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Forecasting Time Series Combining Holt-Winters and Bootstrap Approaches

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Abstract. Exponential smoothing methods are the most used in time series modeling and forecasting, due to their versatility and the vast model option they integrate. Also, within the computing statistical area, Bootstrap methodology is widely applied in statistical inference concerning time series. Therefore, this study's main objective is to analyse Holt-Winters exponential smoothing method's performance associated to Bootstrap methodology, as an alternative procedure for modeling and forecasting in time series. The Bootstrap methodology combined with Holt-Winters methodology is applied to a study case on an environmental time series concerning a surface water quality variable, Dissolved Oxygen (DO). The proposed procedure allows to obtaining better point forecasts and interval forecasts with less amplitude than those obtained by means of the usual methods.

Keywords: Time series, exponential smoothing methods, Holt-Winters method, bootstrap, forecasting, prediction intervals.
PACS: 02.50.Ey, 05.45.Tp

INTRODUCTION

Forecasting in time series, of a certain phenomenon or variable under study, is one of the main purposes for applying time series models. The choice of the forecasting model depends on the data structure and study objectives. Exponential smoothing methods are the most used in time series modeling and forecasting, due to their versatility and the vast model option they integrate. Also, within the computing statistical area, Bootstrap methodology is widely applied in statistical inference concerning time series. Therefore, this study's main objective is to analyze Holt-Winters exponential smoothing method's performance associated to Bootstrap methodology, as an alternative procedure of modeling and forecasting in time series. The methodology is subsequently applied to a study case of time series related to an environment variable concerning surface water quality, Dissolved Oxygen, monthly measured from March 2000 to December 2011, in eight sampling sites of water quality in River Ave's watershed. Following the proposed procedure, forecasting estimates are obtained and then compared with the real values of the observed period left to forecasting. Some measures are used to evaluate the adopted methodology's performance concerning the forecasting quality of the two forecasting methods under consideration (Holt-Winters usual method and the combination of this method with Bootstrap approach). The computational work was done by using the R statistical software [1]. Some of the R packages and functions were used, but new functions needed to be constructed. The Wolt-Winters (HW) method is an extension of the Holt method, and is applied whenever the data behavior is trendy and is seasonal. Relatively to the seasonal type, it can be additive or multiplicative, depending on the oscillatory movement along the time period. In both versions, forecasts will depend on the following three components of a seasonal time series: its level, its trend and its seasonal coefficient. In addition, both are implemented in the *HoltWinters()* function of the *forecast* package in R. The additive version ought to be considered whenever the seasonal pattern of a series has a constant amplitude over time [2]. In such case, the series can be written by $Y_t = T_t + S_t + \varepsilon_t$, where T_t represents the trend (the sum of the level and the slope of the series at time t), S_t is the seasonal component, and ε_t are error terms with mean 0 and constant variance. When a series displays a seasonal pattern characterized by an amplitude that varies with the series level, the multiplicative version is a better choice. In such case, the series can be represented by $Y_t = T_t \times S_t + \varepsilon_t$. The multiplicative and additive Holt-Winters methods have the recursive equations presented in the Table 1. The Bootstrap method introduced in ([3]) provides a way to estimate parameters, approximate a sampling distribution or derive confidence intervals when we have data but do not know the underlying distribution. If the population represented through a probability distribution and its parameters are unknown, the Bootstrap idea is to take

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TABLE 1. The recursive equations of the Holt-Winters methods.

multiplicative H-W	$F_t = \alpha \frac{X_t}{f_{t-s}} + (1-\alpha)(F_{t-1} + b_{t-1}), 0 \leq \alpha \leq 1$ $b_t = \beta(F_t - F_{t-1}) + (1-\beta)b_{t-1}, 0 \leq \beta \leq 1$ $f_t = \gamma \frac{X_t}{F_t} + (1-\gamma)f_{t-s}, 0 \leq \gamma \leq 1$ $\hat{X}_{t+k} = (F_t + kb_t)f_{t+k-ms}, m = 1, 1 \leq k \leq s, m = 2, s < k \leq 2s$, etc.
additive H-W	$F_t = \alpha(X_t - f_{t-s}) + (1-\alpha)(F_{t-1} + b_{t-1}), 0 \leq \alpha \leq 1$ $b_t = \beta(F_t - F_{t-1}) + (1-\beta)b_{t-1}, 0 \leq \beta \leq 1$ $f_t = \gamma(X_t - F_t) + (1-\gamma)f_{t-s}, 0 \leq \gamma \leq 1$ $\hat{X}_{t+k} = F_t + kb_t + f_{t+k-ms}, m = 1, 1 \leq k \leq s, m = 2, s < k \leq 2s$, etc.

(re-)samples $(x_1^*, x_2^*, \ldots, x_n^*)$, drawn with replacement from the original sample $(x_1, x_2, \ldots, x_n)$. Computing prediction intervals are an important part of the forecasting process, intended to indicate the likely uncertainty in point forecasts. The prediction intervals are usually based on the Mean Square Error (MSE) that denotes the variance of the one-step-ahead forecast errors. Prediction intervals (if a normality assumption is verified) for both at one-step-ahead and at m-steps-ahead are given by the following expression [4]:

$$\left[\hat{X}_{k+1} - z_{1-\alpha/2}\sqrt{MSE_1}, \hat{X}_{k+1} + z_{1-\alpha/2}\sqrt{MSE_1}\right], \text{ and } \left[\hat{X}_{k+m} - z_{1-\alpha/2}\sqrt{MSE_m}, \hat{X}_{k+m} + z_{1-\alpha/2}\sqrt{MSE_m}\right]$$

where z is the appropriate percentage point for the standard Normal distribution and $MSE_m = \frac{1}{k-m}\sum_{t=m+1}^{k}(\varepsilon_t^{(m)})^2$ denote the variance of the m-steps-ahead errors. The idea is to look at the bootstrap percentiles rather than the sampling distribution percentiles, and the confidence interval is based on the Bootstrap distribution (i.e., on the percentiles). Different methods are available for the construction of Bootstrap confidence intervals: the percentile method, the percentile-t method, the bias-corrected method [5] and the accelerated bias-corrected method [6]. Suppose F_k is the empirical cumulative distribution function $\{\hat{x}_{n+k}^b, b = 1, \ldots, B\}$, then the prediction interval is given by $\left[F_k^{-1}(\alpha/2), F_k^{-1}(1-\alpha/2)\right]$.

COMBINING HOLT-WINTERS AND BOOTSTRAP METHODS (HW-BOOT)

The methodology adopted in this work considers a time series $(x_1, x_2, \ldots, x_n)$ from a stochastic process. So, there are considered the first observations $(x_1, x_2, \ldots, x_{n-k})$ of the time series to forecast $(x_{n-k+1}, \ldots, x_n)$. The steps for the application of this methodology are as follows:

- Step 1: Apply the additive and multiplicative Holt-Winters method to the time series $(x_1, x_2, \ldots, x_{n-k})$ in order to fit a model to data and to obtain the estimated values, $\hat{x}_t$;
- Step 2: Obtain the residuals $e_t = x_t - \hat{x}_t$, $t = 1, \ldots, n-k$ for both Holt-Winters models and then compute the MSE;
- Step 3: Select the model with the least MSE;
- Step 4: Verify the normality assumption of the residuals of the selected model;
- Step 5: Obtain the empirical ACF and PACF functions for residuals in order to identify a temporal dependence structure, if there exists one;
- Step 6: If the residuals present a temporal dependence structure, adjust an ARIMA to the residual sequence and obtain the ARIMA residuals;
- Step 7: Obtain the Bootstrap resamples of residuals e^* with replacement of the original residuals. If the residuals in Step 6 presented an ARIMA structure process, the resample process with replacement is applied to the residuals of the adjusted model $(e_1^*, e_2^*, \ldots, e_{n-k}^*)$;
- Step 8: Construct the Bootstrap sample of observations: $x_t^* = \hat{x}_t + e_t^*$;
- Step 9: Repeat steps 7 and 8, B times;
- Step 10: By using the B Bootstrap series, obtain the B estimated series (of size $n-k$) and forecast series (of size k). The point estimates/forecasts will be the median of the estimated/predicted values with k_1 fixe, of the $\{\hat{x}_{n+k_1}^b, b = 1, \ldots, B\}$. These estimates/forecasts are obtained by k_1 steps $(k_1 = 1, \ldots, 12)$;
- Step 11: Construct the Bootstrap prediction intervals by the percentile method.

TABLE 2. Additive HW exponential smoothing parameters for monitoring sites.

Monitoring Site	CANT	TAI	RAV	STI	PTR	FER	GOL	VSA
α	0.3499	0.1856	0.1832	0.0717	0.0559	0.1509	0.1559	0.1390
β	0.0135	0.0000	0.0000	0.0000	0.0080	0.0097	0.0158	0.0000
γ	0.2760	0.1833	0.3645	0.3236	0.2874	0.3135	0.1697	0.2835

THE CASE STUDY

Located in the Northwest of Portugal, the River Ave's hydrological basin has an approximate area of 1390 Km2; from its source in Serra da Cabreira to its mouth in Vila do Conde its main river length is 101 Km and its average flow at the mouth is 40 m3/s. The data used in this paper are the monthly series of water quality (Dissolved Oxygen (DO)) measured between March 2000 to December 2011 in a quality monitoring network of 8 monitoring sites: Taipas (TAI), Riba d'Ave (RAV), Santo Tirso (STI), Ponte Trofa (PTR), Cantelães (CANT), Ferro (FER), Golães (GOL), and Vizela Santo Adrião (VSA) (see [7]). The DO concentration is one of the most important quality variables to assess the degree of pollution existent in the surface waters of a river's hydrological basin (low values indicate bad water quality). The period observed was from March 2000 to December 2011 (118 observations in each monitoring site). In order to illustrate both methodologies, Holt-Winters approach and the combining Holt-Winters and Bootstrap methodology (HW-Boot), here will only be presented, with more detail, the modeling process of the time series of Cantelães (CANT) monitoring site, but for each of the 8 times series we obtain the estimation results of the two methodologies. The exploratory analysis of all eight time series indicated the presence of a seasonal pattern. So, we applied the additive and multiplicative Holt-Winters to the first k observations (we considered the period between March 2000 to March 2001, the first year period), and we obtained the initial values for the smoothing parameters. We obtained the residuals $(x-\hat{x})$ and we calculated the MSE in order to compare the forecasting accuracy. The additive Holt-Winters models are those with the best predictive performance. So, we considered for all times series the models obtained by the additive HW method. Thus, the adjusted model was the additive HW

$$X_t = F_t + b_t + f_t + \varepsilon_t, t = 1, \ldots, n,$$

where F_t is the level, b_t is the slope, f_t is the seasonal index, and ε_t the error. In Table 2 are presented the additive HW exponential smoothing parameters for the 8 monitoring sites. After March 2001, $t = 13$, are adopted the update equations of F_t, b_t and f_t. The one-step-ahead prediction, for the period between March 2001 to December 2010, is given by

$$\hat{X}_{t+k} = F_t + kb_t + f_{t+k-12m}$$

where $m = 1$ and $k = 1$. In Figure 1 are represented the original values of the DO time series, the estimates in the modeling period, the forecasts and the prediction intervals for a confidence level of 95%, using the additive Holt-Winters model for the Cantelães monitoring site. The models validation were assessed by means of the residuals analysis. The independency assumption was assessed, by estimating the autocorrelation and the partial autocorrelation functions of residuals and the assumption that the residuals are identically normally distributed were always verified in the 8 monitoring sites. The Holt-Winters procedure associated to the Bootstrap resampling method was programmed in the R software by taking $B = 2000$ replicates in the residuals resampling process. For both methods (HW and HW-Boot) were computed the forecasts from January to December 2011 in order to assess the methodologies performance, namely by the forecast confidence intervals range. Figure. 1 presents the estimates, forecasts and the Bootstrap confidence intervals since March 2001 to December 2010, and the forecasts and Bootstrap prediction intervals (from January 2011 to December 2011) by considering a confidence level of 95%. By analyzing the predictions intervals it can be verified that the HW-Boot produces intervals with lower ranges than the HW. In order to compare the predictive accuracy of both methods, we considered three forecasting performance criteria: MSE (Mean Squared Error), MAE (Mean Absolute Error), and MAPE (Mean Absolute Percentage Error). Table 3 presents the results. The HW-Boot methodology presents a better performance when applied to the study case in view of the prediction intervals. The main result achieved was that the HW-Boot procedure produces prediction intervals at 95% with lower range than the usual HW procedure. It was found that for the HW method no observation was outside the prediction interval. In the HW-Boot method, 8% of prediction intervals do not contain the true observations. The results obtained show that the usual HW method is conservative in the construction of prediction intervals. Moreover, the HW-Boot method allowed obtaining prediction intervals with smaller range yielding, in spite of an empirical significance close to 5%.

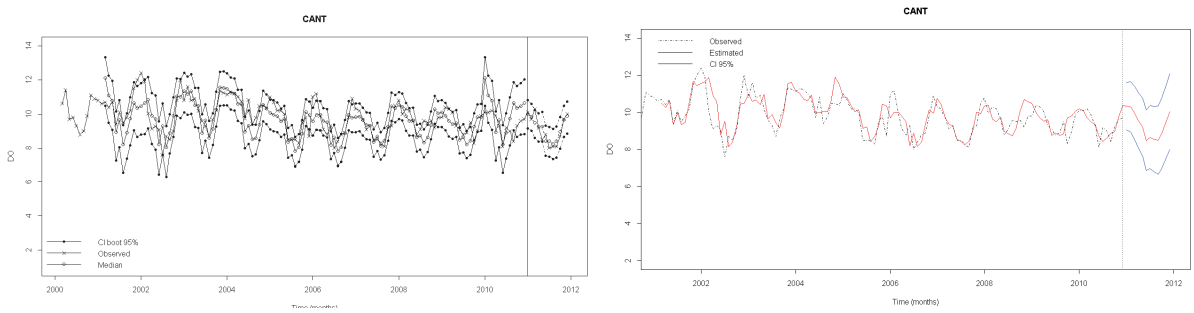

FIGURE 1. (left) Additive HW estimates and forecasts for the DO time series in CANT monitoring site; (right) Estimates, forecasts and Bootstrap confidence interval for the DO time series in CANT monitoring site.

TABLE 3. Forecasting performance evaluation of HW and HW-Boot methodologies (evaluation period: January 2011/December 2011)

criteria	method	CANT	TAI	RAV	STI	PTR	FER	GOL	VSA
MSE	HW	0.181	0.269	0.306	0.296	0.202	0.539	0.175	0.121
	HW-Boot	0.151	0.204	0.336	0.720	0.467	0.196	0.146	0.419
MAE	HW	0.379	0.414	0.490	0.502	0.393	0.656	0.298	0.269
	HW-Boot	0.300	0.345	0.430	0.787	0.614	0.388	0.274	0.587
MAPE	HW	4.299	4.882	5.647	5.974	4.510	7.601	3.341	3.000
	HW-Boot	3.360	4.099	5.057	8.968	6.961	4.412	3.035	6.408

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