USING GEOGEBRA TO STUDY COMPLEX FUNCTIONS

Ana Maria d'Azevedo Breda, José Manuel Dos Santos Dos Santos

Universidade de Aveiro, Instituto GeoGebra Portugal

ambreda@ua.pt, dossantosdossantos@gmail.com

The aim of this workshop to present some of the strategies studied to use GeoGebra in the analysis of complex functions. The proposed tasks focus on complex analysis topics target for students of the 1st year of higher education, which can be easily adapted to pre-university students. In the first part of this workshop we will illustrate how to use the two graphical windows of GeoGebra to represent complex functions of complex variable. The second part will present the use of the dynamic color Geogebra in order to obtain Coloring domains that correspond to the graphic representation of complex functions. Finally, we will use the three-dimensional graphics window in GeoGebra to study the component functions of a complex function. During the workshop will be provided scripts orientation of the different tasks proposed to be held on computers with Geogebra version 5.0 or high.

Keywords: GeoGebra, Complex Functions

INTRODUCTION

The main objective of this workshop is to lead the participants to recognize how and why the use of the proposal tasks are an added value for the understanding and learning of the mathematical content associated with them. The tasks proposed to the participants are the same as the ones designed for students of the first year of engineering and science courses can and should be used as an educational tool in collaborative learning environments. The main advantage in its use in individual terms is the promotion of the deductive reasoning (conjecture / proof).

The applications presented here can trigger exploration in the different teaching degrees. As an example, it can be used for illustrating the Fundamental Theorem of Algebra for pré- university students. These applications can be viewed as an intermediate step for visualizing and understanding Möbius Transformation and their relation with rigid movements of the Riemannian Sphere.

In addition to the time required for the tasks it is also planned to have an extra time for exchanging ideas about the impact of this type of work with students from different educational degrees and attending different courses.

GRAPHIC WINDOWS AND TWO-DIMENSIONAL REPRESENTATION OF COMPLEX FUNCTIONS

Featuring GeoGebra multiple windows, these can be used simultaneously in order to provide a computational environment that interacts with two Cartesian representations, these two GeoGebra windows are an excellent model for the domain and codomain of a complex function (Breda, A., Trocado, A., & Santos, J. ,2013). This session will be proposed various tasks involving the representation of: a) points in the complex plane, a graphical window, and the image display these points by a complex function, the second graphic window; b) points in certain subsets of the complex plane by analyzing the images of these subsets of a complex function, the second graphics window, using the Locus command, and the Trace function of a point; c) use the following command to create grids in the representation of domain points to a complex function analyzing their images.

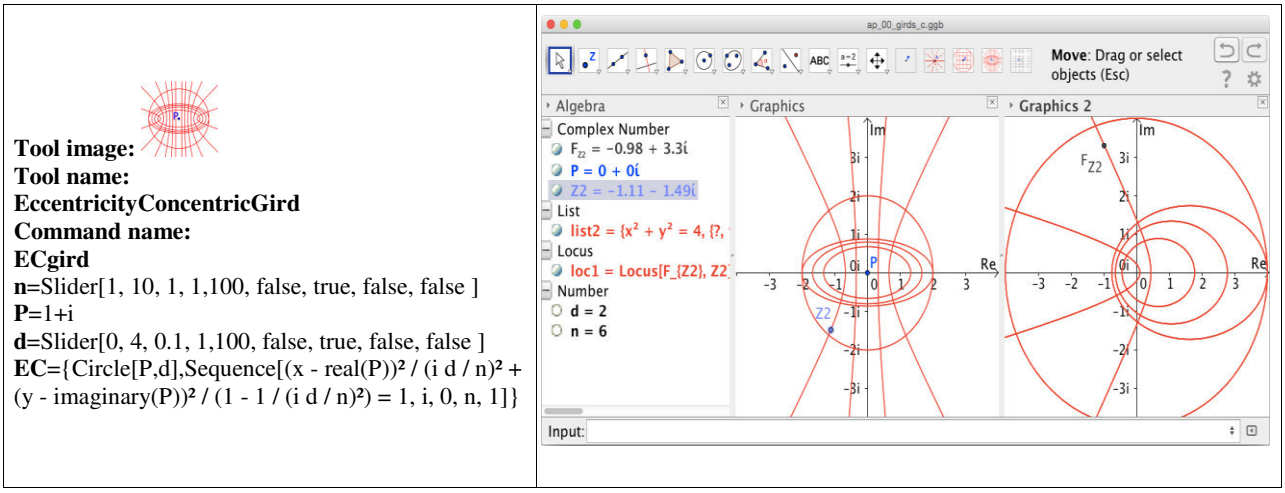
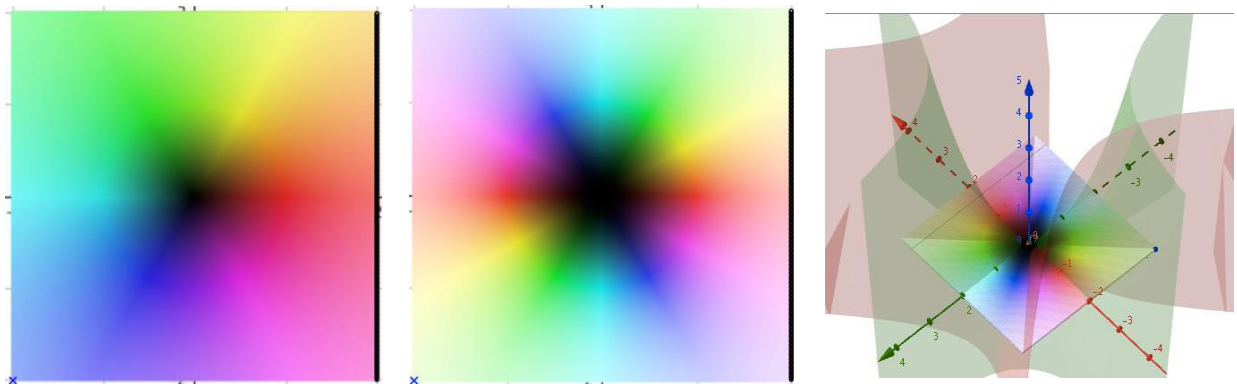


Figure 1. Subset of complex numbers and their image by function $f: C \rightarrow C, f(z)=z^2$.

Figure 1 shows the example code to use to get points on the complex plane a family of conical, represented in the graphics window, as well as the representation of the image of these points by a complex function is in the graphics 2 window of GeoGebra.

Dynamic Colors, Coloring Domains in Representation of the Graphic of a Complex Function in Geogebra

The use of dynamic colors associated with a point allowed Rafael Losada (2009) and Antonio Ribeiro obtain the first representations of fractal images involving complex numbers (Breda, et al, 2013, p. 63). Subsequently, the potential of the dynamic color GeoGebra led to Breda, and Dos Santos (2013) to apply them to obtain color domains thus obtained is the complex variable representation of complex graphic functions (Breda et al, 2013, p. 78).



$I(z) = z \wedge z \in C$ $\qquad$ $f(z) = z^2 \wedge z \in C.$

Figure 2. Coloring Domain: identity function, $f(z) = z^2 \wedge z \in C$ and components maps of f.

This workshop will be presented two essential tasks for Coloring Domains of the complex plane. The first task indicates the strategy for construct a scanner of the plan with GeoGebra, the second adjusting the scanner of plan, developed in the first task, to get the Coloring Domain associated to the graph representation of a complex function, as can be observed two examples the two pictures, on the left, of Figure 2.

3d Graphics Window for the Graphic Representation of Components Maps of a Complex Function

In GeoGebra, the various properties of objects can communicate between different windows and after version 5.0 there is a three-dimensional window, thus we can get the graph representation of each component maps of a complex function. Thus, the last tasks of this workshop will be aimed at graph representation of component maps of a complex function, as well as, analyzing some of its properties. Thus we can simultaneously display various graphical representations (see the third image, the far right of Figure 2) that contribute to improving the visualization of different properties associated with a complex function. In addition, will also be presented, one sample of Coloring Domains applied on the Riemann sphere in order to study Möbius transformations (Breda, A., & Santos, J., 2015).

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