Isometries – a creative approach with Geogebra and iTALC

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Abstract

Creativity is recognized as a basic skill but the educational system has not known, wanted or achieved foster their development. Conceptual renewal on isometries requires new approaches based on mathematically significant task sequences. The use of Dynamic Geometry Environments (DGE) can contribute that Mathematics is considered less hostile and to the development of geometrical competences. These "technological environments" are enhanced by the use of Classroom Management Systems. In this context, a qualitative case study has been developed. We concluded that a complementary technological approach seems to develop geometrical knowledge and skills and an "atmosphere" of cooperation and sharing enhance creativity.

Keywords: Mathematics, Isometries, CMS, Creativity, GeoGebra

Introduction

Creativity is seen as a key to a profitable future. Thus, creative thinking is one of the basic skills, transversal to all areas of knowledge, required for this century (Cropley, 2003). It is therefore essential that Education promotes its development in their students (Adams & Hamm, 2010). However, this is not happening (Robinson & Aronica, 2009).

Several studies (Hiebert 2003; Lu, 2008; Ponte & Serrazina, 2004; Ruthven, 2008) point out that Mathematics remains a subject taught in routinely way. Moreover, some content should be recast. In Portugal, different initiatives tried to reform the curricula. The Basic Education Mathematics Program⁵ - PMEB (Ponte et al., 2007) advocates changes in what and how we learn and teach this subject. In Geometry, which achieves greater importance in the curricula, geometric transformations deserve a central place that also calls for different approach that involves new understandings about isometries and symmetry.

⁵ Although this program has already been repealed in 2013, this research was based on it as it was the official document at the date of completion of the empirical work.
Methodological guidelines of PMEB (Ponte et al., 2007) suggest the use of computer technologies. In fact, learning Geometry using DGE, such as GeoGebra, is quite different from doing so only with traditional instruments in a paper and pencil environment. DGE free their users from mechanical and routine tasks such as measurement and calculations and construction procedures, leaving room for a more active and fruitful work in Geometry.

The construction of learning "technological environments" capable of keeping students engaged on tasks, characterized by being truly collaborative and cooperative is greatly facilitated by the use of Classroom Management Systems - CMS. However, despite all institutional efforts to widespread use of computers, particularly in Mathematics, the use of these tools in our schools remain inadequate with limited impact in the classroom.

Creativity and the use of technological tools, including DGE framed by a CMS, were the main area that leads to this study, oriented by the main question: "In which level the appropriate use of technological tools favors: a stronger ownership of the geometric concepts and skills involved and the development of creativity?"

Theoretical Framework

Creativity is a prerequisite for a high level of development in the new (current) global information society (Adams and Hamm, 2010) and studies on creativity reveal that all individuals are creative (Alencar, 2007). The same author states that a significant factor that interferes with their creative potential is education. However, its development implies deep and extended educational changes (Cropley, 2003). In fact, education has numerous flaws and distortions where "place for exploration, for discovery, for creative thought is reduced and sometimes nonexistent" (Alencar, 2007: 9). Therefore, education should implement favorable practices improve it in their students (Fleith & Alencar, 2005).

Zamir and Leikin (2011) argue that teaching creatively and for creativity can enhance the learning process. Ponte (2005:1) suggests the creation of tasks able to involve students in "mathematically rich and productive activities". These should be challenging: resolution and problem formulation, mathematical explorations and investigations (Vale et al., 2011). This author also states that confronting students with various resolutions, especially from their own colleagues, develops some dimensions of creativity. According to Berger (2012),
mathematical tasks that require "complex and not algorithmic thinking" where students have to determine their own path through the problem, demand students to engage in their exploration using various mathematical concepts, relationships and processes. Computers can free students so they can focus on task conceptual aspects.

The term creativity has a wide variety of definitions but emerges a common idea: the potential to generate original ideas, and therefore unique, as well as useful (Sternberg & Lubart, 1999). In this study, we adopt the definition presented by Torrance (1974), which includes the concepts:

- Fluency - is the ability to generate a great number of ideas and refers to the continuity of those ideas, use of basic knowledge and flow of associations;
- Flexibility - is the ability to produce different categories or perceptions, whereby there is a variety of different ideas about the same problem or thing. It reflects when students show the capacity of changing ideas among solutions;
- Originality - is the ability to create unique, unusual, totally new or extremely different ideas or products. Concerning Mathematics, originality may be manifested when a student analyzes many solutions to a problem, methods or answers and then creates a different one (Silver, 1997; Leikin, 2009; Vale et al., 2012);
- Elaboration - is related to the presentation of a large amount of details in one idea (Adams & Hamm, 2010).

In this perspective, creativity is likely to be assessed. Fluency can be measured by the number of correct responses, solutions, proposed by the student during the same task (Silver, 1997; Conway, 1999). Flexibility can be measured by the number of different categories of solutions that students’ can produce. Originality can be measured analyzing the number of responses in the categories that were identified as original, by comparison with the number of students in the same group that could produce the same solutions.

Mathematics occupies a central role in most advanced societies and in the curricula. Within Mathematics, Geometry has gained further importance (Matos, 2001; NCTM, 2000). Then, emerges in the PMEB (Ponte et al., 2007) as the main purpose for basic education, "Developing students' spatial sense, with emphasis on visualization and understanding of the properties of geometric shapes in plane and space, the comprehension of geometric quantities and their measurement processes, and the use of such knowledge and skills in solving problems in different contexts" (p.36). One of the most significant changes is related to the early introduction of isometric transformations, with a special focus on the concept
of symmetry. This document also suggests that Geometry approach should be based in tasks that provide opportunities to observe, analyze, relate and construct geometric figures and work with them. Open and complex tasks involving isometries should deserve special attention in 5-6th grades, especially those related to reflections and rotations.

But we must attend that difficulties may arise related with some particular transformation. Several studies have concluded that these difficulties decrease in tasks involving rotation with respect to reflection (Jacobson & Lehrer, 2000). Others found that students’ performance was superior in tasks involving translation regarding rotation (Clements et al., 1996). However, some studies on children’s perceptions (Shah, 1969; Moyer, 1978) showed that they considered translation, particularly horizontal, simpler than reflection, which was easier than rotation. Kucheman (1981) found that students considered harder to make rotations when its centre was outside the figure and that the inclination of the axis of reflection constitute a difficulty as well. Schultz and Austin (1983) stated that students seem to have difficulties when the reflection is underpinned by an oblique axis. The complexity of objects also appears to influence negatively the results. According to Clements (2003), children have early notions on symmetry, thus, an approach to this concept should start from their previous experiences. Schattschneider (2009) states that students begin by learning to recognize symmetry by observing various figures, exploring them with mirrors, folding them, turning them and overlapping them. However, there are some variables that interfere with the ability to perceive symmetry of figures (Hershkowitz, 1990): the orientation of the axis of symmetry, the respective position of different parts of the geometric shape and axis (prototypes phenomenon), and age of the students. Gerkins (1975), cited by Clements (2003), considers that the vertical bilateral symmetry is more easily understood than one with a horizontal axis and argues that conceptualisation of symmetry even does not occur in a solid way before 12 years of age.

The tasks proposed must be solved using measuring and drawing instruments, DGE programs and applets, which promote the understanding of geometric concepts and relationships (Ponte et al., 2007). According to NCTM (2008), the use of technology, particularly computers and DGE, is one of the principles for teaching Mathematics that promotes active and meaningful learning by students. But this does not necessarily imply the marginalization of paper and pencil even because the exams are usually held in this
environment (Laborde, 2001). Therefore, it seems to emerge the need of combining the two approaches, exploiting the advantages of each and minimize their disadvantages.

A computer use that promotes powerful learning environments where students can build their knowledge interacting with objects, with each other and with the world will be much richer and more valuable, passing from an instructional perspective to a constructivist one (Valente, 2001). Regarding DGE, NCTM (2000) states that they “[...] can allow experimentation with families of geometric objects, with an explicit focus on geometric transformations. [...] the boundaries of the mathematical landscape are being transformed.” (p.27). The most commonly used DGE have been Cabri-Géomètre and Geometer’s Sktechpad. More recently came out GeoGebra, which is an added value when compared with those applications, as it combines graphical manipulations with their correspondent algebraic representations and calculus (Mehanovic, 2009; Misfeldt, 2009).

Despite obvious benefits, using computerized technology and respective applications can disrupt teaching and learning process (Berliner & Calfee, 1996; Brophy and Good, 1986; Galluch & Thatcher, 2011). Nowadays, many classes take place in ICT rooms. Failure to use a system to manage all activities in the room, for example, block access to websites, restrict use of certain applications or closely follow students’ work can negatively affect their progress. A CMS application could be the answer. The CMS should not only be used to monitor students’ work but as a catalytic tool: (i) increasing student task engagement; (ii) increasing collaboration, cooperation and sharing; (iii) keeping students focused on the task (especially important for students in the early grades); (iv) allowing easily work supervising (Joyce & Schmidl, 2008). There are several commercial solutions on the market. We opted for an open source alternative, the iTALC.

The iTALC (intelligent Teaching and Learning with Computers) allows the teacher to monitor and remotely control any workstation in his class; show a demo - the teacher’s screen is shown on all students’ computers in real time; lock workstations; send text messages; and home schooling.
Method

Within a constructivist paradigm, we selected a qualitative case study (Bogdan & Biklen, 1994), focused around three groups of twelve (the privileged way of working in the classroom) of students of the 5th grade. They were selected because they had different school performances and expectations regarding Mathematics and they were present in every moment of the instructional sequence. At the end of the study, they were eleven years old and had no retentions. The group G1 was comprised of a single female member, appointed by Catarina; G2 by Tiago and Luísa and G3 comprising Gabriela and Francisca⁶.

The development of this study involved structuring an instructional exploratory task sequence (Ponte, 2005; Stein & Smith, 2009), on "reflection, rotation and translation" topic, in the "Geometry" theme. The teacher/researcher took an active part in this study, as he planned and led all the events resulting from this research.

To collect the data were used: i) participant observation by the teacher/researcher, supported by field notes and Logbook; ii) inquiry, through questionnaires and interviews with the case students and iii) a documentary analysis of a variety of documents - students’ tasks resolutions, Initial and Final tests and some official documents produced by the school.

To begin with, we passed an Initial Questionnaire (IQ) to obtain information mainly about their habits and some basic knowledge of computer use, including DGE. Then, we implemented a small test on technological skills, which the results served afterwards to make adjustments in the structure of the didactic intervention. It follows an Initial Test (IT), solved in paper and pencil and with GeoGebra, which was intended to analyze the knowledge that students held on the topic, even if constructed beyond the formal context. Such a review could advise changes to planning and task structure. Subsequently, it facilitated the analysis of the evolution of student performance when compared with the same Final Test (FT) at the very end of the didactic intervention.

Then, in eight sessions, we implemented the didactic intervention consisting in a sequence of seven adapted tasks (Coelho, 2013), with increasing complexity, whether mathematical or technical, previously validated and presented (Cabrita et al., 2011)

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⁶ These are fictional names to preserve students’ true identity.
The first task was set to remind some concepts related with isometric transformations discussed in earlier grades and provide the students with an informal exploration on reflections, rotations and translations. Traditional materials and instrumentation in paper and pencil and DGE environments were used.

The second task focused on reflection. It was designed to evolve from a more informal approach, with traditional materials and instrumentation in a paper and pencil environment, to another, yet informal, with GeoGebra, ending again with paper and pencil in a more formal way.

Task III pursued a different approach. Centered in the rotation, it was intended that students address the concept through GeoGebra and evolve to a paper and pencil environment with increased formal requirements.

The fourth task focused on translation, called for the use of traditional instruments in a paper and pencil environment in a more formal way. A final open-ended task, using GeoGebra, invited them to freely create and show their creativity.

Sliding reflection, approached in the fifth task, was an unknown concept to the students. It consisted in an open-ended task, allowing multiple solutions and introducing a slightly more formal notion on composition of isometries.

Tasks VI and VII (adapted from Cabrita et al., 2011) were related to the concept of symmetry which all students misunderstood with reflection. In these tasks, they evolve from a pencil and paper environment, exploring several images, to a computer assisted one to support their earlier conclusions (see figure 1).

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**Fig. 1** – Exploring rotational symmetries in task VI using GeoGebra

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The classroom environment was mediated by a CMS, which was installed by the teacher/researcher in the ICT room. The teacher’s workstation desktop screen, which
contained the "Master", was constantly been shown in the interactive whiteboard which everyone could see, in real time, what was happening in all computers (see figure 2).

Fig. 2- iTALC “Master” application is teacher’s desktop

In the selection/creation process we aim to find exploratory activities, with several open-ended tasks, whose implementation was arranged in four different phases (Stein et al., 2008). In the first phase, the task was presented orally and some aspects, to be considered relevant or by direct request from the students, were clarified by the teacher. In the next phase, all groups solve their tasks autonomously but under teacher’s supervision, through the CMS when tasks were performed in the computer. In the third phase, the working groups presented their results, either in terms of processes or solutions. Likewise, in computer assisted tasks, they use CMS features to show their own desktop in the interactive whiteboard. Finally, in the fourth phase, conclusions were established yielding small written reports on daily notebook. Students had to present their solutions and the underlying strategies were discussed, in order that everybody reflected on the work done by each pair. The main ideas were registered. By the end of each session, we collected the students’ productions. The field notes were analyzed as soon as possible and they were used to improve the Logbook. All these documents were analyzed before the following session, so that the plan could be changed, if necessary.
After the didactic intervention, we passed the FT and the FQ. This questionnaire was intended, essentially, to gather data on their opinions about the approach on this topic. The IT and FT had double aims: the initial one gave us an image on the knowledge and skills pupils had before the didactic intervention and the final one allowed us to assess what they had learned concerning isometric transformations and symmetry.

All collected data was object of content analysis using categories related to: i) Geometry - isometries and symmetry and ii) dimensions of Creativity – fluency, flexibility and originality.

Results

Direct observation and the analysis of students’ answers to the FQ showed the importance they give to the high technological approach on the topic, as well as nature of tasks and how they were addressed and discussed and their contribution in the development of their creativity.

As suggested by Stein and Smith (2009), teaching for creativity must be, at same time, creative, where the challenging nature of the tasks, based on the formulation and problem solving, exploration and research, can promote creative thinking (Vale et al., 2012).

One variable that assumed great preponderance in this study which seems to play a decisive role was the construction of a classroom “atmosphere” that allowed truly exploratory activities with open-ended tasks and where students feel "safe" from destructive criticism (Fleith & Alencar, 2005).

Using iTALC on an ICT room seems to help build this environment. The main goal was to promote sharing and collaboration among the different actors, holding proper control of a room full of computers connected to de Internet.

In what concerns to the influence in the development of creativity of this approach on the topic using GeoGebra in a CMS monitored environment, is important to note that classroom “atmosphere” had the same features above described. Students actively shared their knowledge and findings. Discussion moments seemed to trigger new motivations and, consequently, new strategies and productions.

The five students also declared in the FQ that observing other students work (most creative ones) motivated themselves to be more creative, although Catarina said that, despite having felt this necessity, could not be more creative. Although, her productions were quite original both in solutions and in adopted strategies (see figure 3).
Also Tiago and Luísa and Francisca and Gabriela reacted in a similar way, predisposing themselves to reassess their approaches. In this cases, and in contrast with Catarina, feedback from colleagues was seriously taken into account, which led them to incorporate often new elements that were absent in their original ideas (see figure 4).

Hence, more original "constructions" progressively arose. This seems to confirm the idea formulated by Levenson (2011) that creativity can be built collectively, while being individually developed. The ability to share, at any time, any approach, process or solution on a computer was provided by iTALC.

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Concerning colour, in strictly mathematical terms, the construction should be monochromatic. Attending that it is a student’s construction of an early educational level its use was accepted in all cases.
Productions of the three selected groups show unique and distinctive features, therefore great originality (see figure 5).

**Fig. 5** - Creative students’ productions in different tasks of the didactic intervention

It was also seen that G2 and G3 tended to have a higher number of more elaborate and original answers when resorted to GeoGebra to solve the tasks. In these cases, use of "paper and pencil" (including traditional instrumentation and manipulatives) seemed to "soak" students in a whirlwind of technical procedures that somehow seemed to prevent them from exploring alternative strategies, thus, limiting their ability to adapt processes. This aspect, very pronounced in G2 and also noticeable in G3, seemed irrelevant for Catarina (G1). This student often used "paper and pencil" in early attempts to rehearse procedures for resolving tasks (see figure 6).

**Fig. 6** – Not requested initial attempt on “pencil and paper” environment

Concerning fluency, it could be observed in several tasks, that all three groups developed several approaches to the same problem, which, in a likely manifestation of flexibility, they adapted to achieve the desired "effects". There seemed to be improvements in three considered dimensions of creativity.
Concerning the impact of this approach developing a stronger ownership of geometric concepts and their application, it could also be seen, earlier in this empirical study, that the three cases showed a very superficial knowledge (or even null), and often conceptual errors on isometries and symmetry. Their final results in the FT were very encouraging.

It seems clear that, facing the results students had achieved at the end of the study, the use of this software is valuable, establishing itself as a powerful tool in the graphical problem solving that allows multiple approaches and solutions (Bardini, Pierce & Stacey, 2004).

Analysis of further responses to the FQ revealed a high degree of agreement on the benefits of using GeoGebra. Any negative aspect wasn’t pointed out. Concerning how the topic had been implemented, some students reported that the program helped them to understand isometries, making Geometry less complex and more fun.

Use of DGE seemed to play an important role, especially in relation to students who had greater difficulties. It was observed that success in solving a task in GeoGebra not always ensured a similar success when it was held on "paper and pencil" environment (see figure 7).

**Fig. 7** – Group 2 difficulties when evolving from DGE to “paper and pencil” environment (task III)

Evolving from "paper" to DGE posed no problem. The reverse was not true. In particular, G2 felt some difficulties making this transition.

All students expressed their agreement or strong agreement when asked if they had considered important to have worked with "paper and pencil" and have used instruments for measuring and drawing. These findings suggest the importance of a complementary approach (Laborde, 2001; Ponte, 2005; Ponte et al, 2007).
Final remarks

The research undertaken suggests that the appropriate use of CMS in highly technological approaches seems to contribute positively to improve teaching and learning of Mathematics. These applications allow students to keep more focused on their tasks and contribute decisively to build a learning environment where cooperation, collaboration and sharing between all actors in the classroom are indeed possible.

Further deeper and extended studies should be performed to better understand the benefits of its use. Variations in age, topics or themes should be introduced. Knowing CMS’ potential for E-learning (when the student can not attend school) also constitutes an unexplored field. To understand in which context in it could be detrimental is another one.

This study also follows the perception that the creation of a "social atmosphere" seems to elicit increases in dimensions of creativity. However, the limitations of this study, relating primarily to its short period of implementation and the extraordinary complexity the phenomenon, do not allow more ambitious conclusions.

Also regarding creativity, the use of DGE, seems to promote the emergence of more creative productions in Geometry. More studies are needed in both range and depth.

If using DGE appears to have a major influence in some dimensions of creativity when working in Geometry, the development of geometric knowledge and skills seems to benefit more from a complementary approach, which combines DGE with “paper and pencil” environments.

Some of these aspects should be, as suggested, target of much more extensive and detailed studies. Their relevant role in teaching and learning Mathematics should have implications in teacher training.

References


