New robust iterative minimum mean squared error-based interference alignment algorithm

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Abstract: Interference alignment (IA) is a promising technique for multiple input multiple output interference channels based systems, achieving the theoretical bound on degrees of freedom. However, these gains are reduced in the presence of imperfect channel state information (CSI), because of quantisation or channel estimation errors. In this Letter, the authors propose a new robust iterative IA minimum mean squared error-based algorithm, which includes these channel errors in the IA design. The results show that the proposed robust IA algorithm outperforms the known IA-MMSE algorithms, for low-to-moderate variance of CSI errors.

1 Introduction

It was shown in [1] that the capacity of an interference channel (IC) for a given user is one half the rate of its interference-free capacity in the high-transmit power regime, regardless of the number of users. One interesting recent scheme to efficiently eliminate the intercell interference and achieve a linear capacity scaling is interference alignment (IA). With this technique, the transmitters align in the unwanted users' receive signals in a subspace orthogonal to the subspace used for that users' data, through the use of appropriate precoders and we can achieve the maximum degrees of freedom (DoF) [2].

A closed-form solution for constant channels is still unknown for more than three users and is not necessarily the best solution for low-to-moderate signal-to-noise ratio (SNR) values [1]. Owing to the difficulty to obtain a closed-form solution for constant channels, some iterative algorithms were proposed using alternating minimisation in minimum mean squared error (MMSE)-based or maximum signal and interference noise ratio-based iterative algorithms [3–5]. In [6], a robust iterative MMSE (IMMSE)-based algorithm was proposed, which takes in consideration the channel estimation errors, motivated by the fact that in the presence of imperfect channel the performance of the MMSE algorithm has some degradation in high SNR.

In this Letter, we propose a new robust iterative, MMSE-based IA technique, where channel errors (because of estimation or quantisation of the channel) are explicitly taken into account to deduce the equaliser and precoder matrices, allowing better performance than with the technique of [6].

2 System model

The system model considered have a K-user multiple input multiple output (MIMO) IC with constant coefficients. It comprises a set of K transmitter–receiver pairs sharing the physical channel, with a given transmitter intending to have its signal decoded only by a single receiver. Without loss of generality, we consider a symmetric case where all receivers and transmitters have M antennas, with M even, and with d streams per user. Since each transmitter is allowed to transmit $d = M/2$ data symbols, this system has $KM/2$ DoF.

Under linear precoding, the received frequency-domain signal is given by

$$\boldsymbol{y}_k = \boldsymbol{H}_{k,k}\boldsymbol{W}_k\boldsymbol{s}_k + \sum_{\substack{j=1\\ j\neq k}}^{K}\boldsymbol{H}_{k,j}\boldsymbol{W}_j\boldsymbol{s}_j + \boldsymbol{n}_k \tag{1}$$

where $\boldsymbol{s}_k$ is the user k data vector of size $M/2\times 1$, with $E[\boldsymbol{s}_k\boldsymbol{s}_k^{\mathrm{H}}] = \boldsymbol{I}_{M/2}$; $\boldsymbol{W}_j \in \mathbb{C}^{MxM/2}$ is the linear precoding matrix computed at transmitter j, normalised such that $\|\boldsymbol{W}_j\|_F^2 = P_{\mathrm{t}}$ and P_{t} is the transmit power at the transmitters; $\boldsymbol{H}_{k,j}$ is the channel between the transmitter j and receiver k of size $M\times M$; and $\boldsymbol{n}_k$ is the additive white Gaussian noise vector at receiver k, that is, $\boldsymbol{n}_k \sim \mathcal{CN}(0,\ \sigma_n^2\boldsymbol{I}_M)$.

The soft estimated symbols associated with the user k are given by

$$\hat{\boldsymbol{s}}_k = \boldsymbol{G}_k\boldsymbol{H}_{k,k}\boldsymbol{W}_k\boldsymbol{s}_k + \boldsymbol{G}_k\sum_{\substack{j=1\\ j\neq k}}^{K}\boldsymbol{H}_{k,j}\boldsymbol{W}_j\hat{\boldsymbol{s}}_j + \boldsymbol{G}_k\boldsymbol{n}_k \tag{2}$$

where $\boldsymbol{G}_k$ denotes the linear receiving filter employed at the receiver k, with dimension of $M/2\times M$.

The closed-form MMSE and the IMMSE algorithms for IA are presented in [3]. The IMMSE algorithm outperforms the closed-form MMSE by relaxing the need for perfect alignment, while minimising the signal's sum mean square error (MSE), that is, it minimises the expected sum of the norms between each $\hat{\boldsymbol{s}}_{k,l}$ and $\boldsymbol{s}_{k,l}$ for all k, which can be formulated by

$$\begin{aligned}&\min\sum_{k=1}^{K}E\left\{\left\|\hat{\boldsymbol{s}}_k-\boldsymbol{s}_k\right\|^2\right\},\quad \text{s.t. } \left\|\boldsymbol{W}_j\right\|_F^2 = P_{\mathrm{t}},\\ &j\in\{1,\ \dots,\ K\}\end{aligned} \tag{3}$$

In this Letter, we propose a robust IMMSE IA algorithm, where the MMSE is explicitly minimised by considering the channel errors committed during quantisation or estimation of channel state information (CSI). The overall channel frequency-domain matrix, when taking into account the channel errors, can be modelled as $\hat{\boldsymbol{H}}_{i,j} = \boldsymbol{H}_{i,j} + \boldsymbol{e}_{i,j},\ i,\ j = 1,\ \dots,\ K$, where $\hat{\boldsymbol{H}}_{i,j}$ represents the overall quantised channel matrix and $\boldsymbol{e}_{i,j}$ is the overall channel error matrix. By replacing $\boldsymbol{H}_{i,\ j}$ by $\hat{\boldsymbol{H}}_{i,j} - \boldsymbol{e}_{i,j}$ in (3), we obtain the MSE as a function of the quantisation error

$$\begin{aligned}\Im_{\mathrm{MSE}} = \sum_{k=1}^{K}\Bigg(&\mathrm{tr}\left(E\left[\sum_{j=1}^{K}\boldsymbol{G}_k\left(\hat{\boldsymbol{H}}_{k,j}-\boldsymbol{e}_{k,j}\right)\boldsymbol{W}_j\boldsymbol{W}_j^{\mathrm{H}}\left(\hat{\boldsymbol{H}}_{k,j}-\boldsymbol{e}_{k,j}\right)^{\mathrm{H}}\boldsymbol{G}_k^{\mathrm{H}}\right]\right)\\ &-2\,\mathrm{tr}\left(E\left[\boldsymbol{G}_k\left(\hat{\boldsymbol{H}}_{k,k}-\boldsymbol{e}_{k,k}\right)\boldsymbol{W}_k\right]\right)+\sigma_n^2\mathrm{tr}\left(\boldsymbol{G}_k\boldsymbol{G}_k^{\mathrm{H}}\right)+M/2\Bigg)\end{aligned} \tag{4}$$

This expression is then used to solve the optimisation problem, whose solution is derived through the Karush–Kuhn–Tucker conditions

$$\begin{cases} \dfrac{\partial L\left(\boldsymbol{W}_j,\ \boldsymbol{G}_k,\ \lambda_j\right)}{\partial \boldsymbol{W}_j} = 0 \\ \dfrac{\partial L\left(\boldsymbol{W}_j,\ \boldsymbol{G}_k,\ \lambda_j\right)}{\partial \boldsymbol{G}_k} = 0, \quad k,\ j = 1,\ \ldots,\ K \\ \dfrac{\partial L\left(\boldsymbol{W}_j,\ \boldsymbol{G}_k,\ \lambda_j\right)}{\partial \lambda_j} = 0 \end{cases} \tag{5}$$

where λ_j is the Lagrange multiplier associated with the power constraint of transmitter j and the Lagrangian function is given by

$$L\left(\boldsymbol{W}_j,\ \boldsymbol{G}_k,\ \lambda_j\right) = \Im_{\text{MSE}} + \sum_{j=1}^{K} \lambda_j \left(\operatorname{tr}\left(\boldsymbol{W}_j \boldsymbol{W}_j^{\text{H}}\right) - P_{\text{t}}\right) \tag{6}$$

After lengthy mathematical manipulations, we obtain the solution for both the equaliser and precoder matrices

$$\boldsymbol{W}_j = \left(\sum_{k=1}^{K} \hat{\boldsymbol{H}}_{k,j} \boldsymbol{G}_k \boldsymbol{G}_k^{\text{H}} \hat{\boldsymbol{H}}_{k,j}^{\text{H}} + \lambda_j \boldsymbol{I}_M + \sigma_h^2 \operatorname{tr}\left(\sum_{k=1}^{K} \boldsymbol{G}_k \boldsymbol{G}_k^{\text{H}}\right) \boldsymbol{I}_M\right)^{-1} \hat{\boldsymbol{H}}_{j,j}^{\text{H}} \boldsymbol{G}_j^{\text{H}} \tag{7}$$

$$\boldsymbol{G}_k = \boldsymbol{W}_k^{\text{H}} \hat{\boldsymbol{H}}_{k,k} \left(\sum_{j=1}^{K} \hat{\boldsymbol{H}}_{k,j} \boldsymbol{W}_j \boldsymbol{W}_j^{\text{H}} \hat{\boldsymbol{H}}_{k,j}^{\text{H}} + \sigma_n^2 \boldsymbol{I}_M + \sigma_h^2 \operatorname{tr} \sum_{j=1}^{K} \left(\boldsymbol{W}_j \boldsymbol{W}_j^{\text{H}}\right) \boldsymbol{I}_M\right)^{-1} \tag{8}$$

where σ_h^2 denotes the variance of the channel errors. Naturally, when this variance tends to zero, (7) and (8) tends to the conventional equaliser and precoder matrices, respectively.

The iterative procedure is identical to the conventional IA-MMSE-based algorithms:

1. Fix $\boldsymbol{W}_j$ arbitrarily for all j.
2. Calculate matrix $\boldsymbol{G}_k$.
3. Find λ_j that solve $\operatorname{tr}\left(\boldsymbol{W}_j^{\text{H}} \boldsymbol{W}_j\right) = P_{\text{t}}$ for $j = 1, 2, \ldots, K$.
4. Update $\boldsymbol{W}_j$ and $\boldsymbol{G}_k$ with the obtained λ_j.
5. Repeat steps (2)–(4) until convergence or a predefined number of iterations is reached.

It should be pointed out that this algorithm has the same initial procedure than the robust proposed in [6], but we obtain a different expression for the equaliser matrix.

3 Performance evaluation

In this section, we evaluate the performance of the proposed robust IMMSE IA algorithm and its performance is compared with the conventional IMMSE approach and with the robust proposed in [6]. Our scenario has $K = 3$ transmitters, cooperating to transmit information to $K = 3$ receivers sharing the same resources. All terminals are equipped with four antennas. The modulation used is quadrature phase shift keying (QPSK), the channel coefficients are constant and we consider 16 iterations for the iterative algorithms, similarly to [6].

The performances of these algorithms are presented in terms of the average bit error rate as a function of E_b/N_0 (Fig. 1) with E_b denoting the average bit energy and N_0 denoting the one-sided noise power spectral density. Several channel error variances are considered, equal to 10 and 20 dB. Moreover, we present the cases of perfect CSI and when the channel error variance is equal to the noise variance (since it is often observed that the channel estimation errors vary with the noise power).

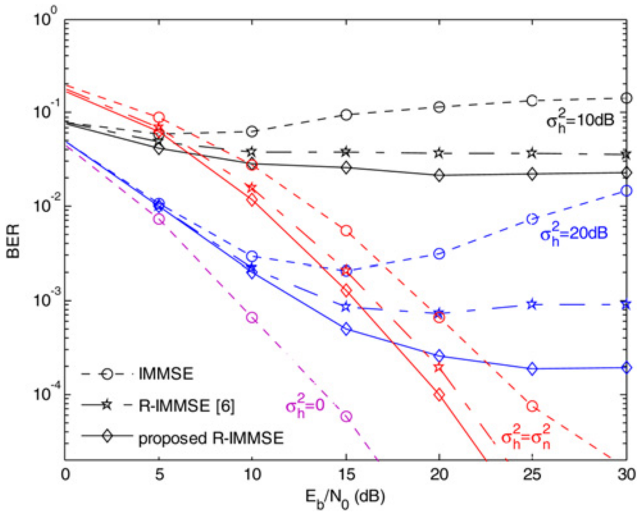

Fig. 1 *Performance of the proposed robust IMMSE and other reference MMSE-based algorithms for IA technique*

We observe that the proposed scheme clearly outperforms the iterative algorithm and the robust recently proposed in [6]. In addition, we can observe that the performance gains achieved with this algorithm are higher for medium-to-high SNR regime, since this is the region where the channel errors strongly affect the systems performance.

4 Conclusion

In this Letter, we proposed a new robust IMMSE-based IA algorithm, where the IA design takes in consideration channel errors obtained in channel estimation and/or quantisation. As observed in the results obtained, our proposed algorithm clearly outperforms the previously proposed IMMSE ones (for robust and non-robust cases), mainly for medium and high SNR.

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