I. INTRODUCTION

Inflaton and dark matter candidates in particle physics models share several common features, both being typically assumed to be weakly interacting and neutral fields. The inflaton scalar field must interact weakly with itself and other degrees of freedom in order to ensure the required flatness of the associated scalar potential, which could be spoiled by large radiative corrections [11]. Similarly, dark matter particles should form a stable non-relativistic and non-luminous fluid at late times that accounts for the observed galaxy rotation curves [2] and the large scale structure in the universe as inferred from Cosmic Microwave Background [3, 4] and weak-lensing observations [5, 6] (for a recent review see [7]). Both inflation and dark matter are essential features in the modern cosmological paradigm and cannot be accounted for within the present framework of the Standard Model of particle physics. It is therefore interesting to consider the possibility that the same field accounts for both accelerated expansion in the early universe and the hidden matter component at late times.

Scalar fields have the interesting property of mimicking fluids with different equations of state depending on the regime typically considered. For a homogeneous scalar field \( \phi \) with potential \( V(\phi) \), we have:

\[
\rho_\phi = \frac{1}{2} \dot{\phi}^2 + V(\phi), \quad \rho_\phi = \frac{1}{2} \dot{\phi}^2 - V(\phi). \quad (1.1)
\]

Hence, on the one hand a slowly-varying field, \( \dot{\phi}^2/2 \ll V(\phi) \) acts as an effective cosmological constant, which is the regime typically considered in canonical inflationary models. On the other hand, a field oscillating about the minimum of its potential where \( V(\phi) \sim m^2_{\phi} \dot{\phi}^2/2 \) behaves as non-relativistic matter, with \( \langle \dot{\phi}^2/2 \rangle = -\langle V(\phi) \rangle \) such that \( p_\phi \ll \rho_\phi \) [8]. These two regimes will generically be present in inflationary potentials, which further suggests a common framework for inflation and dark matter. The main difficulty in achieving such a unified description lies, however, in the fact that inflation must end with a transition to a radiation-dominated universe, in order to recover the standard “Big Bang” evolution at least before the freeze-out of light nuclear abundances takes place [9]. An efficient transfer of energy between the inflaton field and radiation generically requires the former to decay into light degrees of freedom following the period of inflationary slow-roll [10], even though other non-perturbative processes such as parametric resonance amplification could contribute significantly to the reheating process [11]. A plausible alternative is also that non-equilibrium dissipative processes continuously source a radiation bath during inflation that smoothly takes over as the field exits the slow-roll regime, a possibility generally known as warm inflation [12, 13].

Nevertheless, efficient reheating does not imply the complete decay of the inflaton field and the possibility that a stable remnant is left after inflation has been considered in a few studies in the literature [14–18]. In this work, we propose a concrete realization of this generic idea in quantum field theory, where the decay of the inflaton is truly incomplete, occurring only for a finite period after the end of the slow-roll regime.

We describe a simple mechanism based on standard Yukawa coupling between the inflaton and fermion fields (and potentially their superpartners), endowed with an appropriate symmetry that protects the full decay of the inflaton field, and discuss the parametric regimes where inflaton decay into such fermions is incomplete. We also discuss the embedding of this generic mechanism in concrete inflationary models and possible scenarios for the interactions between the fermion fields and the Standard Model degrees of freedom that allow for the presence of the latter in the post-inflationary thermal bath. We also show that the inflaton remnant is not necessarily in the form of a coherent condensate of bosonic particles and that, in particular, for not too small couplings the evap-
oration of this condensate is inevitable. In this case, the thermalized inflaton particles eventually decouple from the radiation bath and their abundance freezes out, yielding a thermal inflaton relic with properties similar to other WIMP candidates \[20\]. We suggestively denote this as the "WIMPlaton" scenario.

In this work, we consider two dynamically distinct scenarios. Firstly, we will consider a minimal model with a single dynamical scalar field that simultaneously drives inflation, reheats the universe through incomplete decay and leaves a stable non-relativistic remnant. Secondly, we consider a (supersymmetric) hybrid inflation model with an additional dynamical waterfall sector, which is responsible for ending inflation and reheating of the universe. In this case, we show that despite reheating being ensured by a different field, the inflaton field must decay in order for radiation to fully come to dominate the energy balance in the universe. As in the minimal model, this decay may nevertheless be incomplete and we discuss the parametric regimes in which the inflaton remnant constitutes a suitable dark matter candidate in the hybrid framework.

This work is organized as follows. In the next section we discuss the minimal model of inflaton dark matter from incomplete decay, starting with its basic properties and dynamics and then discussing the possibility of condensate evaporation, its embedding in a consistent inflationary model and different phenomenological possibilities for reheating the Standard Model degrees of freedom. In Section III we discuss these aspects in the hybrid realization of inflaton dark matter. Finally, in Section IV we summarize our main results and conclusions.

II. MINIMAL MODEL

A. Basic properties and dynamics

The minimal model for inflaton dark matter considers a single dynamical (real) scalar field, the inflaton $\phi$, with a potential energy $V(\phi)$ such that a period of slow-roll can occur for some field range. We take the inflaton field to be coupled to fermion fields $\psi_+$ and $\psi_-$ through standard Yukawa terms and impose a discrete symmetry $\mathbb{Z}_2 \subset \mathbb{Z}_2 \times S_2$ on the Lagrangian such that the scalar inflaton transforms under the $\mathbb{Z}_2$ group as $\phi \rightarrow -\phi$ and the fermions are simultaneously interchanged by the permutation symmetry $\psi_+ \leftrightarrow \psi_-$. This yields for the resulting Lagrangian density:

$$
\mathcal{L} = \frac{1}{2} \partial^\mu \phi \partial_\mu \phi - V(|\phi|) + \bar{\psi}_+ (i \gamma^\mu \partial_\mu - m_f) \psi_+ + \bar{\psi}_-(i \gamma^\mu \partial_\mu - m_f) \psi_- - \bar{\psi}_+ \psi_+ + \bar{\psi}_- \psi_- ,
$$

where we note that, as a result of the discrete symmetry, the two fermions have the same tree-level mass $m_f$ but opposite Yukawa coupling to the inflaton field. The action of the discrete symmetry is restricted to the inflaton-fermion sector, such that all other fields, including the Standard Model fields, are invariant under this symmetry.

This symmetry implies that the inflaton potential is an even function of the field and we assume that it is unbroken at the minimum of the potential, which must therefore lie at the origin, $\phi = 0$. As a result, since no other terms linear in $\phi$ except for the above Yukawa terms are allowed by the discrete symmetry, the only possible decay channels of the inflaton at the minimum are $\phi \rightarrow \psi_+ \psi_+$. If the inflaton mass at the minimum is given by $m_\phi^2 = V''(0) > 0$ and we require $m_f > m_\phi/2$, these decays will be kinematically forbidden for $\phi = 0$ and the inflaton will be stable when it settles at the minimum of its potential. Hence, the discrete symmetry and the mass hierarchy ensure the stability of the inflaton at late times, such that it will contribute to the present dark matter abundance.

The two fermion fields are indistinguishable apart from their coupling to the inflaton, which leads to a mass splitting away from the origin:

$$
m_\pm = |m_f \pm h\phi| .
$$

This implies that, although inflaton decay into these fermions is forbidden at late times as the field approaches the origin, that need not be the case if the amplitude of field oscillations after inflation is sufficiently large. In particular, decay will be allowed for field values satisfying:

$$
|m_f \pm h\phi| < m_\phi/2 .
$$

This implies, in particular, that decay is kinematically allowed for field amplitudes $|\phi| \gtrsim m_f/h$. The partial decay widths associated to the two fermionic decay channels are then given by:

$$
\Gamma_\pm = \frac{h^2}{8\pi} m_\phi \left(1 - \frac{4m_\pm^2}{m_\phi^2}\right)^{3/2} ,
$$

with $\Gamma_\phi = \Gamma_+ + \Gamma_-$. Note that due to the opposite sign of the Yukawa couplings, the inflaton will alternately decay into each fermion species as it oscillates between negative and positive values. The inflaton equation of motion as it oscillates about the minimum of its potential is then given by:

$$
\ddot{\phi} + 3H \dot{\phi} + \Gamma_\phi \phi + m_\phi^2 \phi = 0 ,
$$

which, upon multiplying by $\dot{\phi}$ is equivalent to:

$$
\dot{\rho}_\phi + 3H (\rho_\phi + p_\phi) = -\Gamma_\phi \dot{\phi}^2 .
$$

The term on the right hand side gives the rate at which energy density is transferred from the inflaton field into the fermions $\psi_\pm$. Let us assume that the fermions quickly
thermalize, an assumption that we will check \textit{a posteriori}, in the process exciting $g_*$ relativistic degrees of freedom and forming a radiation bath at temperature $T$, with energy density $\rho_R = (\pi^2/30)g_* T^4$. Note that since the fermion masses oscillate due to the varying inflaton field, they only contribute periodically to the number of relativistic degrees of freedom. These may also include the inflaton for $T \gtrsim m_\phi$ and other species such as the Standard Model particles. Note that the latter must be excited before the cosmological synthesis of light nuclear elements takes place, as we discuss in more detail in section IID. For simplicity, we consider a fixed value of $g_*$, which is not a bad approximation since, as we will show later on, our results exhibit only a mild dependence on this parameter.

Energy conservation then implies that the energy lost by the inflaton field in Eq. (2.6) is gained by the radiation bath, which then follows the dynamical equation:

$$\dot{\rho}_R + 4H\rho_R = \Gamma_\phi \dot{\phi}^2.$$  

(2.7)

Since both the inflaton and the radiation contribute to the energy density in the universe, we may write the Friedmann equation as:

$$H^2 = \frac{\rho_\phi + \rho_R}{3M_p^2} = \frac{1}{2}\dot{\phi}^2 + \frac{1}{2}m_\phi^2\phi^2 + \rho_R \frac{3}{3M_p^2}. \quad (2.8)$$

Eqs. (2.6), (2.7) and (2.8) then form a complete set of differential equations that can be solved for given choices of the parameters ($m_\phi, m_f, h$) and initial conditions. The non-linearity of the equations precludes a complete analytical description of the problem, although as we discuss below a few simple analytical estimates can be performed, but it is straightforward to integrate them numerically. It is useful to write the fermion tree-level mass as:

$$m_f = \frac{m_\phi}{2} (1 + \delta), \quad (2.9)$$

such that the decay is kinematically forbidden (allowed) at late times for $\delta > 0$ ($< 0$).

Figure 1 shows an example of the results obtained by numerically solving the inflaton-radiation equations for $m_\phi = 10^{-3}M_p$, $\delta = 0.1$ and $h = 1$. In this example we take as initial conditions at $t = t_i = 10^{-2}m_\phi^{-1}$, $\phi(t_i) = M_p$, $\dot{\phi}(t_i) = 0$ and $\rho_R(t_i) = 10^{-10}\rho_\phi(t_i)$, although the results are essentially unchanged as long as the field value is sufficiently large before oscillations begin for the decay to be kinematically allowed as discussed above and the radiation energy density is initially negligible, as should be expected after 50-60 e-folds of inflation.

The plot in Figure 1(a) illustrates the main dynamical features that are generically obtained. The inflaton field begins to oscillate about the origin with frequency $m_\phi$ for $t \sim m_\phi^{-1}$ and behaves initially as cold dark matter, $\rho_\phi \propto a^{-3}$, being the dominant energy component such that $a \propto t^{2/3}$ and $H = 2/3t$. While decay is blocked before the onset of oscillations, since $m_\pm \sim h|\phi| \gg m_\phi$, it

![FIG. 1. Results of the numerical integration of the inflaton-radiation dynamical equations for $m_\phi = 10^{-3}M_p$, $\delta = 0.1$ and $h = 1$, showing the time evolution of (a) the inflaton (solid blue curve) and radiation (solid red curve) energy densities; (b) the inflaton-to-entropy ratio; and (c) the inflaton decay width (solid blue curve) compared to the Hubble parameter (solid green curve). The blue dashed curves in (a) and (c) give the evolution of the inflaton energy density in the absence of decay and the maximum value of the decay width, respectively. All quantities are given in Planck units such that $M_p = 1$.](attachment:image.png)
becomes kinematically allowed as soon as the field goes through the origin. Since in this example the amplitude of the field oscillations largely exceeds the tree-level masses, with \(h\Phi \gg m_f\), decay occurs for two narrow field ranges close to (and on both sides of) the origin. The decay width then corresponds to the series to periodic narrow peaks shown in Figure 11(c), with maximum value \(\Gamma_{\phi}^{\max} = \left(h^2/8\pi\right)m_\phi\). While initially \(\Gamma_{\phi}^{\max} \ll 3H\) as illustrated in Figure 11(c), such that the inflaton’s energy density remains essentially unaffected by the decay into fermions, the source term in the radiation equation quickly becomes significant, leading to a jump in the value of the radiation energy density. The latter remains approximately constant until the inflaton’s energy density is sufficiently redshifted. When they become comparable in magnitude, the inflaton effectively decays and becomes as cold dark matter, eventually taking over the radiation as the dominant component at later times.

The oscillating inflaton then becomes stable and once velocity values. For \(\psi\rightarrow\Phi\), the source term in the radiation equation due to decay by an amount:

\[
\Delta \rho_{R,\text{decay}} \simeq \frac{\Gamma_{\phi}^{\max}}{2} (m_\phi \Phi)^2 (2\tau_d) \simeq \frac{h}{8\pi} m_\phi^3 \Phi ,
\]

where we have taken into account the decays into both \(\psi_+\) and \(\psi_−\). When decay is forbidden, radiation simply redshifts with expansion, which counteracts the enhancement due to decay by an amount:

\[
\Delta \rho_{R,\text{Hubble}} \simeq -4H \rho_R (\pi/m_\phi) .
\]

Since \(H = 2/3t = (m_\phi/\sqrt{6M_P})\Phi\), it is easy to see that the amount of radiation produced by inflaton decay can be balanced exactly by Hubble expansion to yield a constant energy density. Equating \(\Delta \rho_{R,\text{decay}} = -\Delta \rho_{R,\text{Hubble}}\) then gives:

\[
\rho_R \simeq \frac{\sqrt{6}h}{32\pi^2} m_\phi^4 M_P ,
\]

which is in very good agreement with the numerical simulations. Noting that, from Eq. (2.9), \(\rho_R \simeq 0\) implies \(4H\rho_R \simeq \Gamma_{\phi}^{\prime 2} \simeq \Gamma_{\phi} \rho_\phi\), we see that that \(\Gamma_{\phi} \sim 3H\) for \(\rho_R \sim \rho_\phi\), so that as observed numerically the inflaton energy density is only reduced significantly when it becomes comparable to the radiation energy density.

From Eq. (2.13) we can easily determine the associated temperature, which remains approximately constant up to inflaton-radiation equality and thus constitutes the reheating temperature:

\[
T_R \simeq \left(\frac{15\sqrt{6}}{16\pi} \right)^{1/4} g_*^{-1/4} h^{1/4} \left(\frac{m_\phi}{M_P}\right)^{3/4} M_P
\]

\[
\simeq 2.7 \times 10^6 \, g_*^{-1/4} h^{1/4} \left(\frac{m_\phi}{\text{1 TeV}}\right)^{3/4} \, \text{GeV} .
\]

From this we conclude, in particular, that the reheating temperature is generically larger than the inflaton mass for:

\[
m_\phi < 5.6 \times 10^{16} \frac{h}{g_*} \, \text{GeV}.
\]

After inflaton-radiation equality, the field decays exponentially fast until its amplitude drops sufficiently for decay into fermions to become inefficient. Numerically, we observe that this occurs before the decay becomes kinematically forbidden, for \(\Phi \lesssim m_f/h\), corresponding roughly to when the (maximum) decay width becomes less than the Hubble rate. Since afterwards the field stabilizes and behaves as non-relativistic matter, it is useful to compare its number density \(n_\phi = \rho_\phi/m_\phi \propto a^{-3}\) with the radiation entropy density:

\[
s = \frac{2\pi^2}{45} g_* T^3 ,
\]

which redshifts in a similar way up to changes in the number of relativistic degrees of freedom, with \(T \propto a^{-1}\). As illustrated in Figure 11(b), the ratio \(n_\phi/s\) stabilizes after decay becomes inefficient, and numerically we obtain the following expression:

\[
\frac{n_\phi}{s} \simeq 0.5 g_*^{−1/4} h^{−3.6} (m_\phi/M_P)^{1.02} f^3(\delta) ,
\]

where for \(0 < \delta \lesssim 1\):

\[
f(\delta) = 1 + 4.8\sqrt{\delta} + 0.5\delta .
\]

If no other processes change the inflaton particle number density in the oscillating field, as we investigate below in more detail, this ratio will remain constant until the present day. Assuming the oscillating inflaton field accounts for all the dark matter in the universe, the present inflaton-to-entropy ratio is given by:

\[
\frac{n_{\phi0}}{s_0} = \frac{3}{4} \frac{T_0}{\Omega_{\phi0} g_{*0}} \frac{\Omega_\phi}{\Omega_{R0} g_{*0}} \approx 10^{-28} \frac{M_P}{m_\phi} ,
\]
where $T_0 = 2.73$ K is the present CMB temperature, while $\Omega_{\phi} h^2 = 0.12$ and $\Omega_{R0} h^2 \approx 4.17 \times 10^{-5}$ are the present abundances of cold dark matter and radiation, respectively [4]. We have also taken into account the present difference between the number of relativistic degrees of freedom contributing to the radiation and entropy densities, $g_{\text{R}} \approx 3.36$, $g_{\text{R}0} \approx 3.91$. Equating Eq. (2.17) and (2.19), we obtain for the inflaton mass:

$$m_\phi \approx 72 g^\prime_\phi 0.12 h^{-1.78} f(\delta)^{1.49} \text{TeV}.$$  

(2.20)

For fermion masses above the kinematical limit and of the order of the inflaton mass, $0 < \delta \lesssim 1$, and taking $g_s = 10 - 100$, we conclude from Eq. (2.14) that the reheating temperature is above 100 MeV for Yukawa couplings $h \gtrsim 10^{-6} - 10^{-5}$, corresponding to inflaton masses $m_\phi \gtrsim 10$ keV. For larger (and arguably more natural) couplings $h \gtrsim 10^{-3} - 10^{-2}$, the inflaton may account for the dark matter in the universe for masses in the GeV–TeV range, similarly to the mass range obtained for WIMP-like thermal relics. Qualitatively, it is easy to understand the parametric dependence of the required inflaton mass. The (incomplete) decay into fermions is more efficient for larger couplings, which affect both the overall value of the decay width and the effective fermion masses, and lighter fermions, yielding a smaller inflaton-to-entropy ratio at late times and thus allowing for larger inflaton masses to match the present dark matter abundance. On the other hand, smaller couplings and heavier fermions lead to a larger inflaton abundance, which may overclose the universe unless the inflaton is sufficiently light.

We note also that there is no gain in considering finely-tuned fermion masses, i.e. $\delta \ll 0$, since even though decay is kinematically allowed for longer as $\delta$ decreases, the decay becomes inefficient when $\Gamma_\phi \sim \dot{H}$, a condition that becomes independent of the tree-level fermion mass in this limit. Although we have restricted the numerical analysis to values of $\delta \lesssim 1$, we expect incomplete decay to be efficient as long as $\Gamma_\phi > \dot{H}$ for inflaton field values satisfying the kinematical condition in Eq. (2.3). In particular, since the decay width takes its maximum value $\Gamma_\phi^{\text{max}} = (h^2/8\pi) m_\phi$ for $h|\phi| = m_f$, i.e. when the fermions are effectively massless, and $\dot{H} \sim m_\phi |\phi|/M_P$ during inflaton-domination, we conclude that decay will be efficient for:

$$m_f \lesssim h^3 M_P,$$  

(2.21)

which allows for quite large fermion masses if the Yukawa coupling is not too suppressed.

**B. Condensate evaporation: the WIMPlaton scenario**

Although the imposed discrete symmetry protects the inflaton from decaying into any other particles except for the fermions $\psi_{\pm}$, the above analysis neglects the effects of additional interactions induced by the Yukawa terms in Eq. (2.1) and which may play an important role as we discuss below.

The classical inflaton field corresponds to a collective state of zero-momentum scalar bosons, assuming that no large field inhomogeneities are formed at the end of the slow-roll inflationary regime. Inflaton particles in this condensate can interact with the fermions that result from its decay and, in particular, these fermions can scatter some of the bosons out of the condensate and promote them to higher-momentum states that become part of the thermal bath. These correspond to scattering processes $\psi_{\pm} (\phi) \rightarrow \psi_{\pm} (\phi)$, where we denote by $\phi$ and $\bar{\phi}$ scalar particles in the zero-momentum condensate and in higher-momentum modes, respectively, and which are mediated through both $s$- and $t$-channel fermion exchange. Moreover, these processes may occur as soon as the field begins oscillating and decay into $\psi_{\pm}$ becomes kinematically allowed, potentially leading to the evaporation of the condensate and the transfer of the inflaton particle number into the thermal bath.

As we have seen earlier, soon after the onset of inflaton oscillations, the temperature of the thermal bath rises sharply to a value that remains approximately constant until inflaton-radiation equality and that corresponds to the reheating temperature in Eq. (2.14). In particular, this temperature exceeds the inflaton and fermion (tree-level) masses in the parameter space region that yields the present-day dark matter abundance. Further assuming that local thermal equilibrium is quickly achieved, as we check below, we may then take the phase-space distributions for inflaton and fermion species in the thermal bath to be the relativistic Bose-Einstein and Fermi-Dirac distributions, respectively. Taking into account the above scatterings and the inverse processes, the net condensate evaporation rate is given by [21]:

$$\Gamma_{\text{evap}} = \frac{1}{n_\phi} \int \prod_{i=1}^{4} \frac{d^3 p_i}{(2\pi)^3 2E_i} \int d^4 p (p_1 + p_2 - p_3 - p_4) \times |\mathcal{M}|^2 \left[ f_1 f_2 (1 + f_3) (1 - f_4) - f_3 f_4 (1 + f_1) (1 - f_2) \right],$$  

(2.22)

where $\mathcal{M}$ is the scattering amplitude for $\langle \phi \rangle (p_1) \psi_{\pm}(p_2) \leftrightarrow \phi (p_3) \psi_{\pm}(p_4)$ and $f_i$ the corresponding phase-space distribution factors. Since the condensate is inherently characterized by large occupation numbers $f_1 \gg 1$, we obtain to leading order for $T \gg m_\phi, m_{\pm}$:

$$\Gamma_{\text{evap}} \simeq \frac{h^4}{12\pi^3} \left( 1 + \log \left( \frac{T}{m_\phi} \right) \right) T,$$  

(2.23)

where we have taken into account the contribution of both fermion species. Note that this is only valid when the fermions are relativistic, while for non-relativistic fermions in local thermal equilibrium the evaporation rate is exponentially suppressed. This means that during the first few oscillations before the inflaton amplitude is significantly reduced, condensate evaporation only occurs during the short periods where decay is also kinematically
allowed. As we have concluded above, for the parameter values yielding the present dark matter abundance we obtain \( T_R \gtrsim m_\phi, m_f \), and since \( m_\perp \sim m_f \) after reheating the above expression holds until either the inflaton or the fermions become non-relativistic. Since in the radiation era \( H \simeq (\pi/\sqrt{90}) g_\ast T^2 / M_p \), condensate evaporation becomes progressively more efficient as the temperature drops. We may then determine a lower bound on the Yukawa coupling such that condensate evaporation is inefficient for \( T \gtrsim m_\phi, m_f \). For comparable masses, we obtain:

\[
\frac{\Gamma_{\text{evap}}}{H} \bigg|_{T=m_\phi} \simeq \frac{h^4}{4\pi^3} \left( \frac{g_\ast}{10} \right)^{-1/2} \left( \frac{M_p}{m_\phi} \right) \lesssim 1 . \tag{2.24}
\]

Using Eq. \( \text{(2.24)} \) for the inflaton mass, we then obtain for \( \delta = 1 \):

\[
h \lesssim 10^{-5} g_\ast^{0.28} . \tag{2.25}
\]

As we had seen above, this is in tension with the lower bound on the coupling required for a reheating temperature above 100 MeV, such that initial conditions for Big Bang Nucleosynthesis (BBN) are already in place after inflaton decay \( \text{22} \). This conclusion is essentially common to all fermion mass values \( m_f > m_\phi / 2 \). Although a more detailed analysis of the Boltzmann equation determining the evolution of the inflaton condensate may be required, this estimate indicates that in the physically interesting parameter range, where the condensate could account for the present dark matter abundance while satisfying the BBN constraint, condensate evaporation is most likely inevitable.

Since evaporation simply transfers the zero-momentum condensate particles into excited states, the conclusion above does not imply that inflaton particles cannot account for dark matter. One can easily check that all 4-body processes involving relativistic inflaton and fermion particles, including scatterings and annihilations, occur at a rate comparable to the evaporation rate obtained in Eq. \( \text{(2.23)} \). Hence, evaporation of the condensate should lead to a bath of fermions and inflaton particles (as well as other species) in local thermal equilibrium. In fact, this may even occur just after the onset of inflaton oscillations, for \( H \lesssim m_\phi \), when the temperature reaches an approximately constant value \( T \gtrsim m_\phi, m_f \) as seen above, for \( h \lesssim 0(1) \) couplings. The condensate’s energy is in this case quickly transferred to the thermal bath, increasing its temperature to a maximum value:

\[
T_R^{\text{max}} = \left( \frac{90}{\pi^2} \right)^{1/4} g_\ast^{-1/4} \sqrt{M_p m_\phi} \simeq 8.5 \times 10^{10} g_\ast^{-1/4} \left( \frac{m_\phi}{1 \text{ TeV}} \right)^{1/2} \text{GeV} . \tag{2.26}
\]

Note that this was obtained for \( H = m_\phi \), which constitutes only an upper bound since fermion particles are only produced for \( H \lesssim m_\phi \) as shown by our numerical simulations. The true reheating temperature will then lie between the value obtained in Eq. \( \text{(2.14)} \) and this maximum value if condensate evaporation occurs before it decays significantly. The reheating temperature is given by Eq. \( \text{(2.14)} \) if condensate evaporation only occurs in the radiation era, which corresponds to Yukawa couplings \( h \lesssim 10^{-4} \) for \( \delta \lesssim 1 \) and \( g_\ast = 10 - 100 \). Note that in this parametric regime thermalization of the fermions in the plasma can only be efficient if their interactions with other degrees of freedom are stronger than those induced by the Yukawa terms, while for \( h \gtrsim 10^{-4} \) the latter occur sufficiently fast to maintain local thermal equilibrium, as we analyze in more detail in section IID.

After reheating, inflaton and fermion particles will be kept in local thermal equilibrium by annihilation and elastic scattering processes. Once these become inefficient, the abundance of inflaton particles will freeze out, as for other conventional WIMP dark matter candidates. Assuming this occurs when both the inflaton and the fermions are non-relativistic, the relevant (fermion t-channel) annihilation cross section is given by:

\[
\sigma_{\phi \phi} \simeq \frac{h^4}{8\pi m_\phi^2} , \tag{2.27}
\]

which is independent of the fermion mass in this limit. Following the standard calculation for the thermal relic abundance of a decoupled non-relativistic species, we obtain for the inflaton mass:

\[
m_\phi \simeq 1.4 h^2 \left( \frac{\Omega_{\phi h_0^2}}{0.1} \right)^{1/2} \left( \frac{g_\ast}{10} \right)^{1/4} \left( \frac{x_F}{25} \right)^{-3/4} \text{TeV} , \tag{2.28}
\]

where \( g_\ast F \) denotes the number of relativistic degrees of freedom at freeze-out and \( x_F = m_\phi / T_F \), with \( T_F \) denoting the freeze-out temperature. This is somewhat smaller than the mass values obtained assuming the oscillating inflaton condensate survives until the present day, although in a comparable range and exhibiting a similar dependence on the Yukawa coupling.

This then gives us a more realistic dynamical picture of what we suggestively denote as the “WIMPlaton” scenario. After inflation, the scalar inflaton begins oscillating about the minimum of its potential, decaying into fermions in short bursts every oscillation. These may thermalize and excite other degrees of freedom in the plasma, and scatter off the inflaton particles in the condensate, leading to its evaporation. Both decay and evaporation increase the relative abundance of radiation and decrease the amplitude of oscillations, until eventually radiation becomes dominant and inflaton decay is no longer kinematically allowed. The stable inflaton particles remain in thermal equilibrium until the temperature drops below their mass and they decouple from the plasma, their frozen abundance yielding the inferred dark matter component of our present universe.
C. Embedding in a consistent inflationary model

As we have concluded from the analysis above, the inflaton field can account for dark matter in the universe at late times for masses below or around the TeV scale in the WIMPPlaton scenario, with a similar mass range obtained assuming the inflaton condensate does not evaporate. This implies that the inflaton potential cannot be given solely by a quadratic mass term, since the amplitude of CMB temperature anisotropies would yield in this case:

\[
m_{\phi} \approx \frac{\sqrt{6\pi^2\Delta_\phi}}{N_\epsilon} M_P \approx 1.4 \times 10^{13} \left( \frac{60}{N_\epsilon} \right) \text{GeV},
\]

(2.29)

which for 50 – 60 e-folds of slow-roll inflation largely exceeds the TeV scale. However, our analysis assumed only that \(m_\phi\) is the inflaton mass as it oscillates about the minimum of the potential at the origin, while the effective inflaton mass can be much larger if slow-roll occurs for significantly larger field values where self-interactions become important. For example, the discrete \(\mathbb{Z}_2\) symmetry allows for quartic self-interactions such that:

\[
V(\phi) = \frac{\lambda}{4!} \phi^4 + \frac{1}{2} m^2 \phi^2,
\]

(2.30)

with the quartic term dominating for \(|\phi| > \sqrt{12/\lambda m_\phi}\). Since it increases the inflaton mass, the quartic self-interaction would make the decay into fermions more efficient and speed up the reheating process. Our analysis is nevertheless valid if the quartic term is subdominant for the field values \(|\phi| \sim m_f/h\) at which decay into fermions occurs, which requires:

\[
h \gtrsim \sqrt{\frac{\lambda}{12}} \left( \frac{m_f}{m_\phi} \right) \approx 3 \times 10^{-8} \left( \frac{m_f}{m_\phi} \right),
\]

(2.31)

where we have used \(\lambda \approx 10^{-14}\) as imposed by the COBE normalization for inflation with a quartic potential. This is easily satisfied if the fermions are not much heavier than the inflaton given the more stringent bounds on the Yukawa coupling discussed earlier. The addition of a quartic term is, however, not sufficient to produce a consistent model of inflation, since it predicts a too red-tilted spectrum for curvature perturbations and a tensor-to-scalar ratio already outside the bounds obtained by Planck [23] and BICEP2 [24].

A consistent spectrum may be achieved in warm inflation scenarios [22], where the coupling between the inflaton and other sectors plays a significant role during inflation. For example, renormalizable couplings between the inflaton and additional fermionic and bosonic species may lead to dissipation of the inflaton’s energy into radiation during the slow-roll period, leading to an observationally consistent spectrum if inflaton particle states thermalize with the other relativistic degrees of freedom [25]. Since this naturally leads to radiation becoming the dominant component at the end of the slow-roll regime, the post-inflationary evolution will necessarily differ from the dynamical picture discussed in this work. It would nevertheless be interesting to investigate whether the inflaton can act as a stable dark matter candidate at late times in this context, although this lies outside the scope of the present discussion.

Another interesting possibility is the inclusion of a non-minimal coupling to the gravitational sector, in particular a coupling between the inflaton and Ricci scalar of the form \(\xi \phi^2 R\), which is compatible with the discrete \(\mathbb{Z}_2\) symmetry. The resulting inflationary scenario yields a perturbation spectrum that smoothly interpolates between the minimal quartic model and the Starobinsky model as the non-minimal coupling constant increases. On the one hand, the latter is characterized by a low tensor-to-scalar ratio and a spectral index \(n_s = 0.96 – 0.97\) in agreement with the Planck results; on the other hand, a small non-minimal coupling constant is preferred to obtain a non-negligible tensor-to-scalar ratio. We refer the reader to [26, 27] for a more detailed discussion of these scenarios, since here we are mainly interested in the post-inflationary dynamics. The effect of the non-minimal coupling on the effective scalar potential in the Einstein frame becomes negligible for \(\xi m_\phi^2/M_P^2 \lesssim 1\), such that consistency of our analysis implies

\[
h \gtrsim \sqrt{\frac{m_f}{M_P}},
\]

(2.32)

which is generally less stringent than Eq. (2.31) for masses in the TeV range and \(\xi < 10^{16}\).

Finally, an important aspect in embedding the interactions in Eq. (2.31) within a consistent inflationary model is the fact that the discrete \(\mathbb{Z}_2 \times S_2\) symmetry does not protect the scalar potential from radiative corrections. In particular, the Yukawa interactions induce loop-corrections of the Coleman-Weinberg form, which for large inflaton field values take the leading form:

\[
\Delta V_f \approx -\frac{h^2 \phi^2}{16 \pi^2} \left( \log \left( \frac{h^2 \phi^2}{\mu^2} \right) - \frac{3}{2} \right)
\]

(2.33)

and therefore induce an effective quartic term in the potential. The effect of this term does not necessarily spoil the predictions of the non-minimally coupled quartic model, as discussed in [28], although one must ensure that the observed normalization of the perturbation spectrum is obtained. While for \(\xi \ll 1\) the effective quartic coupling must have approximately the same value as in the minimally coupled case, which requires \(h \lesssim 10^{-3}\), significantly larger values can be accommodated for large non-minimal couplings.

Radiative corrections can, however, be significantly reduced in supersymmetric scenarios, and a supersymmetric version of the model in Eq. (2.31) with a \(C_2 \subset \mathbb{Z}_2 \times S_2\) symmetry can be easily obtained by considering a superpotential of the form:

\[
W = \frac{h}{2} \Phi \left( Y_1^2 - Y_2^2 \right) + \frac{m_f}{2} \left( Y_1^2 + Y_2^2 \right) + \frac{m_\phi}{2} \Phi^2 + \frac{\lambda}{2} \Phi^2 Z,
\]

(2.34)
where the inflaton and fermions $\psi_{\pm}$ are embedded within
the chiral superfields $\Phi$ and $Y_{\pm}$, respectively, and the aux-
iliary superfield $Z$ induces the quartic term in the infla-
ton potential (noting that the discrete symmetry forbids
cubic inflaton terms in the superpotential). Supersym-
metry then cancels the leading contributions of scalars
and bosons to the 1-loop Coleman-Weinberg potential,
which becomes:

$$\Delta V_{SUSY} \approx \frac{\hbar^2}{16\pi^2} \log \left( \frac{\hbar^2 \phi^2}{\mu^2} \right) \Gamma(\phi) . \quad (2.35)$$

This contribution is thus necessarily smaller than the
tree-level potential $V(\phi) \simeq \lambda^2 |\phi|^4/4$ for $h \lesssim 1$, therefore
avoiding the generation of large effective self-interactions
during inflation.

Besides the Yukawa terms considered so far, the super-
symmetric model also yields scalar interactions between
the inflaton and the scalar partners $y_{\pm}$ of the fermions
$\psi_{\pm}$, which apart from SUSY splittings that vanish at
the origin have the same mass $m_{\pm} = [m_f + h\phi]$. Trilin-
ear terms in the scalar potential also lead to the decay
$\phi \rightarrow y_{\pm} y_{\pm}$, with analogous kinematics and comparable
widths to the fermionic decay channels, therefore yielding
a similar incomplete decay of the inflaton as analyzed
above. We note that the incomplete decay dynamics can be
fully described in terms of scalar fields and is there-
fore not exclusive of fermion Yukawa couplings, although
the required form of the scalar mass terms is more nat-
urally motivated within a supersymmetric context. We
also note the existence of quartic terms in the scalar po-
tential which induce the 3-body decay $\phi \rightarrow z y_{\pm} y_{\pm}$, where
$z$ is the scalar component of the $Z$ chiral multiplet. The
associated couplings have opposite signs for $y_{\pm}$ and pro-
portional to $\lambda$, so that they are typically sub-dominant
with respect to the 2-body decays and they are also kine-
matically forbidden at late times, so that they do not
affect our earlier conclusions.

\section*{D. Reheating the Standard Model}

We have so far assumed that the fermions (or scalars as
discussed above) resulting from inflaton decay thermalize
and excite other degrees of freedom, and in particular it
is crucial that Standard Model particles are generated in
the thermal bath at temperatures above $\sim 100$ MeV so
that BBN may occur following the standard freeze-out
dynamics of light nuclear abundances.

As we have briefly discussed above, the fermions them-
selves cannot be treated as fully relativistic degrees of
freedom before the inflaton decays sufficiently, since their
mass varies between small and large values as the infla-
ton oscillates about the origin. For the short periods
when they are light and decay is allowed, thermaliza-
tion through Yukawa interactions can be quite efficient.
For example, fermion-fermion scatterings through infla-
ton $s$–channel exchange occurs at a rate:

$$\Gamma_{\psi\psi} \simeq \frac{9\zeta(3)}{16\pi^2} h^4 T . \quad (2.36)$$

Since after the onset of oscillations $H \lesssim m_{\phi}$ and tak-
ing the temperature value obtained in Eq. (2.14) from
inflaton decay, we have:

$$\frac{\Gamma_{\psi\psi}}{H} > \frac{\Gamma_{\psi\psi}}{m_{\phi}} \simeq 60g_\ast^{-1/4} h^{-1/4} \left( \frac{m_{\phi}}{1 \text{ TeV}} \right)^{-1/4} , \quad (2.37)$$

such that for $O(1)$ couplings Yukawa interactions can lead
to thermalization from the start of oscillations. Recall
that, as seen above, evaporation of the condensate oc-
curs at a comparable rate, so that as the temperature
rises due to evaporation both processes become progres-
sively more efficient. If the Yukawa couplings are more
suppressed, one may also envisage scenarios where for
example the fermions are charged under a gauge sym-
metry that is unbroken at the relevant temperatures and
thermalization occurs through gauge boson exchange for
sufficiently strong couplings.

Outside the inflaton field range for which the fermions
are effectively light, their number density will become
Boltzmann-suppressed, and any interactions will neces-
sarily become inefficient in keeping the fermions in local
thermal equilibrium. We thus expect them to decouple
for most of the oscillation period, transferring their en-
tropy into light degrees of freedom such as the inflaton it-
self or e.g. gauge bosons. If interactions occur faster than
the inflaton oscillation rate $m_{\phi}$, as given in Eq. (2.37) for
the Yukawa scattering processes, fermions will drop in
and out of local thermal equilibrium as they oscillate be-
tween the relativistic and non-relativistic regimes. This
will lead to an oscillating $g_\ast$, but as mentioned at the
start of our discussion this is of little consequence since
our results exhibit only a mild dependence on this pa-
rameter.

Since the inflaton is typically taken as a gauge singlet,
the structure of the Yukawa interactions implies that $\psi_{\pm}$
are non-chiral fermions (either Dirac or Majorana), as op-
posed to the known SM fermions, being thus unlikely that
they are explicitly charged under the SM gauge group.
There are nevertheless several possibilities for exciting
the SM degrees of freedom in the plasma either before or
after radiation comes to dominate.

The simplest case is perhaps that of unstable $\psi_{\pm}$
fermions decaying into a light scalar and a light fermion,
for which the decay width is:

$$\Gamma_{\psi\psi} = \frac{h_{\psi}^2}{16\pi} m_{\pm} , \quad (2.38)$$

where $h_{\psi}$ denotes the associated coupling constant. Note
that this is computed in the fermions’ rest frame, whereas
in the plasma’s frame an additional time dilation factor
$m_{\pm}/T$ suppresses the decay for relativistic fermions.
For a significant part of the energy density in radiation to be
transferred into these degrees of freedom, the fermions
must decay before they become non-relativistic. Thus,
requiring $\Gamma_{\psi^\pm} \gtrsim H$ for $T \gtrsim m_f$ in the radiation era (where $m_\pm \approx m_f$), we obtain the following bound on the coupling:

$$h_f \gtrsim 8 \times 10^{-8} g_{s,f}^{3/4} \left( \frac{m_f}{1 \text{ TeV}} \right)^{1/2},$$

(2.39)

where $g_{s,f}^{}$ is the number of relativistic degrees of freedom at $T = m_f$. In addition, for this to happen before BBN we require $m_f \gtrsim 100 \text{ MeV}$. If the inflaton and fermion masses are comparable, these corresponds to $h \gtrsim 0.01$ according to Eq. (2.28). These light degrees of freedom may correspond to SM particles if, for example, the fermions coupled to the inflaton field correspond to a pair of degenerate sterile neutrinos, which are singlets under the SM gauge group and may decay into a Higgs-lepton pair through Yukawa terms of the form $h_f H \bar{\psi}_\pm$. Note that this requires $m_f > m_H = 125 \text{ GeV}$ \cite{29} and hence $m_\pm < 250 \text{ GeV}$, which is compatible with the present dark matter abundance for couplings $h \lesssim 0.7$ from Eq. (2.28).

If the fermions are stable, another possibility for reheating the SM degrees of freedom is through efficient annihilation. A possible scenario is for the fermions to be charged under a hidden $U(1)_X$ gauge group, which may mediate fermion scatterings and thus improve the thermalization efficiency. This $U(1)_X$ hidden photon may be kinetically mixed with the SM photon or hypercharge gauge boson $Y^\mu$ through a term of the form $F^\mu_\nu F^\nu_\mu$, which may be generated radiatively if there are fields charged under both gauge groups or simply via gravitational interactions, as happens e.g. in string theory. Diagonalization of the gauge kinetic terms then induces a small electric charge for the fermions $\psi_{\pm}$, such that they may annihilate into SM charged particles via $s$-channel photon exchange, $\psi_\pm \bar{\psi}_\pm \rightarrow \gamma \rightarrow qq, ll$. At high temperature the annihilation cross section is given by the Thomson scattering formula and the corresponding interaction rate for relativistic species is then given by:

$$\Gamma_{\text{th}} \approx \frac{4 \zeta(3)}{\pi} \epsilon^2 \alpha^2 N_{ch} T,$$

(2.40)

where $\epsilon$ is the “mili-charge” of the fermions $\psi_{\pm}$, $\alpha$ is the fine-structure constant and $N_{ch}$ is the effective number of charged species in the final state, which for the full SM is $N_{ch} = 20/3$. In the radiation era, since $H \propto T^2$ annihilation becomes more efficient at smaller temperatures. Once $T \gtrsim m_f$, however, the fermion abundance becomes Boltzmann-suppressed and annihilations can no longer be efficient. Thus, requiring that SM species are excited before $\psi_{\pm}$ become non-relativistic, we obtain the following bound on the mili-charge:

$$\epsilon \gtrsim 5 \times 10^{-7} g_{s,f}^{3/4} \left( \frac{N_{ch}}{20/3} \right)^{-1/2} \left( \frac{\alpha^-}{128} \right) \left( \frac{m_f}{1 \text{ TeV}} \right)^{1/2}.$$

(2.41)

This bound is not very stringent for fermion masses in the GeV-TeV range, where the main constraints come from direct collider searches (including the LHC) yielding $\epsilon \lesssim 0.1$ for $1 \text{ GeV} \lesssim m_f \lesssim 100 \text{ GeV}$ and indirect bounds from the CMB anisotropy spectrum, which yield $\epsilon \lesssim 10^{-4}$ for $m_f \lesssim 1 \text{ TeV}$ based on the effect of mili-charged particles on the baryon-photon oscillations (for a gauge coupling $g_X = 0.1$). Note that for masses above the TeV range, mili-charged particles may give a too large contribution to the dark matter abundance in the universe, and more stringent bounds on $\epsilon$ apply in this case (see \cite{30} and references therein). This thus constitutes a promising scenario with potential for experimental probing in the near future.

The discrete $C_2 \subset Z_2 \times S_2$ symmetry protects the inflaton from decaying at late times, thus constituting a viable dark matter candidate. One can consider, however, scenarios where this symmetry is broken and the inflaton is only meta-stable, with a lifetime longer than the age of the universe, $t_0 \sim 14 \text{ Gyrs}$. Note that interactions between the fermions $\psi_{\pm}$ and other light degrees of freedom as in the scenarios outlined above can induce the decay of the inflaton through radiative effects or processes mediated by off-shell fermions. A few examples of these processes are illustrated in Figure 2 and in all cases the contribution of $\psi_+$ and $\psi_-$ cancels if the discrete symmetry is exact.

![FIG. 2. Feynman diagrams for the 2-body decay of the inflaton into (a) gauge bosons, (b) light scalars and (c) light fermions, induced at the 1-loop level by gauge and Yukawa interactions of the $\psi_{\pm}$ fermions. In (d) we also show the 4-body decay of the inflaton induced by the exchange of virtual $\psi_{\pm}$ modes with Yukawa interactions with other light species. For clarity, all light fields are represented by green lines.](image)

As an example, we have considered the case where $\psi_{\pm}$ are unstable, decaying into a light fermion and scalar, which induce diagrams (b)-(d) in Figure 2. Since these processes have comparable magnitudes, we have computed the inflaton decay width for the 1-loop process (b), where it decays into two light scalars. For concreteness we consider the case where $\psi_{\pm}$ are slightly non-degenerate with $m_- = m_f$ and $m_+ = m_f(1 + \Delta)$, for $\Delta \ll 1$. This gives for $m_0 = 2m_f (\delta = 0)$:

$$\Gamma^{(b)}_\phi = \frac{h^2 h_f^4}{64 \pi^5} m_0 \Delta^2.$$

(2.42)
which is more suppressed for larger values of $m_f$.

For $m_\phi$ yielding the correct relic abundance to account for dark matter and using the lower bound on $h_f$ obtained in Eq. (2.28), we obtain the following upper bound on the inflaton lifetime:

$$
\tau_\phi < 15.7 \ g_{*f}^{-1} \left( \frac{m_\phi}{10 \text{ GeV}} \right)^{-4} \left( \frac{\Delta}{0.01} \right)^{-2} \ \text{Gyrs},
$$

where we have taken the reference values for the present dark matter abundance and freeze-out parameters in Eq. (2.43). Hence, we conclude that significant violations of the discrete symmetry can still yield a sufficiently long-lived inflaton for $100 \text{ MeV} \lesssim m_\phi \lesssim 10 \text{ GeV}$, such that the fermions $\psi_\pm$ decay before BBN and while they are still relativistic. For heavier inflaton particles, values of $\Delta$ below the percent level are required, signaling that the discrete symmetry most hold to a high degree of accuracy in this regime.

It is thus clear from the examples above that if the inflaton field can only decay to the $\psi_\pm$ fermions for a finite period following the end of the slow-roll regime, becoming (meta-)stable at late times, it may account for the dark matter in the universe while allowing for successful reheating of the SM particles and setting the necessary conditions for BBN.

### III. HYBRID MODEL

#### A. Basic properties and dynamics

An alternative framework to the one considered in the previous section is the supersymmetric hybrid model. In this scenario, the inflaton decay products need not include or interact with the Standard Model degrees of freedom, since the additional waterfall sector can be responsible for reheating after inflation.

For the same reasons exposed in the minimal model, the symmetry $C_2 \subset \mathbb{Z}_2 \times S_2$ is imposed on the superfield containing the inflaton and all the superfields that it couples directly to. Hence, for the superpotential to be invariant under the action of this group, the inflaton is, as before, coupled to a pair of superfields $Y_\pm$ which contain the fermions $\psi_\pm$ with masses $m_f$, and to a waterfall sector with a pair of superfields $X_\pm$. The group $C_2$ simultaneously changes $\Phi \rightarrow -\Phi$ and interchanges the superfields $Y_+ \leftrightarrow Y_-$ and $X_+ \leftrightarrow X_-$. The discrete symmetry forbids the linear term in the superpotential that is typically considered in SUSY hybrid inflation to generate the constant vacuum energy driving accelerated expansion. This may nevertheless be generated either by a D-term contribution, or through a non-vanishing F-term coming from a SUSY breaking sector. In addition, in order to ensure the stability of the inflaton at late times, so that it may account for dark matter, the $C_2$ symmetry must be preserved in the ground state, implying equal vacuum expectation values for the scalar components of both waterfall fields. One possibility to satisfy this condition and simultaneously generate a constant vacuum energy is to introduce an additional “driving” superfield, $Z$, which is not charged under the discrete symmetry and is coupled to the waterfall sector, along the lines proposed in [36].

We thus consider a superpotential of the form:

$$
W = \frac{g}{2} \Phi (X_+^2 - X_-^2) + \frac{h}{2} \Phi (Y_+^2 - Y_-^2) + \frac{m_f}{2} (Y_+^2 + Y_-^2) + \frac{\kappa}{2} (Z^2 + X^2 - M^2) + \frac{h}{2} (X_+ + X_-) Q^2 + \ldots ,
$$

where $M$ is a constant mass scale and we have included a coupling between the waterfall superfields and additional chiral superfields $Q$ which give their decay products. The dots indicate additional terms that may be added, involving the inflaton and the superfield $Z$. In particular, if the scalar component of the latter has a sufficiently large mass, either from superpotential terms, soft masses from SUSY breaking in other sectors or non-minimal terms in the Kähler potential, its expectation value will be set to zero both during and after inflation.

The global minimum of the scalar potential will then lie along the real direction $\langle X_+ \rangle = \langle X_- \rangle \equiv \chi / \sqrt{2}$, which preserves the discrete symmetry, and the scalar potential relevant for the inflationary and post-inflationary dynamics has the usual hybrid form:

$$
V(\phi, \chi) = \frac{\kappa^2}{4} (\chi^2 - M^2)^2 + \frac{g^2}{4} \phi^2 \chi^2 + \ldots ,
$$

where $\phi = \sqrt{2}\langle \Phi \rangle$ is the real inflaton scalar field. We recover the usual SUSY hybrid case for $\kappa = g/\sqrt{2}$ and for simplicity we will consider this parametric regime, although our analysis can be extended to the generic case.

Inflation takes place for amplitudes of the inflaton field larger than a critical value, $\phi > \phi_c = M/\sqrt{2}$, such that the waterfall field is held at the origin $\chi = 0$. As the inflaton rolls towards its minimum at $\phi = 0$, its amplitude falls below the critical value and the waterfall field can roll to its true vacuum at $\chi = M$, thus ending inflation. After that point, both fields start to oscillate around its respective minima, triggering the process of reheating the universe into a radiation era.

In this scenario, the inflaton cannot decay into either the bosonic or fermionic components in the waterfall sector due to kinematical blocking. This is easy to see at the global minimum, where $m_\phi = m_\chi = gM$, but extends to all field values. As in the minimal model, the inflaton can decay into the $Y_\pm$ fields and the decay will be incomplete for $m_f > m_\phi/2 = gM/2$. For simplicity, we assume that the scalar components of the $Y_\pm$ fields acquire large soft masses from SUSY breaking and focus on the fermionic decay channels, noting that the inclusion of both channels will not change our conclusions significantly.

The waterfall fields will decay into the $Q$ sector fields and, for similar reasons, we include only the fermionic decay channels in this case as well. We assume that these
fields are light, eventually leading to the complete decay of the waterfall sector and reheating the universe. Note that neither the inflaton nor the waterfall field can be completely stable in order to reheat the universe, since they carry a comparable amount of the energy density after inflation. In particular, if the inflaton were completely stable and behaved as dark matter at all times, the decay of the waterfall field would only convert half of the total energy density into radiation. Its incomplete decay will then reduce the inflaton abundance and hence allow for an efficient reheating once the waterfall field decays. For the inflaton to decay incompletely before the waterfall field, we require \( h \gtrsim h_\chi \), which is the parametric regime on which we will focus henceforth.

The evolution equations driving the post-inflationary dynamics of the inflaton and waterfall fields, as well as their decay products which we assume to quickly thermalize, are then given by:

\[
\ddot{\phi} + 3H \dot{\phi} + g^2 \phi^2 \phi = -\Gamma_\phi \dot{\phi}, \tag{3.3}
\]

\[
\ddot{\chi} + 3H \dot{\chi} + \frac{g^2}{2} (\chi^2 - M^2 + 2\phi^2) \chi = -\Gamma_\chi \dot{\chi}, \tag{3.4}
\]

\[
\dot{\rho}_R + 4H \rho_R = \Gamma_\chi \dot{\chi}^2 + \frac{\Gamma_{\phi}}{\phi} \dot{\phi}^2, \tag{3.5}
\]

where the decay width of the inflaton is \( \Gamma_\phi = \Gamma_+ + \Gamma_- \), with \( \Gamma_\pm \) given by Eq. (2.4), while the decay width of the waterfall field is given by:

\[
\Gamma_\chi = \frac{h_\chi^2 m_\chi}{8\pi}, \tag{3.6}
\]

where we neglect the \( Q \) fermion masses, which we have checked numerically to be a good approximation for couplings \( h_\chi \lesssim 0.1 \) in the parameter range of interest to our discussion.

Eqs. (3.3), (3.4) and (3.5), together with the Friedmann equation,

\[
H^2 = \frac{\rho_{\phi} + \rho_\chi + \rho_R}{3M_P^2} = \frac{1}{2} \dot{\phi}^2 + \frac{1}{2} \dot{\chi}^2 + V(\phi, \chi) + \rho_R, \tag{3.7}
\]

form a complete set of differential equations that can be solved numerically given a set of parameters \( (g, M, m_f, h, h_\chi) \) and initial conditions. Fig. 3 shows an example of the numerical solution for the parameter values \( M = 10^{-2}M_P \), \( g = 10^{-5} \), \( h = 1, h_\chi = 10^{-5} \) and \( \delta = 0.02 \), the latter being defined in Eq. (2.4). In this example we have considered initial conditions such that the fields are close to their values at the end of inflation and have small velocities, \( \phi(0) = 1.0001\phi_c, \chi(0) = 0, \phi'(0) = \chi'(0) = -10^{-4}gM^2 \), with also \( \rho_R(0) = 0 \) since any pre-inflationary radiation should be exponentially diluted by the accelerated expansion. However, we have checked numerically that the results do not show a strong dependence on the choice of initial conditions.

At the beginning of the evolution, both the inflaton and the waterfall field mimic a pair of coupled matter fluids which give a roughly equal contribution to the total energy density. After the incomplete decay of the inflaton becomes efficient, its energy density is transferred into the radiation bath, while the waterfall field evolves as an effective non-interacting matter field in an expanding universe.

Since radiation is diluted more quickly than matter by the cosmological expansion, the universe experiences an era of matter domination until the waterfall field effectively decays and reheats the universe. We then enter the standard radiation-dominated era of Big Bang cosmology, with the oscillating inflaton field remnant behaving as a cold dark matter component.

The temperature after the effective decay of the inflaton, \( T_D \), can be determined analytically following the same reasoning used in minimal model to compute the
reheating temperature, giving:

$$T_D = \left(\frac{15\sqrt{3}}{16\pi^4}\right)^{1/4} g_{*D}^{-1/4} h^{1/4} \left(\frac{m_\phi}{M_P}\right)^{3/4} M_P,$$

(3.8)

where $g_{*D}$ is the effective number of light degrees of freedom when the inflaton effectively decays. Unlike in the minimal model, the reheating temperature is not controlled by the inflaton but rather by the waterfall decay, corresponding to the temperature for which $\Gamma_\chi = H$:

$$T_R \simeq 0.23 g_{*R}^{-1/4} h_\chi (m_\phi M_P)^{1/2},$$

(3.9)

with $g_{*R}$ being the effective number of light degrees of freedom at reheating and where we used $m_\phi = m_\chi$ since the fields are close to the global minimum at this stage.

The computation of the inflaton dark matter abundance after reheating is more involved than in the minimal scenario due to the larger set of parameters in the hybrid scenario and the coupling between the inflaton and waterfall fields. The effect of the parameters of the potential in Eq. (3.3), $g$ and $M = m_\phi/g$ on the final abundance is shown in Fig. 4.

![Diagram](image)

**FIG. 4.** Dependence of the inflaton-to-entropy ratio after reheating multiplied by the inflaton mass in Planck units for different values of the parameters in the potential, $g$, $M$ for $\delta = 0.02$, $h = 1$ and $h_\chi = 10^{-3}$.

For large values of the coupling $g$, the inflaton-to-entropy ratio grows with $g$ since the system is strongly coupled and the waterfall field transfers a significant part of its energy to the inflaton, therefore increasing its abundance. For very low values of the coupling, on the other hand, the inflaton-to-entropy ratio shows a very mild dependence on this coupling. In this case the Hubble parameter $H \sim m_\phi^2/g$ is very large, so that the condition for the incomplete decay to be efficient, $\Gamma_\phi > H$, cannot be maintained for sufficiently long and the abundance is not so drastically reduced. Numerically, we take the endpoint of the curves at small $g$ values such that $V_0^{1/4} = \sqrt{7} M = m_\phi/\sqrt{g} = 10^{16}$ GeV, of the order of the GUT scale, which gives roughly the upper bound on the scale of inflation. The vacuum energy scale therefore diminishes when moving from left to right along each curve of constant $m_\phi$ in Fig. 4.

We will restrict the remainder of our analysis to the values of $g$ for which the incomplete decay exhibits its maximal efficiency, therefore yielding the lowest inflaton abundance for a given mass $m_\phi$. Numerically, we find these values to be given by:

$$g \simeq 0.04 \left(\frac{m_\phi}{M_P}\right)^{0.57} \delta^{-0.08}.$$  

(3.10)

With this relation, we obtain for the inflaton-to-entropy ratio from the numerical results:

$$\frac{n_\phi}{s} \simeq 134 g_{*R}^{-1/4} h_\chi h^{-3.42\beta(\delta)} \delta^{2\gamma(\delta)} \left(\frac{m_\phi}{M_P}\right)^{0.7},$$

(3.11)

$$\beta(h) = 1 - \frac{1}{6} \log_{10} h,$$

(3.12)

$$\gamma(\delta) = 1 + 0.115 \log_{10} \delta.$$  

(3.13)

This expression is equivalent to Eq. (3.4) in the minimal model, with the different powers on the parameters reflecting the more complicated dynamics present in this scenario. Equating (2.19) and (3.11), we then obtain for the inflaton mass yielding the observed dark matter abundance:

$$m_\phi \simeq 442 g_{*R}^{0.15} \left(\frac{h_\chi}{10^{-3}}\right)^{-0.6} h^{-0.6\beta(h)} \delta^{-0.6\gamma(\delta)} \text{GeV}.$$

(3.14)

For couplings $h \sim O(1)$ and $h_\chi \sim 10^{-3} - 10^{-5}$, the inflaton may account for the dark matter in the universe with masses in the GeV-TeV range, similarly to the minimal model, while predicting a reheating temperature well above the BBN constraint. The qualitative dependence of the inflaton mass on the Yukawa coupling $h$ can be understood using the same arguments as in the minimal realization for inflaton dark matter described earlier in this work.

The parameter $h_\chi$ determines the duration of the reheating process, which is due to the decay of the waterfall field. During that process, there is entropy production and as a consequence the abundance $n_\phi/s$ will be further reduced, as shown in Fig. 3 (b). The dilution factor is given by $\Upsilon = S_{eq}/S_R$, with $S$ being the entropy and the subscript “eq” denoting the time at which the $\chi$ thermalized decay products start dominating the radiation bath, at a temperature $T_{eq} > T_R$. The dilution factor is then given by:

$$\Upsilon = \left(\frac{T_R}{T_{eq}}\right)^5 \simeq \frac{5}{3} \left(\frac{T_R}{T_D}\right) \left(\frac{g_{*D}}{g_{*R}}\right),$$

(3.15)

with $T_D$ given in Eq. (3.8). The smaller the coupling $h_\chi$, the longer the reheating process and the smaller $T_R$. 


and the more efficient the reduction of the abundance. Therefore larger inflaton masses are allowed to match the present dark matter abundance.

B. Condensate evaporation: the WIMPlaton scenario

The effect of additional processes induced by the Yukawa coupling $h$ in the evaporation of the condensate of zero-momentum inflaton particles is essentially the same that has been described for the minimal model, with the rate of evaporation given by Eq. (2.28). However, the possibility of the fermions becoming non-relativistic during the matter era, where the dominant contribution to the energy density comes from the waterfall field, gives a different lower bound on the Yukawa couplings such that evaporation is inefficient for $T \gtrsim m_\phi, m_f$:

$$h_\chi \lesssim 5 \times 10^{-10} h \log_{10} h \left( \frac{g_*}{10} \right)^{13/32}. \quad (3.16)$$

Nevertheless, as we found in the minimal scenario of inflaton dark matter, this bound is in tension with the lower bound on the couplings consistent with a reheating temperature above 100 MeV. Therefore, in the region where the condensate may account for the present dark matter and the reheating temperature is consistent with the BBN constraint, condensate evaporation is most likely inevitable. The evaporation process produces a bath of fermions and inflaton particles kept in local thermal equilibrium by annihilation and elastic scatterings that eventually become inefficient, at which point the inflaton abundance freezes out as in the standard WIMP scenario.

If freeze-out occurs after the waterfall field has decayed, i.e. in the radiation-dominated era, we obtain the same value for the inflaton mass as in the minimal model, given in Eq. (2.28). Freeze-out takes place in the waterfall-dominated matter for values of $h_\chi$ satisfying:

$$h_\chi \lesssim 4 \times 10^{-9} h \left( \frac{g_* F}{10} \right)^{3/8} \left( \frac{F}{25} \right)^{-11/8} \left( \frac{\Omega_{\phi0} h^2}{0.1} \right)^{1/4}. \quad (3.17)$$

which then yields an upper bound on the WIMPlaton mass:

$$m_\phi \lesssim 756 h^2 \left( \frac{g_* F}{10} \right)^{1/4} \left( \frac{F}{25} \right)^{-3/4} \left( \frac{\Omega_{\phi0} h^2}{0.1} \right)^{1/2} \text{GeV}, \quad (3.18)$$

where the annihilation cross section is given by Eq. (2.27). In Fig. 5 we summarize on the plane $(h, h_\chi)$ the different possibilities for inflaton dark matter in the hybrid model. In both regions where the freeze-out of the inflaton abundance occurs either in the radiation or waterfall era, the inflaton can account for the present dark matter abundance for masses in the GeV-TeV range with the reheating and freeze-out temperatures being well above the limit imposed by BBN. This shows that the WIMPlaton scenario introduced earlier is not an exclusive feature of the minimal model, with a single dynamical field, but also occurs in other models of inflation with additional dynamical fields.

While the inflaton mass values corresponding to the observed dark matter abundance are not very different in the two realizations that we have analyzed, in the hybrid scenario there are novel phenomenological possibilities. In particular, the inflaton decay products need not interact with the SM degrees of freedom, since these may be excited only after the decay of the waterfall field. Either the waterfall sector decays directly into SM particles or its decay products interact with some of the latter. We note that the waterfall fields may be charged under gauge symmetries, in which case the relevant terms in the superpotential are of the form $\Phi X_\pm \bar{X}_\pm$, etc, where $X_\pm$ and $\bar{X}_\pm$ transform in conjugate representations of the gauge group. This will then open up new avenues for model-building in inflaton dark matter scenarios besides those described in the minimal model, which lie, however, outside the scope of this work.

C. Embedding in an inflationary model

We have explored in the previous sections the dynamics of the incomplete decay of the inflaton field within
and the observational value $P_\mathcal{R} \simeq 5 \times 10^{-5}$ \[23\] to obtain the relation:

$$m_\phi \gtrsim 2.5 \times 10^{15} |\eta| \text{GeV}, \quad (3.20)$$

for $\phi \gtrsim M$, where $m_\phi = gM$ is the inflaton mass at the minimum. Since the scalar spectral index $n_s \simeq 1 + 2\eta \simeq 0.9603 \pm 0.0073$ at 68%CL \[23\] in these scenarios, we conclude that $m_\phi \lesssim \text{TeV}$ cannot yield an observationally consistent model. Typically we have from the normalization of the spectrum $M \sim 10^{13} - 10^{16}$ GeV, and then the WIMPlaton scenario requires small couplings $g \sim 10^{-13} - 10^{-10}$, which are responsible for the very flat potential during inflation and the scale invariant spectrum. A non-minimal Kähler potential can yield the observed spectral index for lower values of the coupling, although at the expense of a slight increase in $M$ \[38, 39\], \[59\], which again makes it difficult to achieve the required WIMPlaton mass values.

An interesting possibility that can realize inflation at low energy scales is to inflate along the waterfall trajectory, which has been shown to yield a sufficiently tilted spectrum for small couplings $g$ within the context of both SUSY and non-SUSY hybrid models \[40, 41\]. Although it has been recently realized that isocurvature contributions yield a too large enhancement of the amplitude of the spectrum unless $N_s \lesssim 60$, the WIMPlaton scenario typically favors a long reheating period, with e.g. $N_s \sim 30 - 40$ being allowed by the data \[42\].

As for the minimal model, a simple way to realize the WIMPlaton scenario is to inflate in a chaotic rather than in a vacuum-dominated regime \[43, 44\]. In particular, a quartic self-coupling $\lambda^2 \phi^4/4$ can be easily induced by a superpotential coupling between the inflaton and the driving field, $\lambda\Phi^2 Z/2$ as in Eq. \[2.31\]. We then have:

$$\frac{\lambda^2 \phi^4}{V_0} \sim 6 \times 10^{16} \left( \frac{\lambda^2}{10^{-14}} \right) \left( \frac{1 \text{ TeV}}{m_\phi} \right)^2 \left( \frac{M_\mathcal{P}}{M} \right)^2 \left( \frac{\phi}{M_\mathcal{P}} \right)^4,$$

so that the quartic term will easily dominate over the vacuum term for super-planckian values and the typical parameters required by the normalization of the spectrum and the WIMPlaton scenario. We note that the vacuum term will come to dominate the energy at small field values, but for $\phi \gtrsim \phi_c$ we have

$$\eta \sim \lambda^2 \frac{\phi^2}{M^2 m_\phi} \gtrsim 10^2 \frac{M_\mathcal{P}^2}{m_\phi} \gg 1,$$ \quad (3.22)

such that slow-roll inflation never takes place in the small field regime. Inflation may then occur entirely in a chaotic regime, with a non-minimal coupling to gravity or warm inflation yielding observables within the Planck window as described for the minimal model.

We note that, in supergravity models, such chaotic inflation scenarios can be obtained by considering a non-minimal Kähler potential for the inflaton, while taking the canonical one for the other superfields in the model, in particular the $Z$ field. One possibility is to consider a Kähler potential with a shift symmetry \[46\], e.g. $K(\Phi, Z, \ldots) = (\Phi + \Phi^\dagger)^2/2 + ZZ^\dagger + \ldots$, with inflation taking place along the imaginary component of the scalar inflaton.

### IV. CONCLUSION

In this work we have shown that the decay of the inflaton following the inflationary slow-roll regime can be incomplete, such that successful reheating is achieved while leaving a stable remnant that can account for the observed dark matter in the universe. This is achieved by coupling the inflaton field $\phi$ to a pair of fermions $\psi_\pm$ (and/or similarly scalar fields) while imposing a discrete symmetry that simultaneously changes $\phi \rightarrow -\phi$ and interchanges the two fermions, corresponding to the discrete subgroup $C_2 \subset \mathbb{Z}_2 \times S_2$. This symmetry forbids all inflaton decay channels except for $\phi \rightarrow \psi_\pm \psi_\pm^\dagger$ if it is preserved in the vacuum state. The inflaton is thus stable at late times if the bare fermion mass $m_f > m_\phi/2$, where $m_\phi$ denotes the inflaton mass at the origin. However, since the physical fermion mass is field-dependent, inflaton decay may occur during the initial oscillations while the field amplitude is sufficiently large.

We have then shown that this partial decay is sufficient for a thermalized bath of radiation, which includes the fermions and potentially other degrees of freedom, to become the dominant component. Since the oscillating inflaton condensate does not decay completely, it can account for the observed dark matter in the universe for masses parametrically within the GeV-TeV range in scenarios with a single dynamical scalar field. However, by
estimating the scattering rate of zero-momentum inflaton particles off thermalized fermions, we concluded that the oscillating condensate will most likely evaporate in parametric regimes where the reheating temperature is above the threshold required for Big Bang Nucleosynthesis. The stable inflaton particles then reach a thermalized state which eventually decouples from the cosmological radiation bath and freezes out as a standard WIMP.

In this WIMPlaton scenario, inflaton masses must also lie in the GeV-TeV range to account for the observed dark matter abundance. While these mass values may \textit{a priori} seem too low to yield the correct amplitude for the primordial spectrum of curvature perturbations, we have shown that the inflaton mass can be much larger during the slow-roll period than at the minimum of the potential. This is for example the case of large-field chaotic models, where the inflaton mass during inflation is set by a quartic self-interaction at super-planckian values and a bare quadratic term close to the origin. Such models are consistent with the latest CMB anisotropy measurements by Planck if e.g. non-minimal couplings to gravity or dissipative effects are included.

In the simplest models with a single dynamical field, the inflaton decay products must interact with the Standard Model (SM) degrees of freedom in order to excite them in the thermal bath. We have explored different possibilities for such interactions, including the case where the inflaton decays into right-handed neutrinos or mili-charged particles in a hidden sector. We have also explored the alternative possibility of hybrid inflation models, where a dynamical waterfall sector, which is also charged under the discrete symmetry, is responsible for reheating the universe. While the incomplete decay of the inflaton is still required to allow the relativistic decay products of the waterfall field to dominate the energy balance, in such scenarios the inflaton decay products need not couple directly to the SM fields, which opens up new phenomenological possibilities.

Since the entropy produced by the waterfall decay dilutes the inflaton condensate’s abundance, it could in principle allow for larger masses accounting for the correct dark matter abundance. However, condensate evaporation will also most likely occur in the viable parametric regimes, such that a WIMPlaton scenario with masses in the GeV-TeV range is again the most probable outcome in hybrid models. A chaotic inflation regime could also provide a consistent inflationary model in this case, although there exist viable scenarios for low-scale inflation such as along the waterfall direction.

A possible alternative that may generically allow for small inflaton masses is the curvaton scenario \cite{20,21}, where the fluctuations of an additional field that is light during inflation dominate the total curvature perturbations when decaying. In the standard realization of this scenario, the curvaton is a late decaying field, and indeed this would bring some additional dilution of the inflaton abundance due to entropy production. While this may relax the bound on the inflaton mass in the condensate scenario, it would have practically no influence when the condensate evaporates, and the same limits on the WIMPlaton mass apply.

Our results show that there exist concrete particle physics models where a single dynamical field can drive inflation in the early universe and account for cold dark matter at late times. This is an appealing feature from the theoretical perspective, since it allows one to address two of the most important problems in modern cosmology within the same simple model. Moreover, we have found that consistent cosmological scenarios typically require the masses of both the inflaton and the particles it interacts directly with to lie in the GeV-TeV range, opening up the possibility of testing both inflation and dark matter at present particle colliders such as the LHC, as well as direct or indirect dark matter searches. Finally, this scenario also singles out particular classes of inflationary models that can be tested with CMB experiments such as Planck, which are now reaching unprecedented levels of precision. We thus hope that our work motivates further exploration of the phenomenological and observational consequences of the proposed generic mechanism in its several possible implementations.

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