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Thermodynamic properties of asymptotically anti-de Sitter black holes in $d = 4$ Einstein–Yang–Mills theory

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ABSTRACT

We investigate the thermodynamics of spherically symmetric black hole solutions in a four-dimensional Einstein–Yang–Mills-SU(2) theory with a negative cosmological constant. Special attention is paid to configurations with a unit magnetic charge. We find that a set of Reissner–Nordström–Anti-de Sitter black holes can become unstable to forming non-Abelian hair. However, the hairy black holes are never thermodynamically favoured over the full set of abelian monopole solutions. The thermodynamics of the generic configurations possessing a noninteger magnetic charge is also discussed.

1. Introduction

Black holes are non-perturbative objects whose existence appears to be an unavoidable consequence of general relativity (and its various extensions). Moreover, as is often stated in the literature, the black holes (BHs) are the quantum gravity counterparts of the hydrogen atom in ordinary quantum mechanics. Thus some basic results derived at the semiclassical level, like the existence of Hawking radiation together with an intrinsic BH entropy are expected to be very basic features that any putative quantum theory of gravity will have to take into account.

As a result, the subject of BH thermodynamics has enjoyed a constant interest over the last four decades. BH solutions in anti-de Sitter (AdS) spacetime background have been considered also in this context. For example, as shown by Hawking and Page [1], the presence of a negative comological constant makes it possible for a BH to reach stable thermal equilibrium with a heat bath. Moreover, according to the AdS/CFT conjecture [2], BH solutions with AdS asymptotics would offer the possibility of probing the nonperturbative structure of some conformal field theories.

The study of thermodynamics of BH solutions violating the no hair conjecture is particularly interesting. This conjecture states that the only allowed characteristics of a stationary BH are those associated with the Gauss law, such as mass, angular momentum and U(1) charges [3]. Apart from a pure mathematical interest, the BHs with hair may be useful for probing not only quantum gravity, but also may play an important role in the context of the AdS/CFT correspondence.

The first (and still the best known) example of hairy BH solutions is that in Einstein–Yang–Mills (EYM) theory. Moreover, this example can be regarded as canonical in the sense that other hairy solutions usually share a number of common characteristics with the EYM case (a review of hairy non-Abelian BHs solutions with a cosmological constant $\Lambda \geq 0$ can be found in [4]). BHs with non-Abelian (nA) hair in AdS background have also been extensively studied, starting with the pioneering work [5]. These EYM solutions possess a variety of interesting features which strongly contrast with those of the asymptotically flat spacetime counterparts in [6]. For example, stable BHs with a magnetic charge are known to exist even in the absence of a Higgs field (see [7] for a review of these solutions).

Moreover, considering such configurations is a legitimate task, since the gauged supergravity models (of interest in AdS/CFT context) generically contain the EYM action as the basic building block.

The main purpose of this work is to address the issue of the thermodynamical behaviour of the AdS solutions with a spherical horizon topology in EYM-SU(2) theory, a problem which, to our knowledge, has not yet been addressed in a systematic way. Special attention is paid to configurations sharing the asymptotics with the Schwarzschild-AdS and the (embedded Abelian) Reissner–Nordström-AdS BHs. Also, for simplicity, we shall restrict our study to configurations featuring magnetic fields only.

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Our results show that the EYM solutions possess a variety of new features, the generic picture depending on the value of the magnetic charge as well as on the ratio of the four-dimensional gravitational constant to the Yang–Mills coupling. Moreover, as we shall see, the thermodynamical properties of the solutions depend also on the topology of the horizon, the case of spherical configurations being special.

2. The solutions

The hairy BHs discussed in this work are solutions of the Einstein–Yang–Mills-SU(2) equations with a negative cosmological constant $\Lambda = -3/L^2$,

$$R_{\mu\nu} - \frac{1}{2}Rg_{\mu\nu} - \frac{3}{L^2}g_{\mu\nu} = 8\pi G\left(F^{(a)}_{\mu\alpha}F^{(a)}_{\nu\beta}g^{\alpha\beta} - \frac{1}{4}g_{\mu\nu}F^{(a)}_{\alpha\beta}F^{(a)\alpha\beta}\right),$$
$$D_\mu F^{\mu\nu} = 0. \tag{1}$$

We are mainly interested in spherically symmetric solutions, with magnetic fields only, a case which can be studied within the following ansatz

$$ds^2 = \frac{dr^2}{N(r)} + r^2(d\theta^2 + \sin^2\theta d\phi^2) - \sigma^2(r)N(r)dt^2,$$
$$\text{with} \quad N(r) = 1 - \frac{2m(r)}{r} + \frac{r^2}{L^2}, \tag{2}$$

for the metric, and

$$A = \frac{1}{2\hat{g}}\left[w(r)\tau_1 d\theta + (\cos\theta\tau_3 + w(r)\sin\theta\tau_2)d\phi\right], \tag{3}$$

for the gauge fields (with $\hat{g}$ the gauge coupling constant and τ_a the Pauli matrices). Then the field equations (1) reduce to

$$m' = \alpha^2\left(Nw'^2 + \frac{V^2(w)}{2r^2}\right), \quad \sigma' = 2\alpha^2\sigma\frac{w'^2}{r},$$
$$w'' + \left(\frac{N'}{N} + \frac{\sigma'}{\sigma}\right)w' + \frac{wV(w)}{r^2N} = 0 \tag{4}$$

(with $V(w) = (1 - w^2)$), where we have defined the coupling constant

$$\alpha^2 = \frac{4\pi G}{\hat{g}^2}. \tag{5}$$

We want the metric (2) to describe a nonsingular, asymptotically AdS spacetime outside a horizon located at $r = r_H > 0$ (here $N(r_H) = 0$ is only a coordinate singularity where all curvature invariants are finite). There are two explicit solutions satisfying these assumptions. If $w^2(r) \equiv 1$, then

$$N(r) = \left(1 - \frac{r_H}{r}\right)\left(1 + \frac{r^2}{L^2} + \frac{rr_H}{L^2} + \frac{r_H^2}{L^2}\right), \quad \sigma(r) = 1, \tag{6}$$

which is just the Schwarzschild-AdS (SAdS) metric, with vanishing YM curvature and mass $M = \frac{r_H}{2}(1 + \frac{r_H^2}{L^2})$. A different solution is found for $w(r) \equiv 0$, with

$$N(r) = \left(1 - \frac{r_H}{r}\right)\left(1 + \frac{r^2}{L^2} + \frac{rr_H}{L^2} + \frac{r_H^2}{L^2} - \frac{\alpha^2}{rr_H}\right), \quad \sigma(r) = 1. \tag{7}$$

This describe the embedded Abelian magnetic Reissner–Nordström-AdS (RNAdS) metric, with mass $M = \frac{1}{2r_H}(\alpha^2 + r_H^2(1 + \frac{r_H^2}{L^2}))$, and unit magnetic charge.

The general solutions are constructed numerically. However, one can write also an approximate expression close to the horizon and for large-r; the first terms in a near-horizon power series expansion in $r - r_H$ read

$$m(r) = \frac{r_H(r_H^2 + L^2)}{2L^2} + \alpha^2\frac{(1 - w_H^2)^2}{2r_H^2}(r - r_H) + \ldots,$$
$$\sigma(r) = \sigma_H + \frac{2\sigma_H\alpha^2 r_H L^4 w_H^2(1 - w_H^2)^2)}{\left(3r_H^4 + r_H^2L^2 - \alpha^2L^2(1 - w_H^2)^2\right)^2}(r - r_H) + \ldots,$$
$$w(r) = w_H + \frac{r_H L^2 w_H(w_H^2 - 1)}{3r_H^4 + r_H^2L^2 - \alpha^2L^2(1 - w_H^2)^2}(r - r_H) + \ldots, \tag{8}$$

with r_H, w_H and σ_H input (positive) parameters fixing the Hawking temperature and event horizon area of the solutions (with the entropy $S = A_H/4G$)

$$T_H = \frac{1}{4\pi}\frac{\sigma_H\left(3r_H^4 + r_H^2L^2 - \alpha^2L^2(1 - w_H^2)^2\right)}{r_H^3L^2}, \quad A_H = 4\pi r_H^2. \tag{9}$$

A similar expression can also be written for $r \to \infty$ as a power series in $1/r$, with

$$m(r) = M - \frac{\alpha^2(2J^2 + L^2(1 - w_0^2)^2)}{2L^2r} + \ldots,$$
$$\sigma(r) = 1 - \frac{1}{2r^4}\alpha^2J^2 + \ldots, \quad w(r) = w_0 - \frac{J}{r} + \ldots, \tag{10}$$

with three free parameters: M, which fixes the ADM mass of the solutions, w_0 which gives the (SU(2)-valued) magnetic charge $\mathcal{Q}_M = Q_M\frac{\tau_3}{2\hat{g}}$ (where $Q_M = 1 - w_0^2$) and J – an order parameter which provides a measure of non-Abelianess.

Note also that the equations (4) possess two scaling symmetries: (*i*) $\sigma \to \lambda\sigma$ and (*ii*) $r \to \lambda r$, $m \to \lambda r$, $L \to \lambda L$, $\alpha \to \lambda\alpha$ (with λ a positive scaling parameter). The symmetry (*i*) was used to set $\sigma(r) \to 1$ as $r \to \infty$. The symmetry (*ii*) can be used to fix the value of the AdS radius L or the value of the coupling constant α; in this work we set $L = 1$ and treat α as an input parameter (also, to simplify the expression of some quantities, we set $G = 1$).

Restricting to a canonical ensemble (which is the natural one in the presence of a magnetic charge), we study solutions holding the temperature T_H and the magnetic charge Q_M fixed. The associated thermodynamic potential is the Helmholtz free energy

$$F = M - T_H S. \tag{11}$$

When different solutions exist at fixed (T_H, Q_M), the configuration with lowest F is the one that is thermodynamically favoured. Also, the condition for (local) thermodynamic stability of BHs in the canonical ensemble is

$$C = \left(\frac{\partial M}{\partial T_H}\right)_{Q_M} = T_H\left(\frac{\partial S}{\partial T_H}\right)_{Q_M} > 0. \tag{12}$$

As recently found in [8], the generic hairy solutions satisfy the first law of thermodynamics

$$dM = \frac{\sigma_H}{2r_H^2L^2}\left(3r_H^4 + r_H^2L^2 - \alpha^2L^2(1 - w_H^2)^2\right)dr_H + \frac{\alpha^2w_0J}{L^2}dw_0$$
$$= T_H dS + \Phi dQ_M, \tag{13}$$

where $\Phi = -\alpha^2J/(2w_0L^2)$ is the magnetostatic potential.

The most remarkable feature of the AdS EYM BHs is perhaps that the value of the parameter w_0 in (10) (*i.e.* the value of the magnetic charge) is not fixed *apriori*. That is, for given α and any horizon size (as set by the input parameter r_H), solutions are found

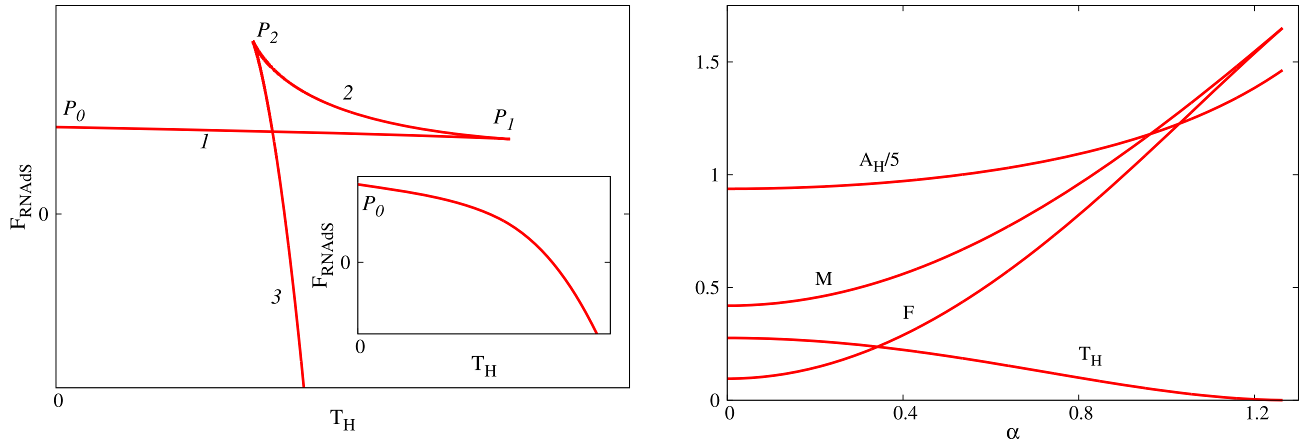

Fig. 1. *Left:* The free energy is shown as a function of temperature for generic embedded Abelian Reissner–Nordström-AdS black holes with $0 < \alpha < L/6$, in which case one notices the existence of three branches of solutions. The generic picture for $\alpha > L/6$ is displayed in the inset, with the existence of a single branch of solutions. *Right:* The mass, free energy, temperature and area of the unstable Reissner–Nordström-AdS black holes where a branch of non-Abelian solutions emerges is shown as a function of α.

for intervals of w_0 and not only $w_0^2 = 1$, as for $\Lambda = 0$ (note however, that the allowed range of w_0 decreases as α increases [5,9,10]).

Moreover, as for the better known case of asymptotically flat hairy BHs [6], the solutions here are also indexed by the node number of the magnetic potential.

In this work we shall consider nodeless (for $w_0 > 0$) and one node solutions (for $w_0 < 0$) only. The configurations with higher number of nodes represent excited states and are therefore ignored in what follows. Moreover, the solutions with nodes are unstable in linearized perturbation theory. A detailed discussion of these aspects together with a large set of references can be found in [7].

3. The thermodynamics of non-Abelian black holes

3.1. The unit magnetic charge solutions

Among all sets of nA BHs, of particular interest are those solutions possessing a unit magnetic charge, *i.e.* with $w_0 = 0$ in the far field expansion (10). The existence of such configurations provides an obvious violation of the no-hair conjecture, since two distinct solutions are found for the same set of global charges. That is, apart from the gravitating Dirac mononopole configuration (7) with $w(r) \equiv 0$, there is also an (intrinsic nA) configuration possessing a nontrivial profile of the magnetic potential $w(r)$.

At this point it is instructive to briefly review the thermodynamics of the magnetically charged RNAdS solutions[1] (7). The free-energy *vs.* temperature behaviour of these solutions is summarized in Fig. 1 (left). For any $\alpha > 0$, a branch of RNAdS BHs emerges at $P_0\big(0, \frac{L}{2\sqrt{3}}p_-(1+p_+^2)\big)$, a point which corresponds to extremal BHs with a nonzero area $A_H(T_H = 0) = \frac{2\pi L^2}{3}p_-^2$ (where we note $p_\pm = \sqrt{\sqrt{1+(2\sqrt{3}\frac{\alpha}{L})^2} \pm 1}$ and $q_\pm = \sqrt{1 \pm \sqrt{1-(\frac{6\alpha}{L})^2}}$. For $0 < \alpha < L/6$, this branch continues to the point $P_1\big(\frac{1}{\pi L\sqrt{3}}(q_- + \frac{1}{q_-}), \frac{L}{6\sqrt{6}}q_-(1+q_+^2)\big)$, where a cusp occurs signaling a second branch of solutions extending backward in T_H, with an increasing F (the corresponding value of the mass at P_1 is $\frac{L\sqrt{2}}{3\sqrt{3}}q_-^2$).

This secondary branch ends at $P_2\big(\frac{1}{\pi L\sqrt{3}}(q_+ + \frac{1}{q_+}), \frac{L}{6\sqrt{6}}q_+(1+q_-^2)\big)$, where another cusp is found, with the occurrence of a third branch of solutions (the corresponding value of the mass at P_2 is $\frac{L\sqrt{2}}{3\sqrt{3}}q_+^2$). This branch intersects the first one at some $T_H^{(c)}$ (with the occurrence of a "swallowtail" shape) and extends to arbitrarily large T_H (note that the free energy vanishes for $T_H = \frac{\sqrt{2}}{\pi L}(p_+ + \frac{1}{p_+})$ (a point where $M = \frac{L}{3\sqrt{2}}p_+(1+p_+^2)$) and becomes negative for larger values of T_H.

The first branch solutions dominate the thermodynamic ensemble for $T_H < T_H^{(c)}$; however, for higher temperatures, the free energy is minimized by the solutions on the third branch. However, the relative size of the second branch decreases with α, the points P_1 and P_2 coinciding for $\alpha = L/6$, where the second branch of solutions disappears. Then, for $\alpha > L/6$ only one single branch of solutions is found, see the inset in Fig. 1 (left) (from there, it is also clear that, for any α, at low enough temperatures there can only be one solution).

The picture valid for the nA solutions is different. First, unit magnetic charge solutions are found for $0 < \alpha < \alpha_{max}$ only, with $\alpha_{max} \simeq 1.264$. Another striking feature is the existence of a soliton limit of the BHs,[2] which is approached as $r_H \to 0$, where $T_H \to \infty$, $A_H \to 0$ and $F \to M > 0$.

Moreover, we notice that the nA solutions exist above a minimal value of $T_H > 0$ only. In particular, no extremal BHs with nA hair is found, which agrees with the recent results in [13].

Also, as seen in Fig. 2, for a given temperature, the free energy of all solutions is minimized by a RNAdS solution. For $\alpha < L/6$, this RNAdS solution is located on the first branch (for $T < T_H^{(c)}$) or on the third branch (for higher temperatures). Thus we conclude that all unit magnetic charge nA solutions are globally thermody-

[1] The thermodynamics of RNAdS solution has been discussed from a slightly different perspective in [11], for electrically charged BHs; however, due to the existence of electric–magnetic duality in $d = 4$ Einstein–Maxwell theory, the results there apply to magnetically charged RNAdS solutions as well.

[2] An exact particle-like solution with $w_0 = 0$ is known for $\alpha = 0$ (*i.e.* the probe limit–YM fields in a fixed AdS background), and has a magnetic potential $w(r) = 1/\sqrt{1+\frac{r^2}{L^2}}$ [12]. Moreover, one can write a perturbative solitonic solution (with unit magnetic charge) of the EYM eqs. (4) by considering a perturbative expansion in α^2 around the global AdS background. For example, the first order corrected metric functions are $m(r) = \frac{3\alpha^2}{4}\left(\frac{1}{L}\arctan(\frac{r}{L}) - \frac{r}{L^2+r^2}\right)$, $\sigma(r) = 1 - \alpha^2\frac{L^2}{2(L^2+r^2)^2}$.

This solution captures some of the basic features of the general nonperturbative configuration and can also be extended to higher orders in α; however, so far we could not identify a general pattern; moreover, the expressions for the functions become increasingly complicated.

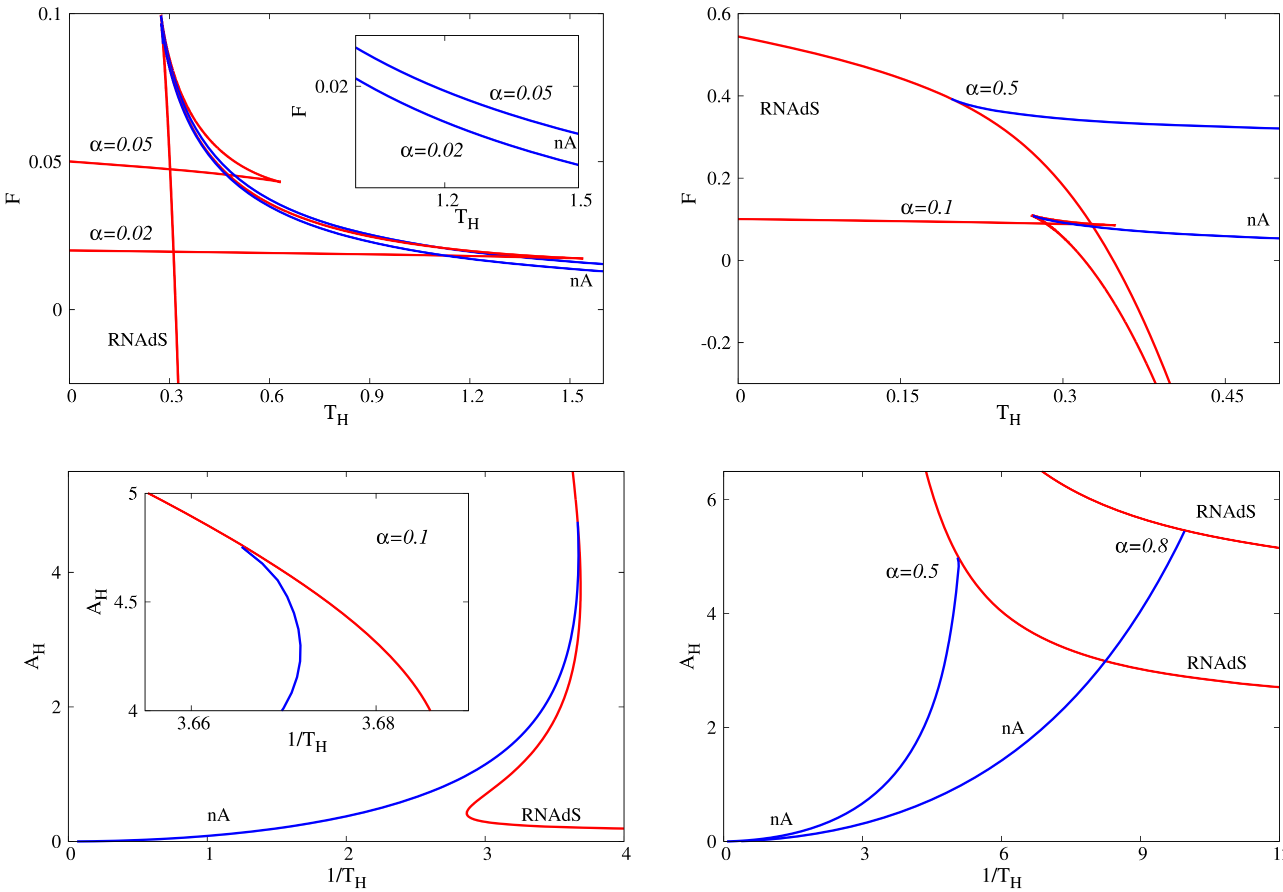

Fig. 2. The free energy and the horizon area are shown as a function of temperature for embedded Abelian (red curves) and non-Abelian (blue curves) solutions with unit magnetic charge and several values of α. (For interpretation of the references to color in this figure legend, the reader is referred to the web version of this article.)

namically unstable (despite the fact that their specific heat can be positive for some range of r_H).

Remarkably, one finds that, as the minimal value of T_H is approached, the branch of nA solutions joins smoothly a critical RNAdS solution with the same value of α, such that the magnetic gauge function $w(r)$ becomes identically zero. This bifurcation can also be viewed as indicating that the embedded RNAdS BH presents an instability with respect to static nA perturbations, for a critical value of the horizon radius, a feature that, to our best knowledge, has not been noticed before in the literature This instability can be studied within the same Ansatz (3), by considering values of the magnetic gauge potential $w(r)$ close to zero everywhere, $w(r) = \epsilon W(r)$, and a fixed RNAdS background. The perturbation $W(r)$ starts from some nonzero value at the horizon and vanishes at infinity, being a solution of the linear equation

$$(N(r)W'(r))' + \frac{W(r)}{r^2} = 0, \tag{14}$$

with $N(r)$ given by (7). Although the above equation does not appear to be solvable in terms of known functions, one can write again an approximate solution near the horizon and at infinity. The general solution is constructed numerically. Given $0 < \alpha < \alpha_{max}$, this reduces to finding the critical value of r_H such that the function $W(r)$ has the proper behaviour. Some parameters of the critical RNAdS solutions are shown in Fig. 1 (right). For $0 < \alpha < L/6$, the unstable RNAdS solutions are located on the second and third branches in a small region around the point P_2 (see Fig. 1 (left)). Further increasing α, the unstable solutions move to smaller temperatures, the point P_0 being approached for $\alpha \to \alpha_{max}$, with $W(r) \equiv 0$ in that limit. Also, an analytic estimate for the critical horizon radius of the RNAdS BHs, $r_H(\alpha)$, can be found by matching at some intermediate point the expansion of $W(r)$ (and its first derivative) at the horizon, to that at infinity. Surprisingly, it turns out that the simple expression $r_H = \frac{\sqrt{\alpha L}}{3^{1/4}}$ provides a good approximation for most of the numerical data (typically with several percent error).

Let us also remark that the nA configurations cannot be the end points of this instability, since, as noticed above, they are not thermodynamically favoured over all sets of Abelian configurations. Although further work is necessary, we conjecture that the branch of hairy BHs would represent intermediate states only, with a phase transition to (final) RNAdS configurations.

3.2. Solutions with a vanishing magnetic charge

Another case of interest corresponds to solutions with $w_0^2 = 1$ in the far field asymptotics (10) (*i.e.* with a vanishing magnetic charge) which possess, however, a non-vanishing magnetic field in the bulk.[3]

The picture we have found is rather different in this case. First, these hairy BHs do not emerge as perturbations[4] of the SAdS so-

[3] These are the natural AdS counterparts of the well-known asympotically flat solutions in [6], which necessarily have $w_0^2 = 1$. Also, note that these are unstable configurations with respect to small perturbations, the magnetic potential possessing at least one node.

[4] That is, one can show that the linearized YM equation in (4) with $w(r) = -1 + \epsilon W_1(r)$ in a SAdS background does not possess solutions which are finite at $r = r_H$ and vanish asymptotically.

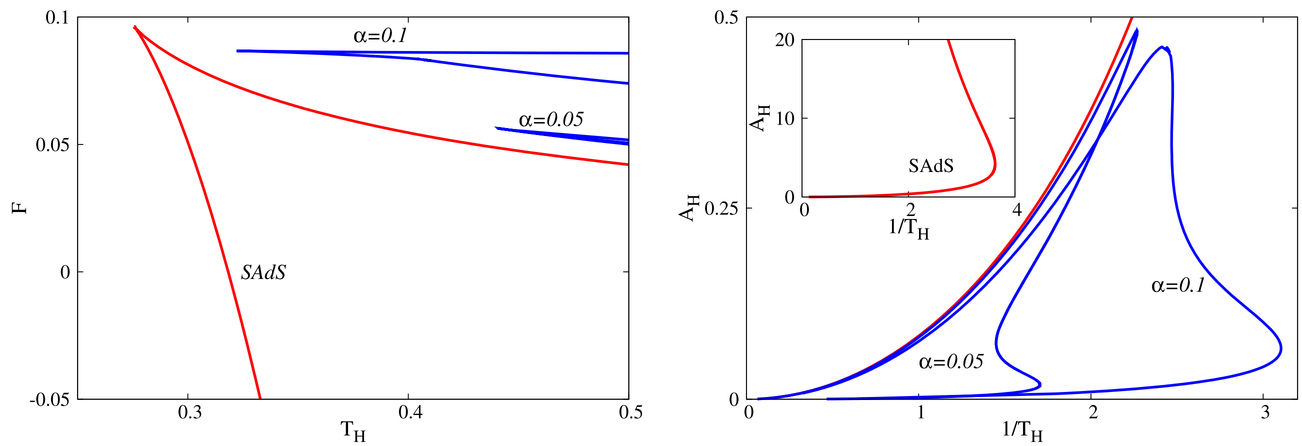

Fig. 3. The free energy and the horizon area are shown as a function of temperature for the vacuum Schwarzschild-AdS black holes (red curves) and non-Abelian (blue curves) solutions with vanishing magnetic charge and two values of α. (For interpretation of the references to color in this figure legend, the reader is referred to the web version of this article.)

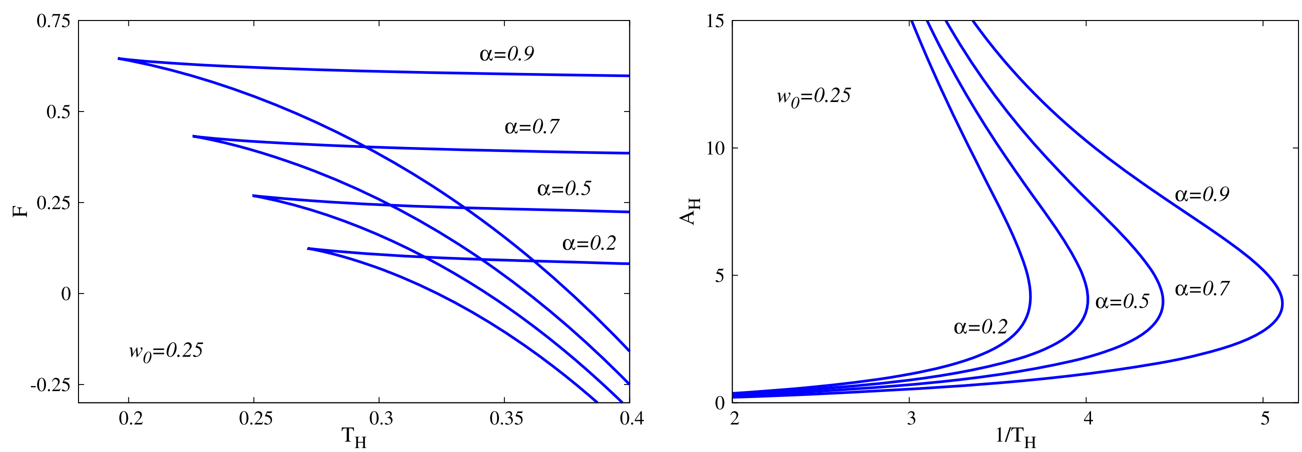

Fig. 4. The free energy and the horizon area are shown as a function of temperature for generic non-Abelian black solutions with a noninteger magnetic charge and several values of α.

lutions [6]. Instead of that, one finds two branches of solutions, which, in an $F(T_H)$ diagram form a cusp for some minimal value of $T_H > 0$. As seen in Fig. 3, this minimal value of T_H decreases with α (the well-known $F(T_H)$ diagram for the SAdS black holes is also shown there; note that the vacuum solutions do not possess a dependence on α). Moreover, the large SAdS solution is always thermodynamically favoured, minimizing the free energy.

Here it is interesting to consider the $\Lambda \to 0$ limit of these EYM BH solutions. Then, by using the data in [6], one can easily verify that the difference between the free energy of a hairy BH and the Schwarzschild solution with the same temperature is always positive. The asymptotically flat EYM solutions are known to be unstable in perturbation theory [4]. Then their thermodynamics also suggests that they should decay to a Schwarzschild vacuum BH.

3.3. The general case

The generic solutions have $w_0^2 \neq 1$ and $w_0 \neq 0$, possessing a nonvanishing (and noninteger) magnetic charge.[5] As shown in Fig. 4, the thermodynamics of the generic solutions resembles that of the SAdS BHs, with the existence of two branches of solutions. However, in this case, the small BHs branch emerges from a solitonic solution with a vanishing horizon area and a nonzero mass, extending backwards in T_H. Then a cusp occurs for a minimal temperature, with the emergence of a branch of large BHs. The free energy is minimized by the secondary branch solutions, which possess also a positive specific heat, $C > 0$. Also, note that a similar picture has been found for other values of w_0 apart from the one in Fig. 4.

4. Further remarks

It is well known that the RNAdS BH solutions possess a variety of interesting thermodynamical features, see *e.g.* [11]. As expected, we have found that enlarging the gauge group of the matter fields to SU(2) leads to an even more complicated picture. For example, as seen already in other cases when the Einstein–Maxwell system is considered as part of a larger theory (see *e.g.* [14]), we have found that the (unit magnetic charge) RNAdS BH can become unstable to forming hair at low temperatures. This results in new branches of hairy solutions of the larger theory.[6] However,

[5] Strictly speaking, these EYM BHs do not violate the no hair conjecture (we recall that the RNAdS solution possesses a unit magnetic charge only, when viewed as solution of the EYM theory).

[6] A related example is the case of asymptotically flat, magnetically charged RN BHs embedded in Einstein–Yang–Mills–Higgs theory, which develop nA hair for some range of the parameters [15]. For $\Lambda < 0$, it is the asymptotic structure of spacetime which effectively replaces the Higgs field.

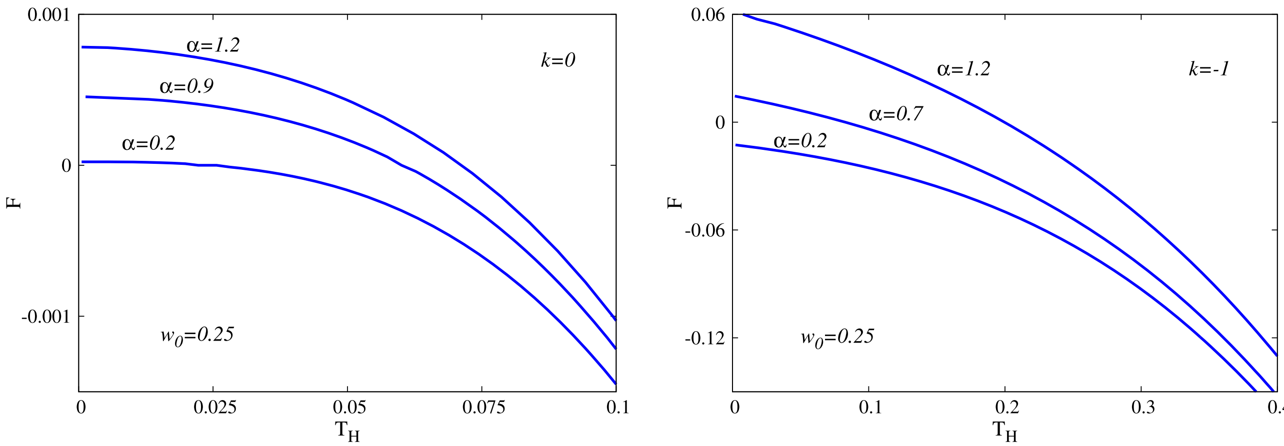

Fig. 5. The free energy density is shown as a function of temperature for EYM black holes solutions with planar ($k=0$) and hyperbolic ($k=-1$) horizon topology and several values of α.

our results show that the hairy BHs are never thermodynamically favoured over the full set of RNAdS solutions.

There are various possible extensions of the results discussed above. First, our preliminary results indicate the existence of static axially symmetric BH solutions with nA hair. These solutions are the counterparts of the particle-like EYM configurations discussed in [16] being constructed by using a similar approach, and possess an event horizon of spherical topology, which, however, is not a round sphere. They are found for an axially symmetric generalization of the magnetic YM ansatz (3) with two positive discrete parameters n and m (which are the azimuthal and polar winding numbers, respectively), the spherically symmetric ansatz (3) being recovered for $n=m=1$. These BHs possess as well a continuous parameter w_0 (which is a generalization of the constant w_0 entering the asymptotics (10)), which fixes the magnetic charge of the solutions [16].

Again, of particular interest are (i) the configurations whose far field asymptotics of the YM fields describe a gauge transformed charge-n Abelian multimonopole, and (ii) the configurations with a vanishing net magnetic flux. The axially symmetric solutions we have constructed so far in a more systematic way, have $m=1$ and $n=2,3,4$ and thus represent deformations of the configurations in Section 2, sharing their basic properties. In particular, as expected, we have found that the charge-n RNAdS solution is always thermodynamically favoured over the nA ones. We hope to report elsewhere on these aspects.

A rather different picture is found when considering '*topological black hole*' generalizations of the solutions in Section 2. In this case, the two-sphere in the metric ansatz (2) is replaced by a two-dimensional space of negative or vanishing curvature[7] (see *e.g.* [18] for vacuum solutions), the corresponding metric ansatz being

$$ds^2 = \frac{dr^2}{N(r)} + r^2(d\theta^2 + f_k^2(\theta)d\phi^2) - \sigma^2(r)N(r)dt^2,$$

where $N(r) = k - \frac{2m(r)}{r} + \frac{r^2}{L^2}$, with k a discrete parameter which fixes the topology of the horizon. For $k=1$, $f_1 = \sin\theta$ and the metric ansatz (2) is recovered; the solutions with $k=0$ have $f_0 = \theta$ and are planar BHs, approaching asymptotically a Poincaré patch of the AdS spacetime; finally, the solutions with $k=-1$ possess a hyperbolic horizon, with $f_{-1} = \sinh\theta$. The corresponding general-k YM ansatz has been displayed in [17] and looks very similar to (3), with

$$A = \frac{1}{2\hat{g}}\left[w(r)\tau_1 d\theta + (\frac{df_k(\theta)}{d\theta}\tau_3 + f_k(\theta)w(r)\tau_2)d\phi\right].$$

The equations of the model are still given by (4), with $V(w) = k - w^2(r)$ in the general case.

A study of the general-k case shows that the EYM BHs with a spherical event horizon topology ($k=1$) are special. First, a soliton limit of the BHs exists only in this case [17]; also, the embedded Abelian solution can possess an instability for $k=1$ only. Moreover, the EYM BHs with a planar or hyperbolic horizon topology are intrinsic nA, since they cannot approach asymptotically a vacuum SAdS or a RNAdS configuration (note that the magnetic gauge potential $w(r)$ is a nodeless function for $k \neq 1$). In fact, an embedded Abelian solution exists for $k=\pm 1$ only, *i.e.* $w(r) \equiv 0$ implies $m(r) = M - \alpha^2 k^2/(2r)$, $\sigma(r) = 1$, which is the (planar) SAdS BH for $k=0$. Also, no reasonable YM vacuum state exists for $k=-1$.

As shown in Fig. 5, in strong contrast to the $k=1$ case, the EYM BHs with a non-spherical horizon topology exhibit a single branch of solutions.[8] Moreover, the $k=0,-1$ solutions are (locally) thermally stable, since $C>0$ (similar results have been found for other values of the magnetic charge, as fixed by the parameter w_0 in the asymptotic expansion (10)).

As an avenue for further research, it would be interesting to extend the analysis in this work to BH dyons, *i.e.* solutions featuring both magnetic and electric nA fields. The study of a special class of such EYM black brane solutions (*i.e.* with a planar horizon, $k=0$) has led to the discovery of holographic superconductors, describing condensed phases of strongly coupled, planar, gauge theories in $d=3$ dimensions [19,20] (note that in this case the magnetic field vanishes on the boundary, such that the hairy solutions share the asymptotics with the electrically charged RNAdS black branes). We expect the EYM solutions with a spherical or hyperbolic event horizon topology to exhibit a rather similar pattern. Moreover, it is likely that the interpretation of EYM black branes as holographic superconductors [19,20] remains valid for a nonspherical topology of the horizon (Note that in the general case the dual theory is defined in a $d=2+1$ dimensional spacetime with a line element $ds^2 = L^2(d\theta^2 + f_k^2(\theta)) - dt^2$). Also, as noticed above, the generic EYM solutions present a nonvanishing magnetic flux at infinity. Therefore, the study of these configurations should give some information about the behaviour of a holographic superconductor in an external nA magnetic field. We hope to return elsewhere with a discussion of some of these aspects.

7 However, such configurations possess the same amount of symmetry for any topology of the horizon.

8 For the data in Fig. 5, we set to one the area of the (θ,ϕ)-sector of the metric.

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