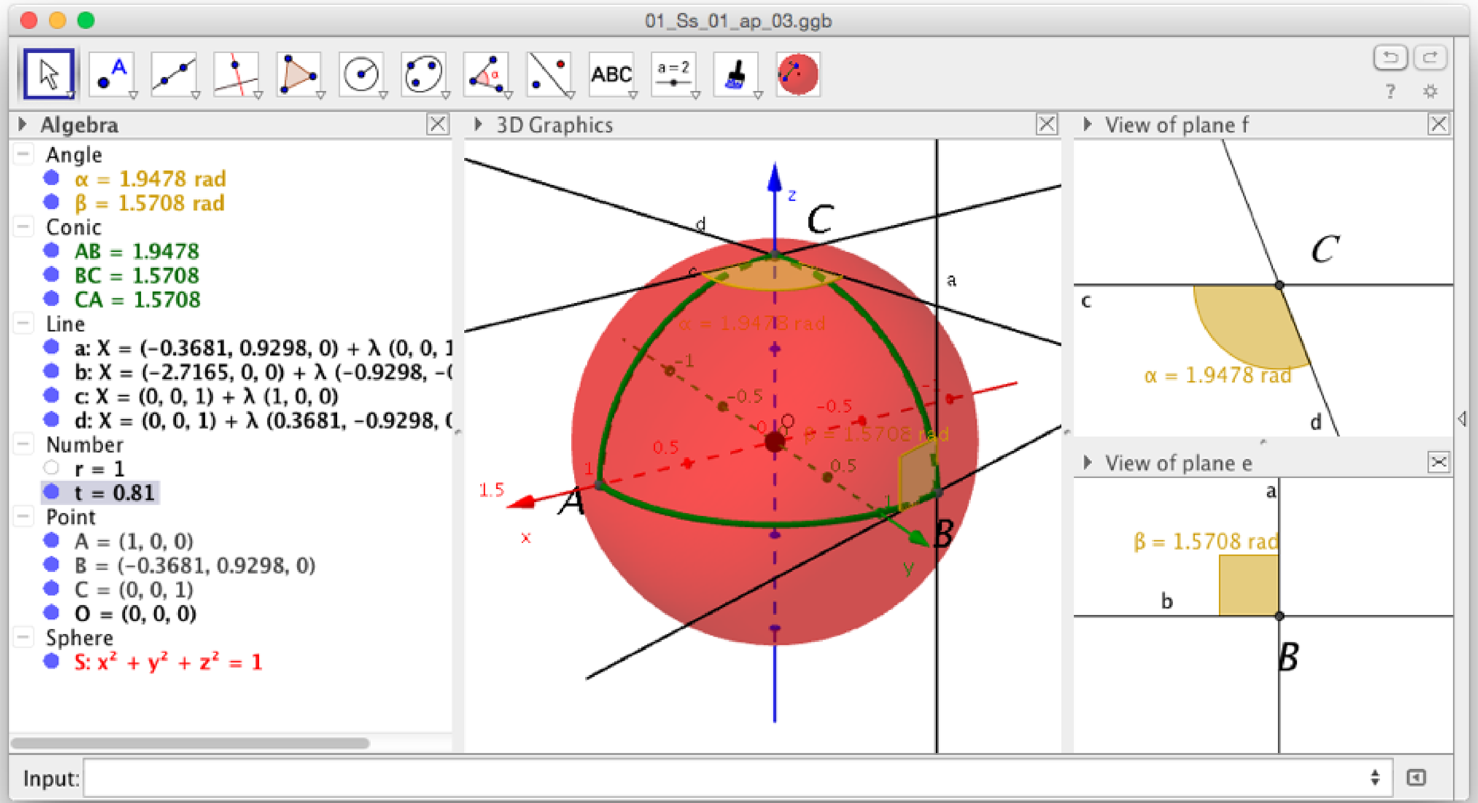
Trata-se de um triângulo equilátero onde todos os lados esféricos medem π/2 (1,50708; 5CD), e em que os seus ângulos são todos retos, isto é medem π/2 rad.

4. Triângulos esféricos, ângulos



01_Ss_01_t.ggb
01_Ss_01_it.ggt

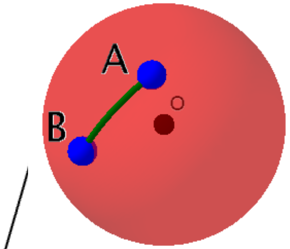
Triângulo esférico ABC, com o ponto C situado num dos polos relativos ao equador definido por A e B.

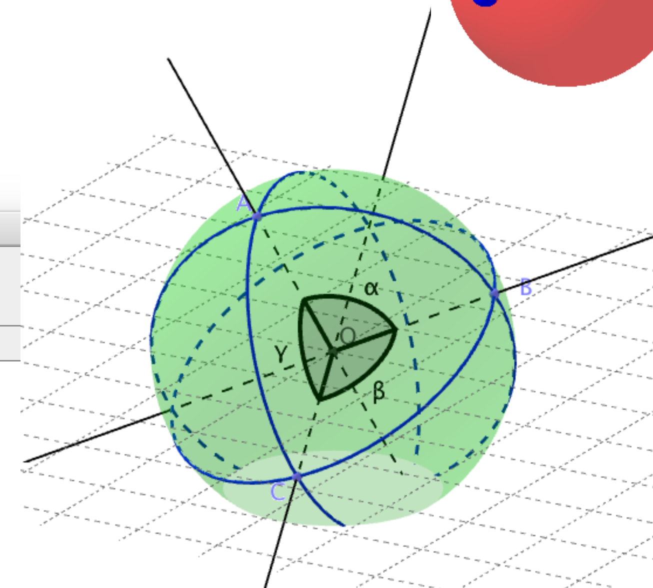


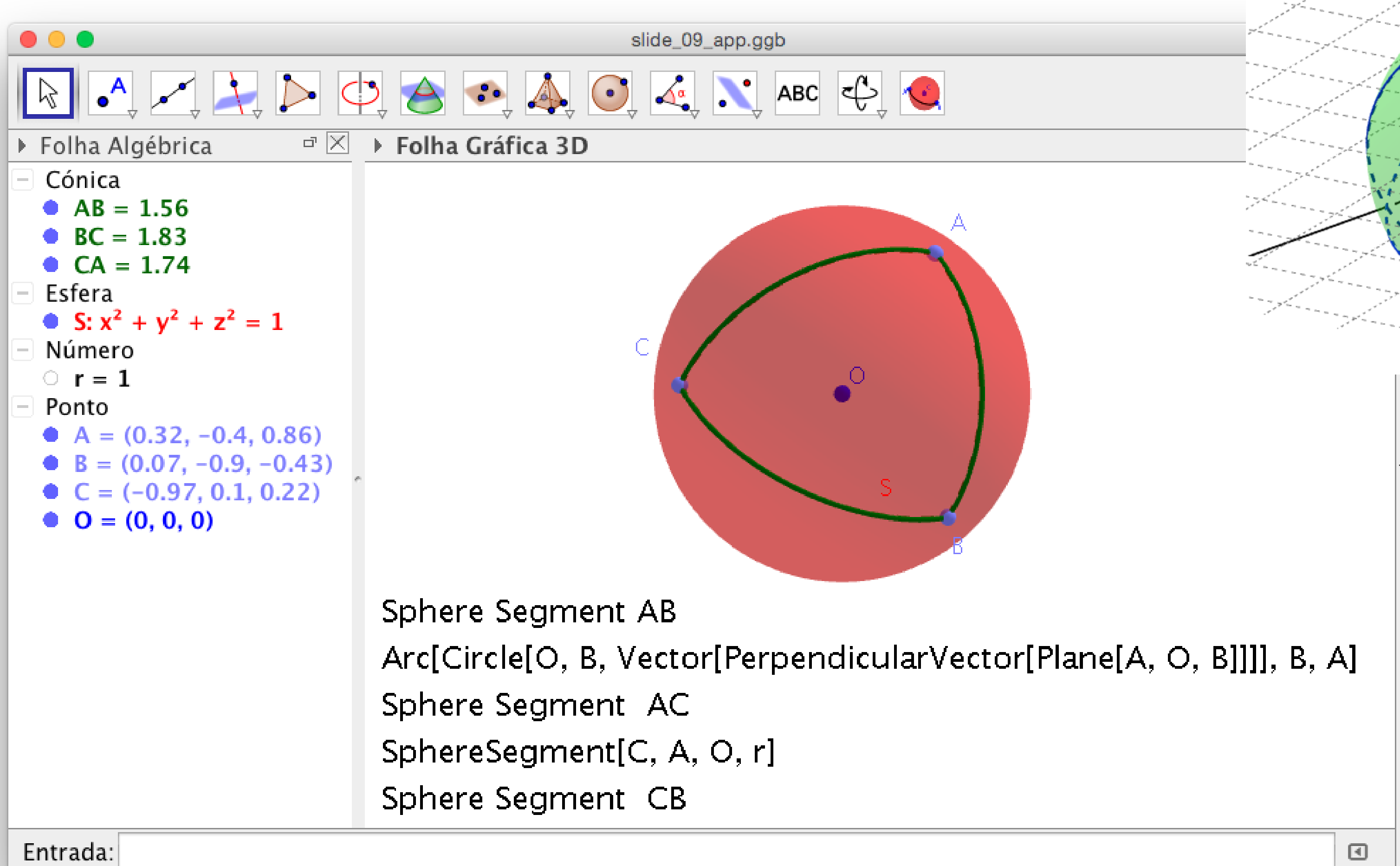
01_Ss_01_ap_03.ggb

Trata-se de um triângulo isósceles em que dois dos seus lados medem π/2 (1,50708; 5CD), e dois dos seus ângulos são retos, medem π/2 rad.

5. Um triângulo genérico na esfera

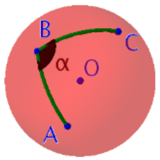


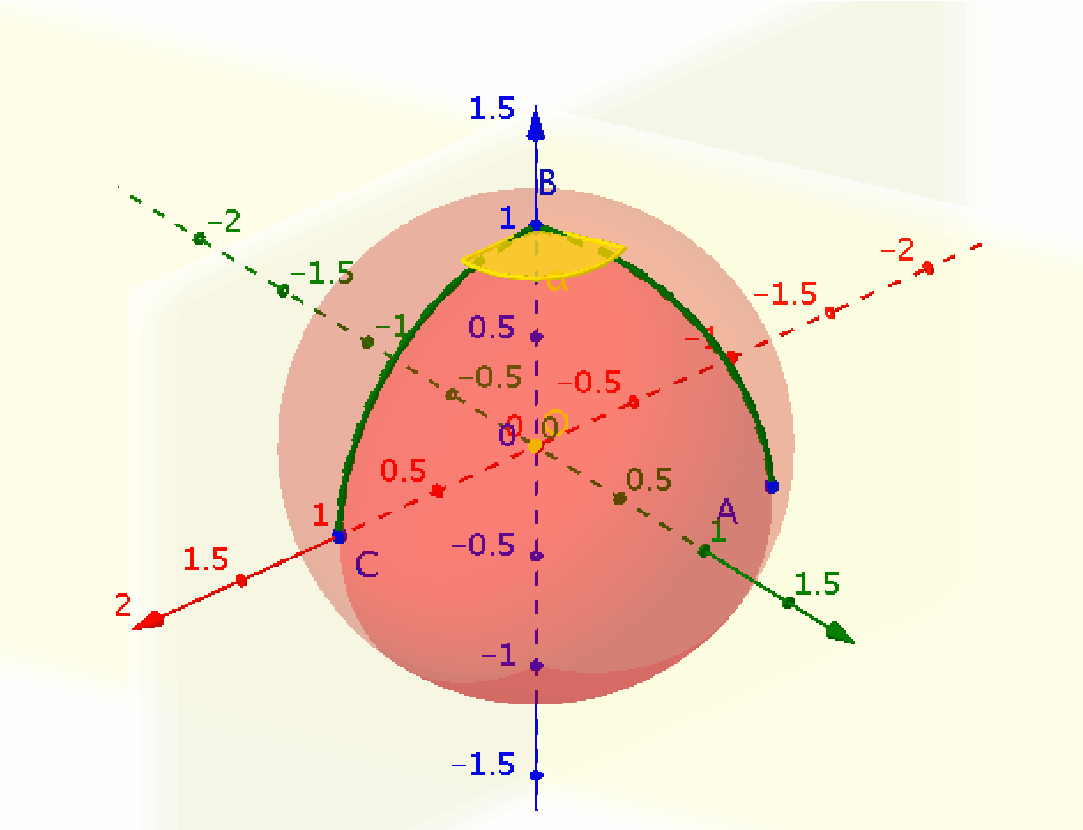




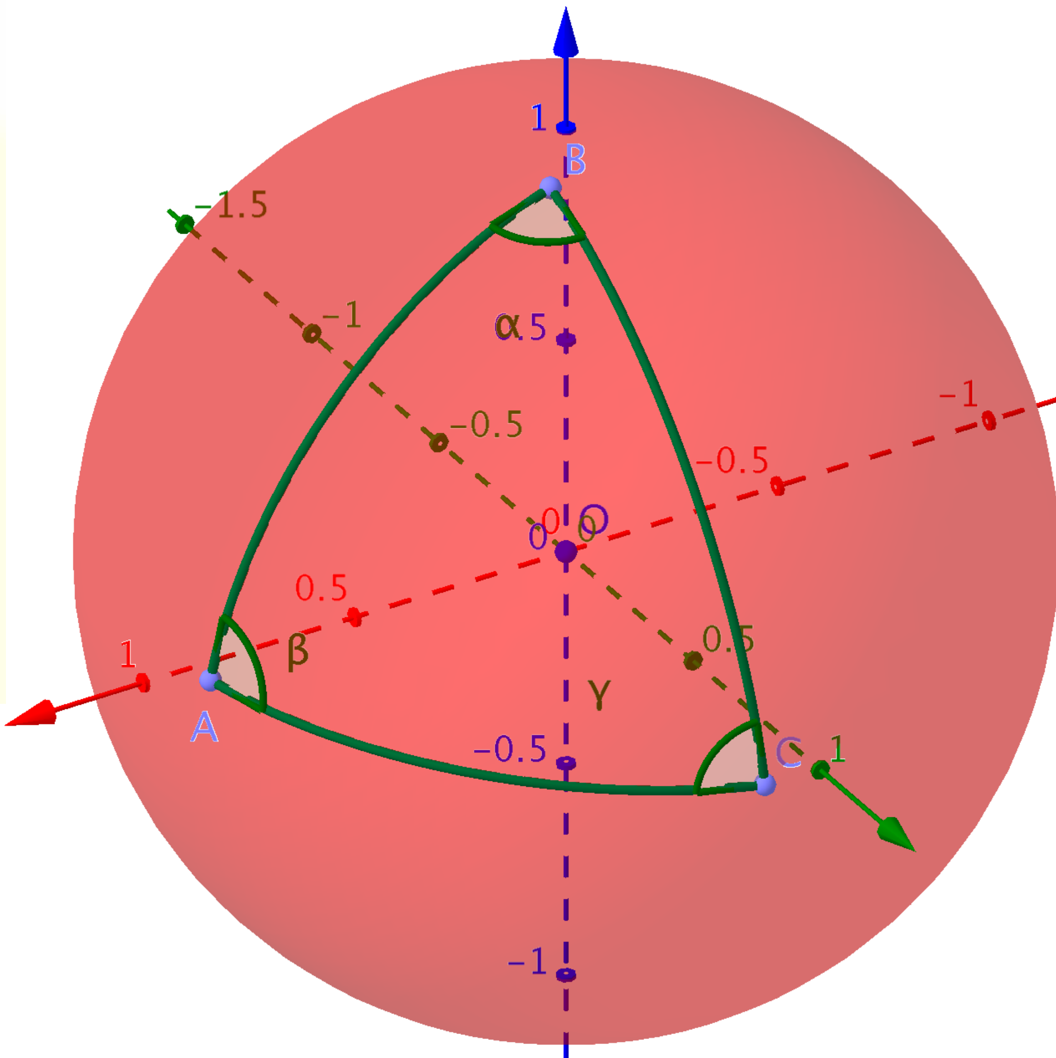
slide_09_app.ggb

4. Ângulos entre dois segmentos ...





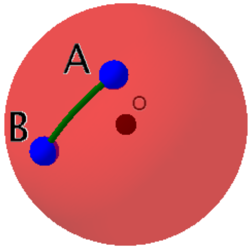
slide_11_app_01.ggb

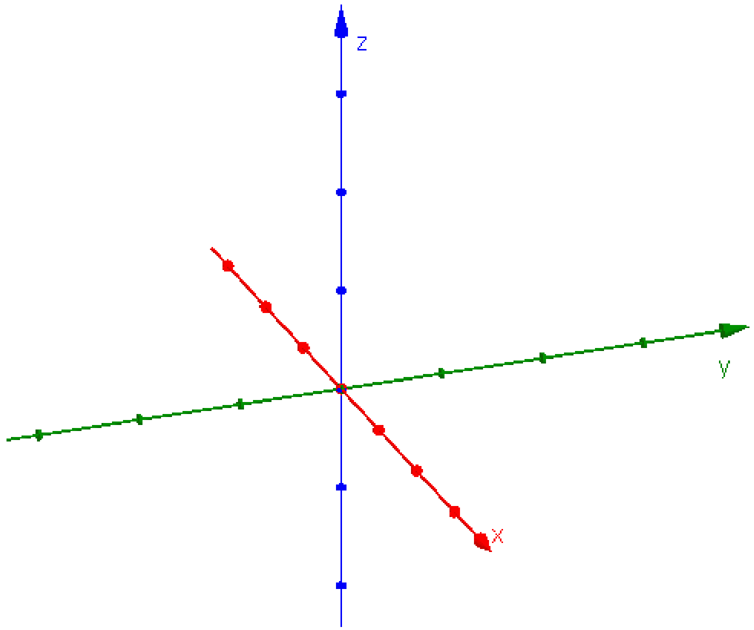


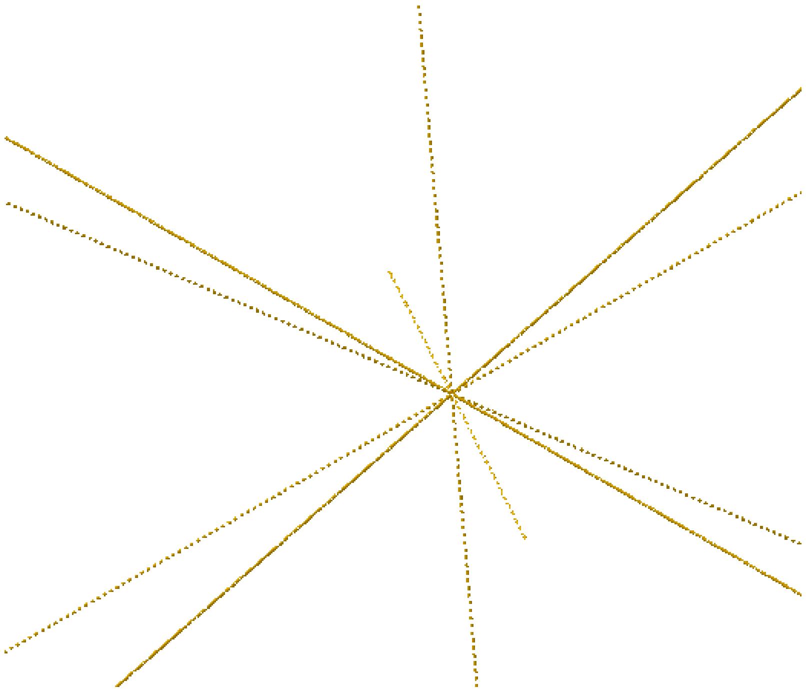
slide_11_app_02.ggb

$$\alpha+\beta+\gamma=72.13^\circ+84.89^\circ+76.99^\circ=234.02^\circ$$

5. Outros polígonos esféricos



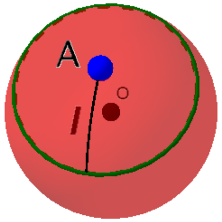




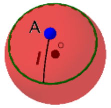
slide_10_app_01.ggb

slide_10_app_02.gg
b

7. Compasso esférico



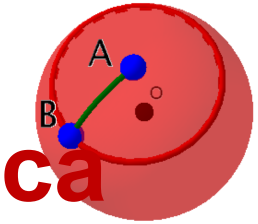
Questão: Dados dois pontos, A e B, construir o lugar geométrico dos pontos da esfera que distam de A a medida l.

, **SphereCompass[A, l, O, r]**

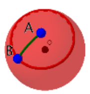
This tool construct the locus of points equidistant of A at the spherical distance l.

Tool Name	**Sphere Compass**
Command Name	**SphereCompass**
Syntax	**SphereCompass[A, l, O, r]**
Tool Help	**Given A, distance l, and centre and radius of sphere, draw the locus of points equidistant of A at the spheric distance l.**
Icon	
Script	r=1 O=(0,0,0) S=Sphere[O, r] A=PointIn[S] l=1 IntersectConic[S, Sphere[A, Rotate[A, l rad, O, Plane[A, line[O,IntersectPath[x + 0z = 0, y + 0z = 0]]]]]]
File	**03_ScAl_01_t.ggb**

6. “Circunferências” na geometria esférica

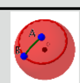


Pontos equidistantes de A e à distância esférica AB

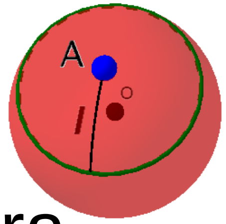


,SphereEquidistantPoints[A, B, O, r]

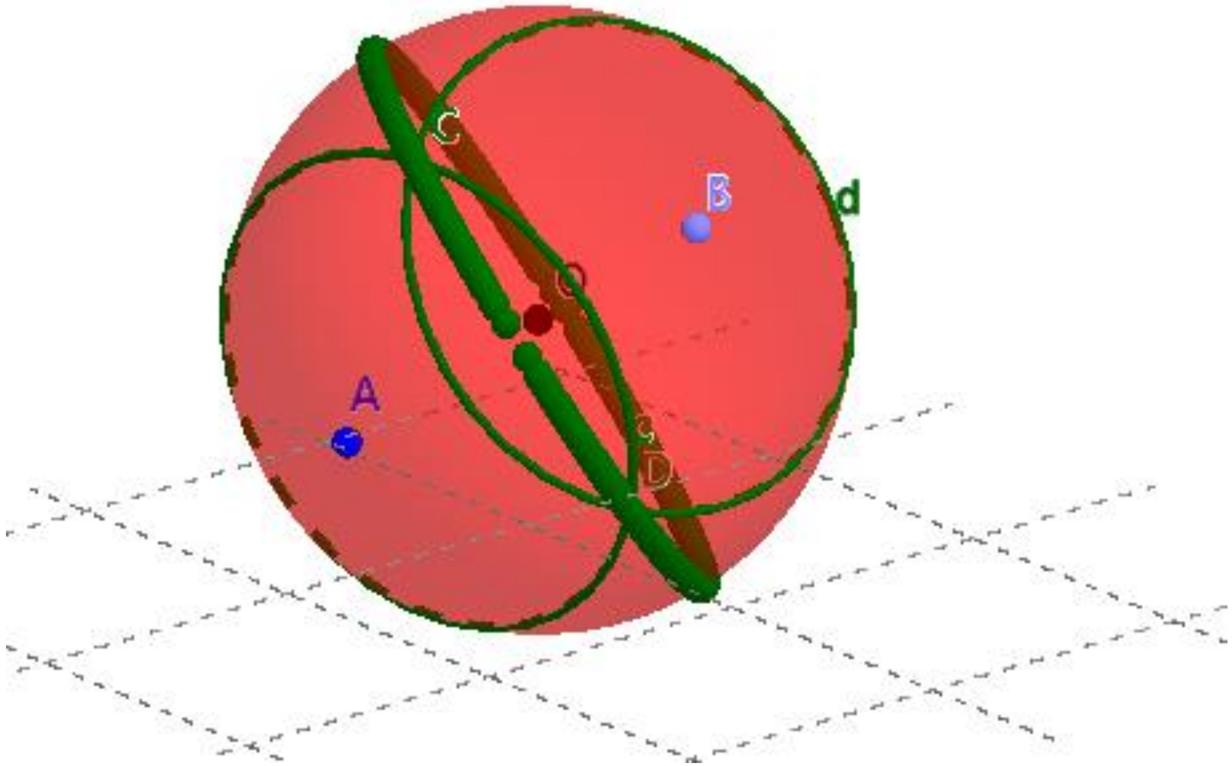
This tool construct the locus of points equidistant of A at the spherical distance AB.

Tool Name	Sphere Compass
Command Name	SphereCompass
Syntax	SphereCompass[A, l, O, r]
Tool Help	Given A, distance l, and centre an radius of sphere, draw the locus of points equidistant of A at the spherical distance l.
Icon 	
Script	r=1O=(0,0,0)S=Sphere[O, r] A=PointIn[S] l=1 IntersectConic[S, Sphere[A, Rotate[A, l rad, O, Plane[A, line[O,IntersectPath[x + 0z = 0, y + 0z = 0]]]]]]
File	03_ScAl_01_t.ggb

8. Compasso esférico e mediatrizes

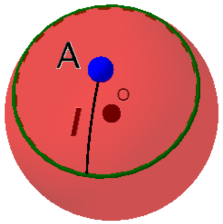


Pontos equidistantes de dois pontos fixos na esfera

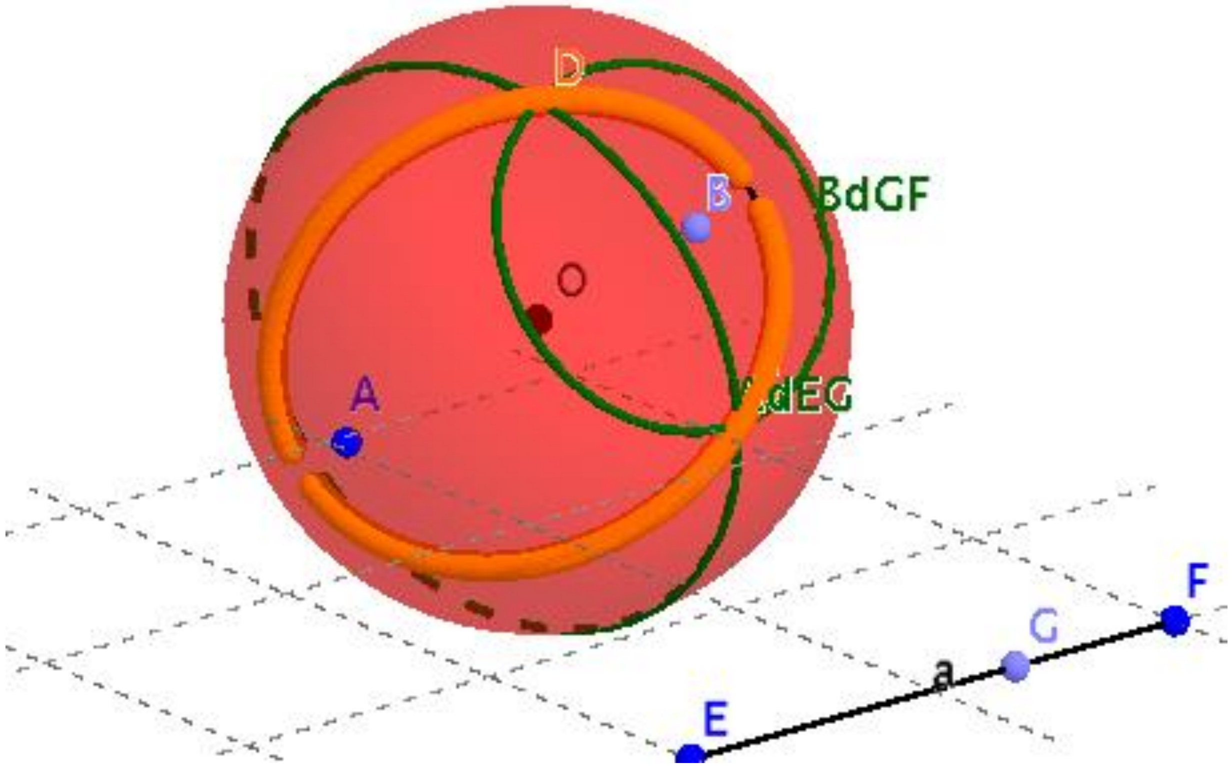


slide_14_app.ggb

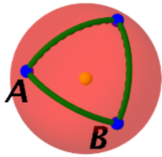
9. Compasso esférico e elipse



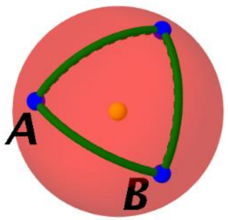
Uma elipse na geometria esférica



slide_15_app.ggb



10. Triângulo equilátero esférico genérico

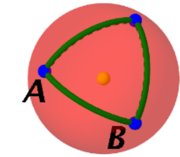
Command	icon
A=PointIn[S] B=PointIn[S] C=Intersect[IntersectConic[Sphere[B, A], S], IntersectConic[Sphere[A, B], S], 2] ABC={SphereSegment[A, B, O, r], SphereSegment[B, Intersect[IntersectConic[Sphere[B, A], S], IntersectConic[Sphere[A, B], S], 2], O, r], SphereSegment[Intersect[IntersectConic[Sphere[B, A], S], IntersectConic[Sphere[A, B], S], 2], A, O, r]}	
SphericEquilateralTriangle[C, B, O, r]	

A ferramenta **SphericEquilateralTriangle[C, B, O, r]** foi concebida para a construção de triângulos esféricos equiláteros.

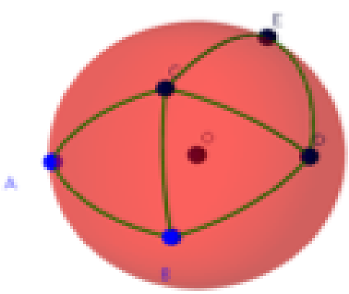
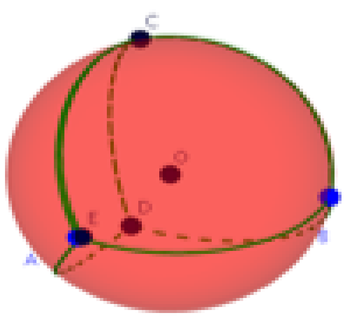
Aplicações:

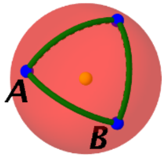
1. Explorar as pavimentações esféricas por triângulos equiláteros;

2. … … …

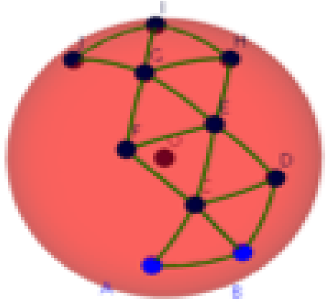
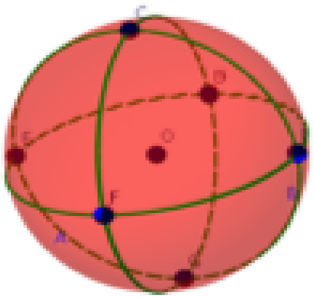


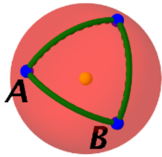
11. Pavimentar com 3 triângulos

N.º de triângulos equiláteros	... sobre um hemisfério	Medida do segmento AB	... a fechar sobre a esfera	Medida do segmento AB
3		1,05 rad		1,908 rad (v.a.) arcos(-1/3) ou 2 artan($\sqrt{2}$) (v.e.)

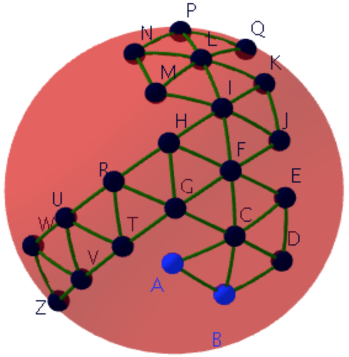
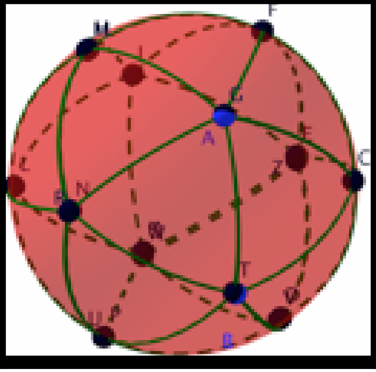


12. Pavimentar a esfera com 8 triângulos

N.º de triângulos equiláteros	... sobre um hemisfério	Medida do segmento AB	... a fechar sobre a esfera	Medida do segmento AB
8		0,64 rad		1,574 rad (v.a.) $\pi/2$ (v.e.)



13. Pavimentar com 20 triângulos

N.º de triângulos equiláteros	... sobre um hemisfério	Medida do segmento AB	... a fechar sobre a esfera	Medida do segmento AB
20		0,40 rad		1,107 rad (v.a.) $2\,\text{arcsin}\left(\frac{\sqrt{5-\sqrt{5}}}{\sqrt{10}}\right)$ 2 arcsin(√(5-√5)/√10) (v.e.)

13. Pavimentar com losangos

Pavimentação dodecaédrica não regular, por “quadriláteros” losangos não rectângulos.

Intersect[xAxis, S]

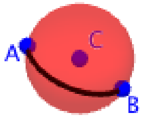
Intersect[yAxis, S]

Intersect[zAxis, S]

Intersect[Line[(0, 0, 0), (1, 1, 1)], S]

Intersect[Line[(0, 0, 0), (-1, -1, 1)], S]

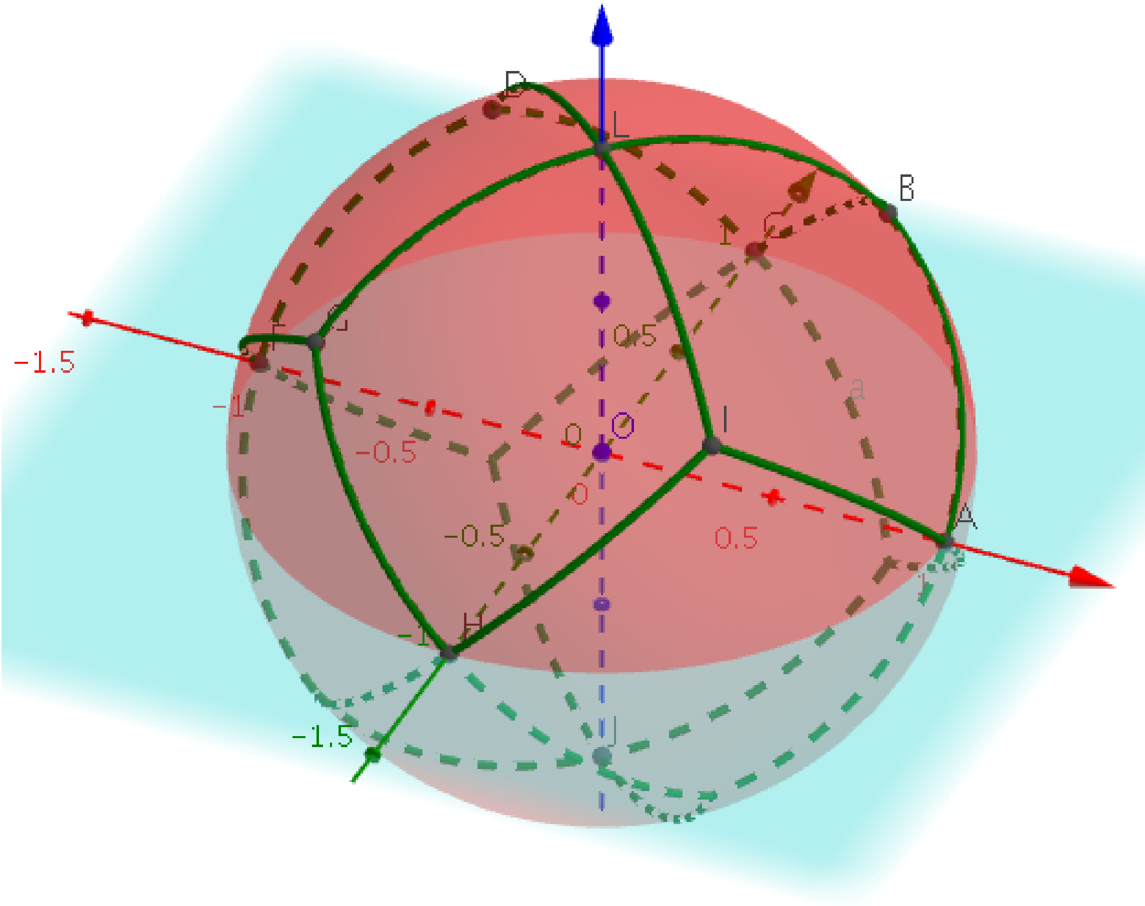
Intersect[Line[(0, 0, 0), (1, -1, 1)], S]



AB=SphereSegment[A, B, O, r]

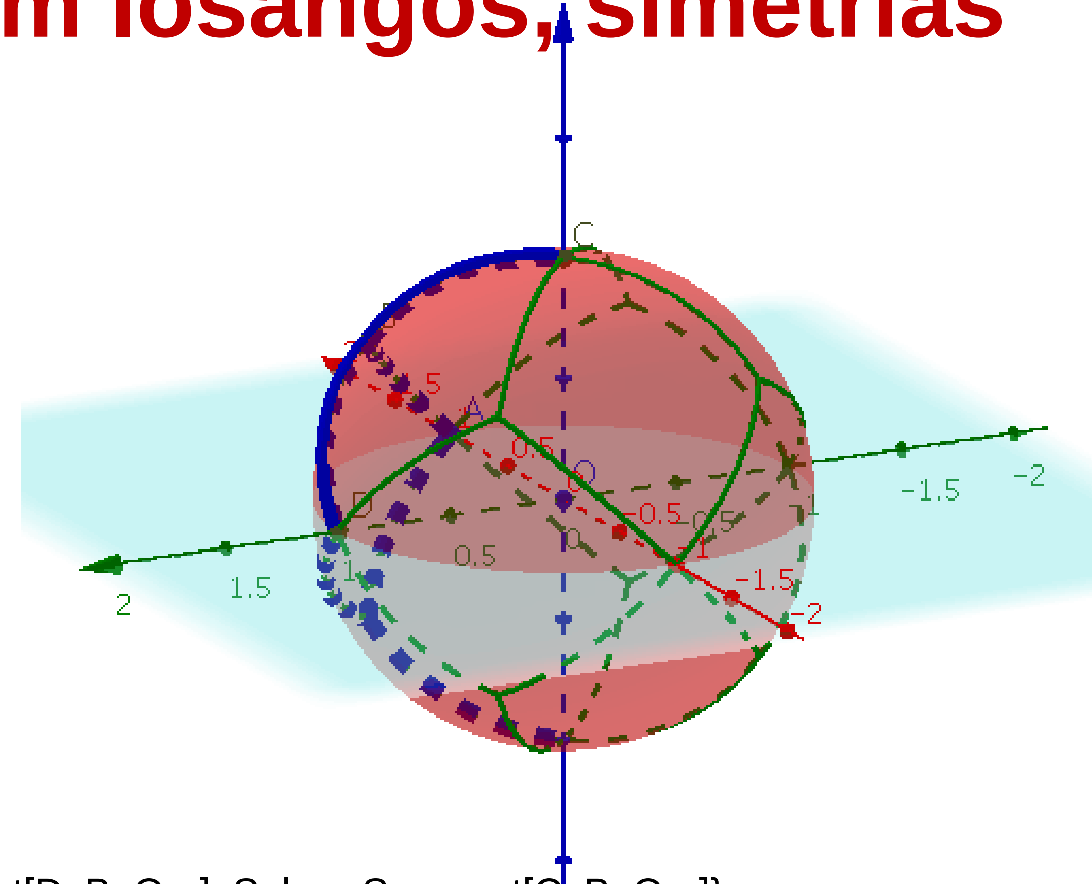
z = 0

Reflect[AB, a]



slide_20_app_losang.ggb

13. Pavimentar com losangos, simetrias



A=(r,0,0)

B=Intersect[Ray[(0, 0, 0), (1, 1, 1)], S,2]

C=(r,0,0)

D=(0, r, 0)

mod={SphereSegment[A, B, O, r], SphereSegment[D, B, O, r], SphereSegment[C, B, O, r]}

Rotate[mod, pi / 2, zAxis]

Rotate[mod, pi, zAxis]

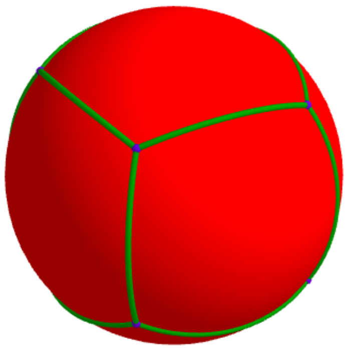
Rotate[mod, 3pi / 2, zAxis]

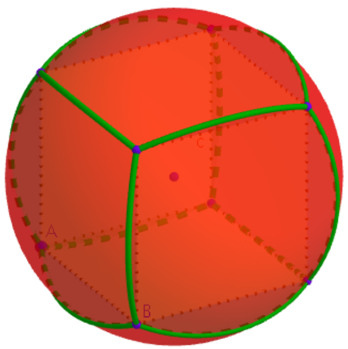
Reflect[half, z=0]

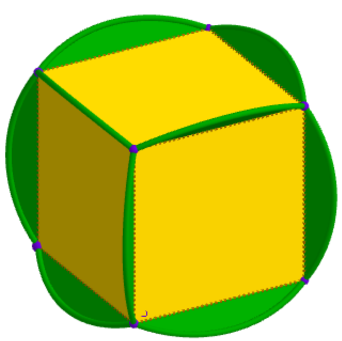
half=Sequence[Rotate[{SphereSegment[A, B, O, r], SphereSegment[D, B, O, r], SphereSegment[C, B, O, r]}, i* pi / 2, zAxis], i, 0, 3]

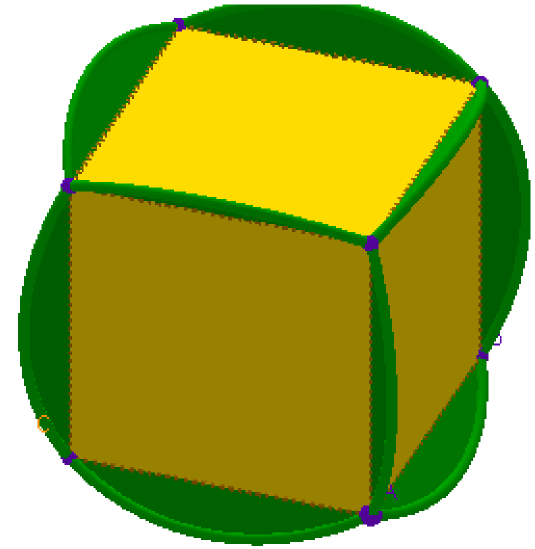
slide_21_app_losang.gg

14. Pavimentação da esfera e o cubo



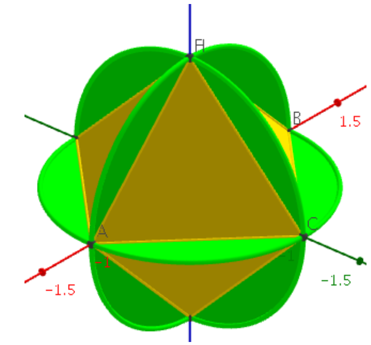


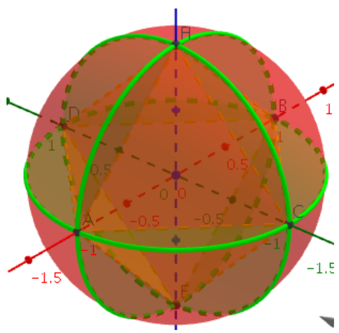


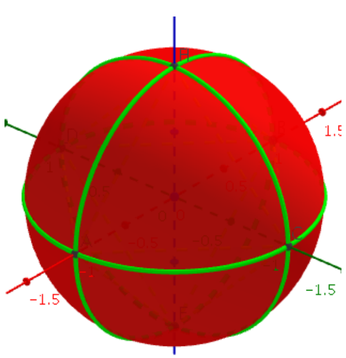


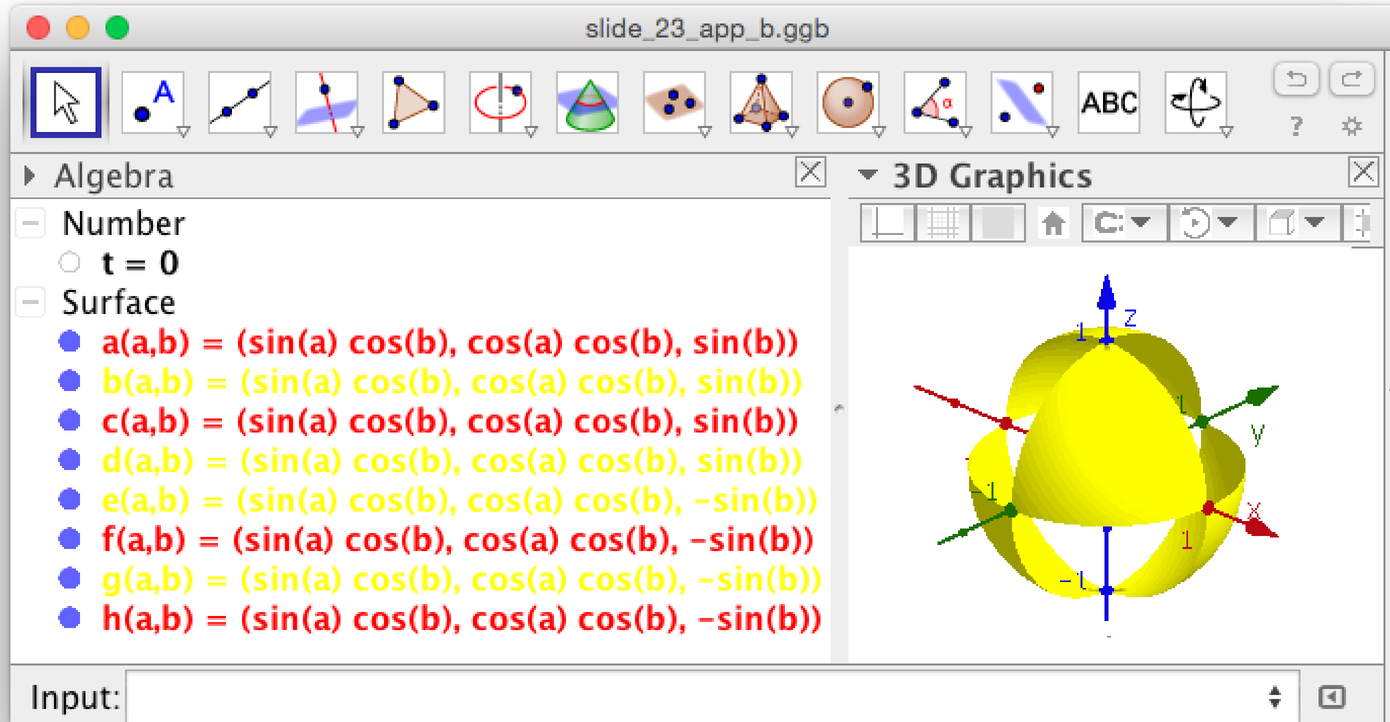
slide_22_app_square.gg

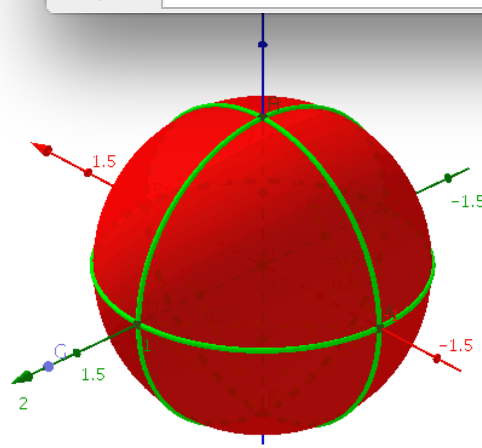
15. Pavimentação da esfera e octaedros







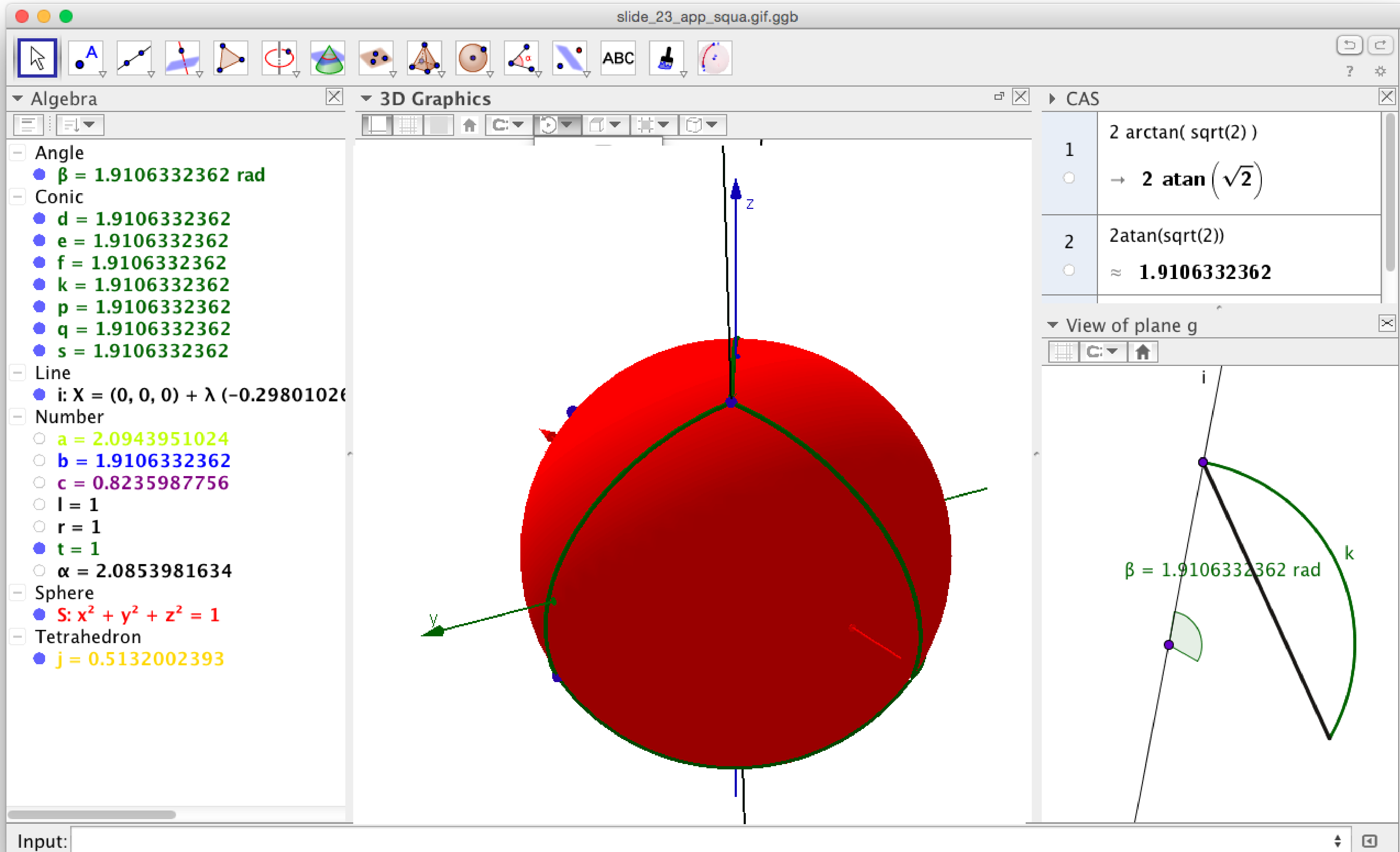




slide_23_app_a.ggb

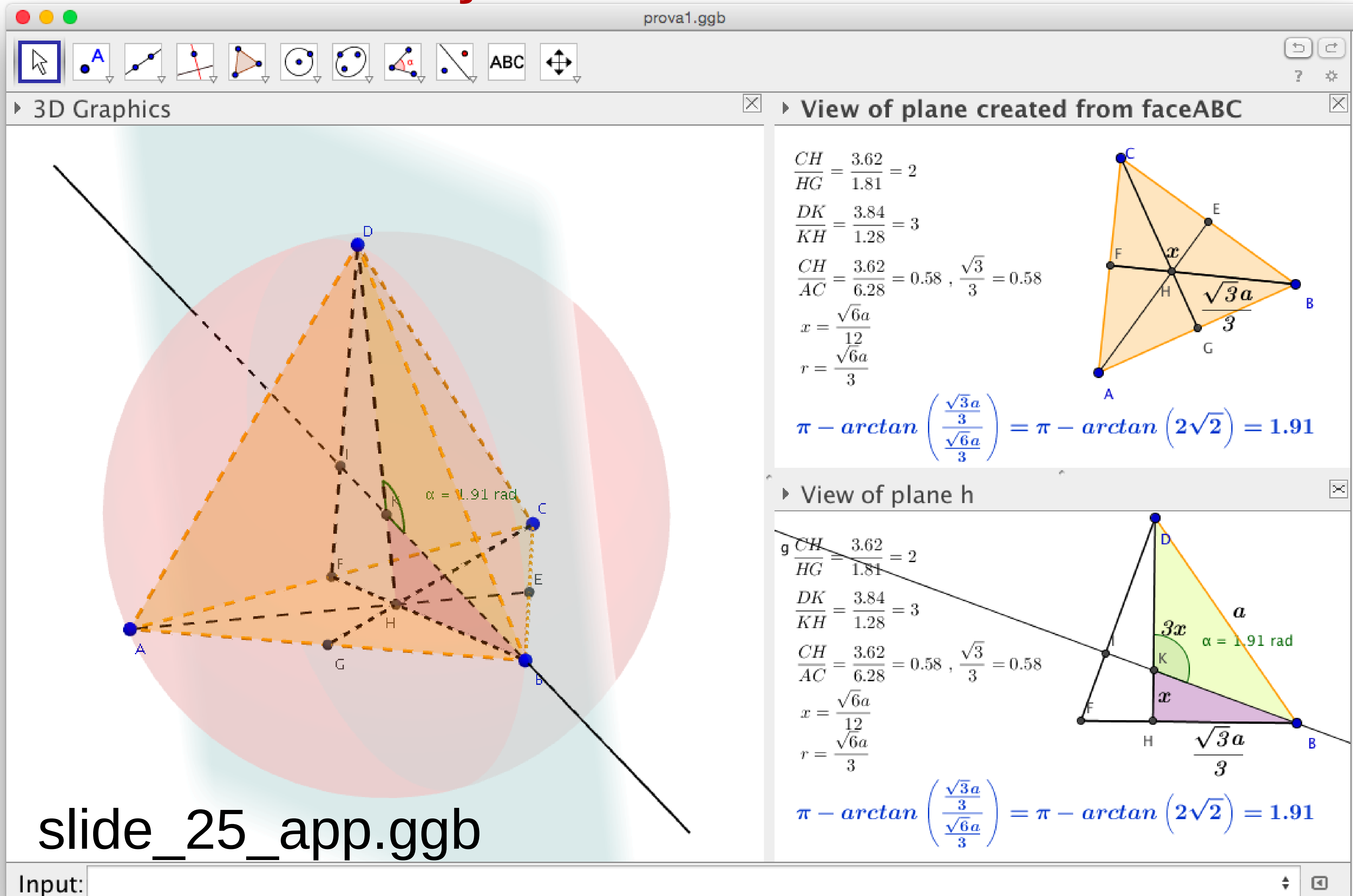
slide_23_app_b.ggb

16. Pavimentação da esfera e tetraedro



slide_24_app.ggb

16. Pavimentação da esfera e tetraedro



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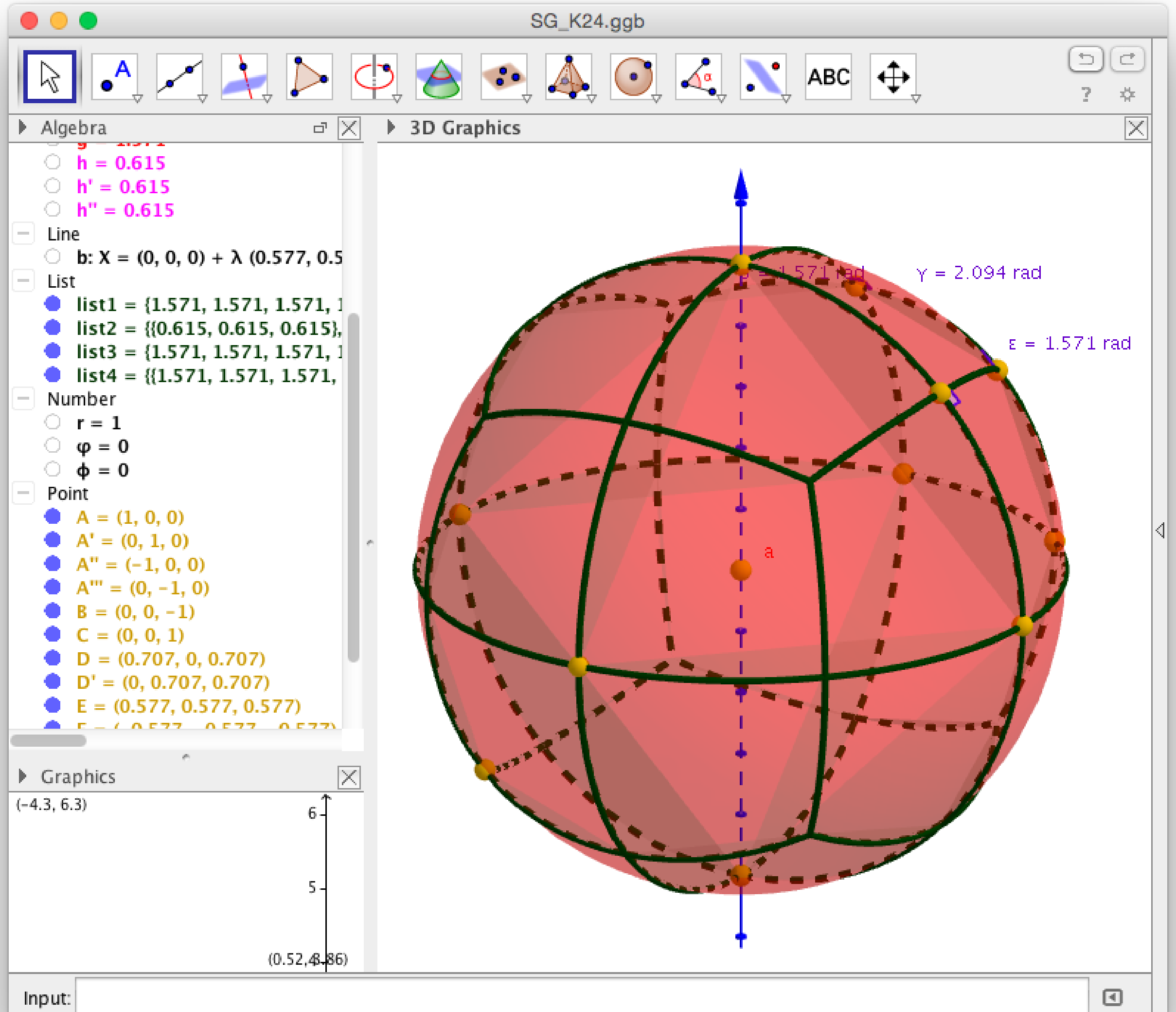
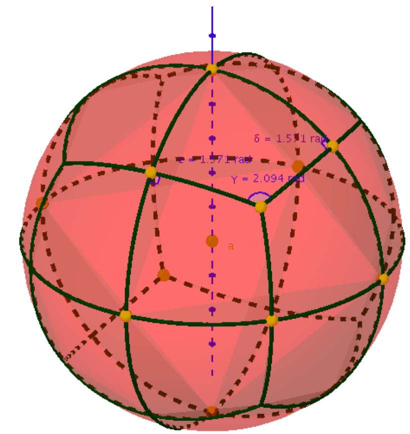
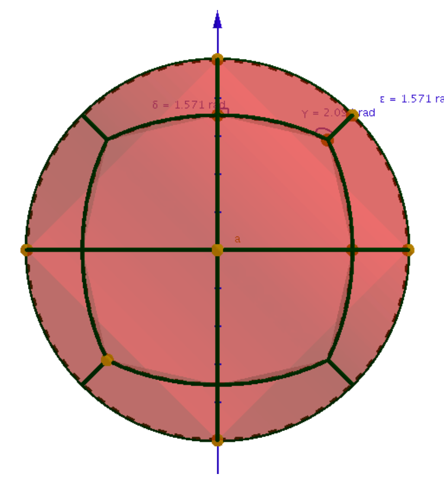
17. Pavimentações Monoédricas da esfera





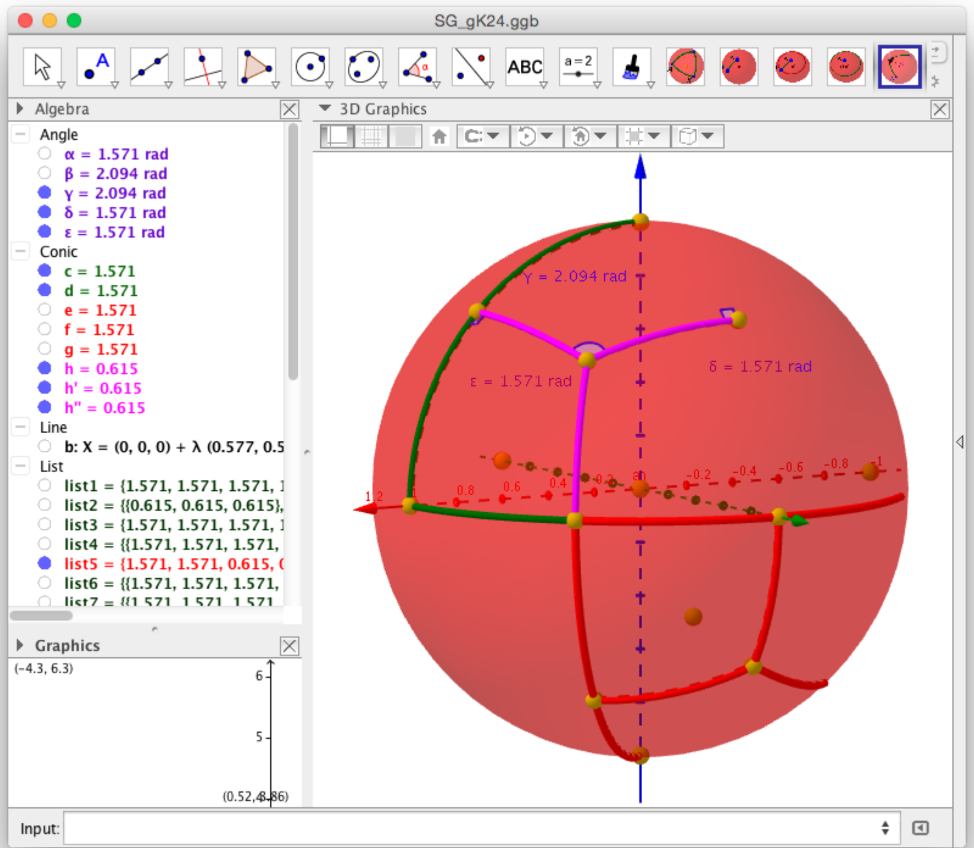


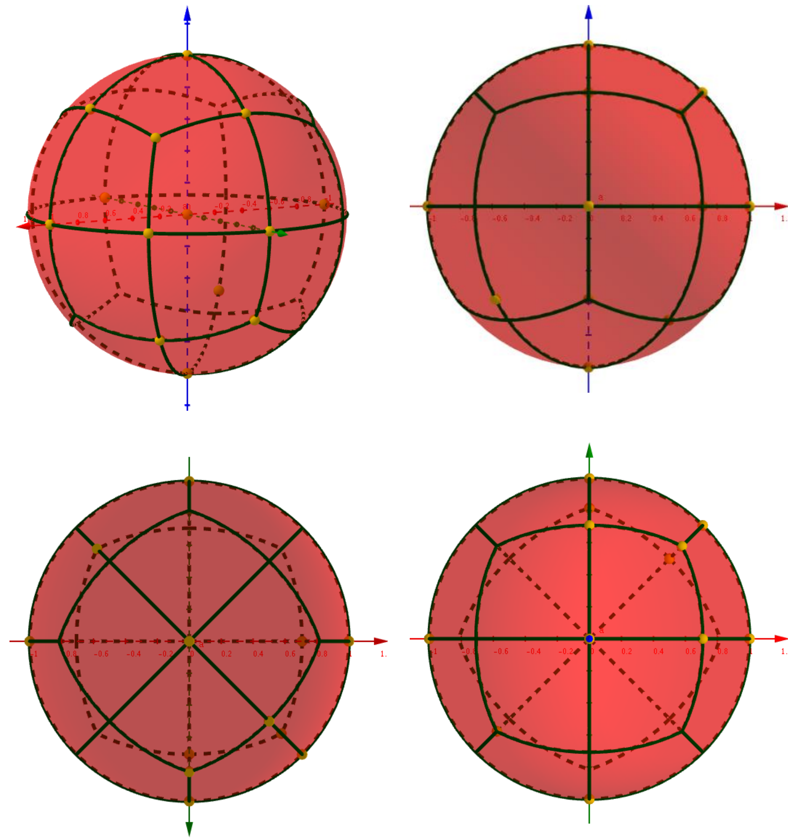
K24

SG_K24.ggb

17. Pavimentações Monoédricas da esfera

SG_gK24.ggb





gK24

Reflect[Rotate[{c, d, h, h', h''}, 0.7854, Line[B, C]], Plane[z = 0]]