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# Visual patterns and the development of creativity and functional reasoning[1]

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**Abstract**

Despite the importance of creativity, its development has not been a priority of Mathematics, especially in what concerns non-gifted students. Students often reveal many difficulties in functional reasoning. Some studies have concluded that an adequate exploration of visual patterns may contribute to overcoming these difficulties. Thus, we developed a qualitative case study aiming to find out if the exploitation of tasks focused on visual patterns contributed to the development of creativity and functional reasoning in 8th grade students. It was found that students improved their performance in tasks whose resolution required the mobilization of functional reasoning and their creativity. Some of the beliefs and/or conceptions about creativity have also evolved positively.

***Keywords:*** Creativity, Visual patterns, Algebra, Functional reasoning

## Introduction

Life in modern society requires that all people be creative, thus capable of producing innovative solutions to the problems they deal with. In fact, creativity is regarded as a transversal competence shared by all content areas, and not only by design, music, or other forms of art. Education in general and Mathematics in particular is not aware of this reality and does not contribute to the development of their students' creativity, because they control their reactions excessively (Robinson and Aronica, 2009).

In Portugal, recent guidelines on education tend to value algebra a lot (Ponte, Serrazina, Guimarães, Breda, Guimarães, Sousa, Menezes, Martins and Oliveira, 2007) and, consequently, functional reasoning. As a matter of fact, developing functional reasoning is one of the most important goals of algebra. Nevertheless, students reveal much difficulty in functional reasoning (Barbosa, 2010; Tanisli, 2011; Warren, 2000).

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According to several authors (e.g. Blanton and Kaput, 2004; Vale, Barbosa, Barbosa, Borralho, Cabrita, Fonseca and Pimentel, 2011; Warren and Cooper, 2008) enabling students to *see* visual arrangements and detect their underlying structure could help them improve their performance in functional reasoning. Moreover, visual representations could also lead to more creative findings (Barbosa, 2010).

These were the basic ideas of a study we conducted, mainly focused on: i) developing creativity, ii) improving functional reasoning and iii) using visual patterns exploration as a tool to develop transversal and specific mathematical abilities. Its main objective was to identify 8th grade students' beliefs and/or conceptions on creativity and to evaluate the impact of the implementation of a didactic sequence about "Sequences and regularities". The didactic sequence comprised tasks which focused on the exploration of visual patterns and the discussion of several solutions found, as well as on the development of creativity and functional reasoning.

## Theoretical Framework

Despite its importance, teachers are aware of the declining interest for Mathematics. For Vale et al. (2011) this reality results mainly from the fact that more and more students face Mathematics as a mere ensemble of procedures one must know by heart and disregard the need to understand the reasons underlying such procedures.

An effective learning of Mathematics though, demands that students are actively engaged in diverse and significant tasks (Doyle, 2007; Stein and Smith, 2009). According to NCTM (2000) students must perform routine tasks, but they should also be involved in good tasks – the ones that make them aware of essential mathematical ideas by presenting them as challenges to overcome.

At the same time, mathematicians and researchers on mathematics education defend that exploring patterns is the essence of Mathematics (e.g. Davis and Hersh, 1995; Devlin, 2002; NCTM, 2000; Orton and Orton, 1999). Several studies conducted show that tasks involving repeating, growth, linear, nonlinear, numeric and figurative or visual patterns promote algebraic thinking, including the symbolism related to it, and the development of the ability to generalize and thus the exercise of functional reasoning (e.g. Amit & Neria, 2008; Lee & Freiman, 2006; Radford, 2008; Rivera & Becker, 2005; Stacey, 1989; Vale and Cabrita, 2008). N*ear* generalization is related to the discovery of the following term by means of counting, using a table, …, which usually involves recursive relations; *distant, explicit* or *functional* generalization involves the discovery of a pattern, including understanding the law of formation and requires the search for functional relationships using functional reasoning (Vale, Pimentel, Cabrita, Barbosa, and Fonseca 2012). D*istant* generalization requires that students find a relationship between elements of the pattern and its position and use this generalization to generate elements elsewhere, i.e., students are motivated to think in patterns as functions instead of merely focusing on the relative variation of the sets.

Thus, tasks focused on patterns exploration, and particularly on visual patterns or patterns in figurative environments, may be an interesting tool for the development of mathematical abilities, as concluded by Vale et al (2011; 2012). Also according to Lee and Freiman (2006) *seeing* a pattern is the first step towards the ability to identify and explore patterns. *Seeing* in different ways implies, for example, the ability to identify disjointed sets composed by elements present in the figure that, once assembled, compose the original figure. This is called *constructive generalization* (Rivera and Becker, 2008). One strategy suggested by Rivera (2007) in this context is supported by a kind of symmetric numbering: students identify the symmetry in the figures that are presented, count the elements in one of the parts and multiply the number of elements of that part by the number of equal parts. *Seeing* may also imply the observation of superposed subsets, counting some of those elements several times and then subtracting them. This is a *deconstructive generalization* (Rivera and Becker, 2008). Intermediate stage involves adding elements to the figure and then subtracting them. Barbosa (2010), based on Rivera and Becker (2008) and Taplin (1995), concludes that students tend to use constructive generalizations more frequently than deconstructive ones, because the latter implies a greater level of visualization.

Creativity is the observable ability to produce something that is original and useful at the same time (Sternberg and Lubart, 1999). It is not exclusive of scientists or artists: we all use it in everyday life (Pehkonen, 1997).

Although there is no consensual definition for creativity, we chose the one presented by Torrance (1974), which includes four elements: fluency, flexibility, originality and elaboration. Fluency is the ability to generate a great number of ideas and refers to the continuity of those ideas, the use of basic knowledge and flow of associations. It can be measured by the number of correct responses or solutions proposed by a student during the same task (Conway, 1999; Silver, 1997). Flexibility is the ability to produce different categories or perceptions, whereby there is a variety of different ideas about the same problem or thing. It is reflected when students show the capacity of changing ideas among solutions. It can be measured by the number of different categories of solutions that a student can produce. Originality is the ability to create unique, unusual, totally new or extremely different ideas or products. It can be measured by analyzing the number of responses in the categories that were identified as original, by comparison with the number of students in the same group that could produce the same solutions. With regard to Mathematics originality may be manifested when a student analyzes many solutions to a problem, methods or answers and then creates a different one (Leikin, 2009; Silver, 1997; Vale et al., 2012). Elaboration is related to the presentation of a large amount of details in one idea (Adams and Hamm, 2010).

While there may be no direct relationship (Martinez and Brizuela 2006), a creative teaching can be very important for the development of students' own creativity (Meissner, 2011). A creative teaching begins with the way the teacher manages the curriculum. At a micro level, it involves the kind of tasks the teacher selects and their sequence. These tasks should be challenging, but accessible to students at the same time (Vale and Cabrita, 2008; Vale et al, 2012) enabling them to gain confidence in their mathematical power. Tasks should be open to allow multiple forms of resolution, which favors flexibility, fluency and originality. The exploration of visual patterns can play a decisive role in this process because, they are challenging, accessible and open (Vale & Cabrita, 2008; Vale et al, 2012). Thus, these tasks can be the basis of a creative teaching for creativity. This teaching presupposes a classroom environment that: recognizes and encourages creative ideas; strengthens self-confidence, curiosity, persistence, independence of thought, and the courage to explore new situations and deal with the unknown; helps students not to be afraid of making mistakes and be criticized (Mann, 2005). It also promotes the exchange of ideas that will prompt students to overcome themselves.

For many people creativity is a gift you are born with; it is something that does not develop over time - you either have it or you don't; creativity requires people to produce something different from new things; creativity is exclusive of arts, so creativity and Mathematics have nothing in common (Pehkonen, 1997). Such representations (opinions, beliefs, conceptions, …) may influence the development of creativity and, therefore, it is urgent to combat these myths.

## Method

The method used to accomplish the investigation was a qualitative one (Bogdan and Biklen, 1994) focused on an exploratory case study (Yin, 2010). The data collection was directed to twenty-five 8$^{th}$ grade students, the whole class and, in particular, to three pairs of students.:

- Manuel and Gonçalo, because Manuel's vision of creativity proved to be completely different from his classmates (in his opinion, creativity is related to the ability to solve everyday problems in a quick, but effective way, as it will be referred later);
- Joana and António, because this pair used unique and more complex methods in the tasks' resolution;
- Margarida and Daniela, because their resolutions of the tasks proposed in the empirical study were quite similar to the remaining pairs of students' resolutions.

We opted to focus on pairs because students were used to work this way in regular classes and because this was the privileged working dynamics throughout the whole didactical sequence underlying the study - *Sequences and regularities.* The study developed did not demand any extra role from students, i.e., they worked as they always worked during regular classes performing all activities proposed by the teacher/researcher in the natural environment of regular classes, related with a curricular didactic unit. The teacher/researcher had an active involvement in this study as she planned, conducted, recorded, analyzed and evaluated all events related with the empirical study.

The main sources of data collection were: i) participant observation by the teacher/researcher, supported by audio and photographic records of the work done in class, field notes and logbook, ii) inquiry, through questionnaires and interviews with the case students and iii) a documentary analysis of a variety of documents - the students' tasks resolutions, the test implemented at the beginning and the end of the study and some official documents produced by the school.

To begin with, the teacher/researcher passed a questionnaire, divided into two parts: i) characterization and ii) beliefs and/or conceptions on creativity in Mathematics. The second part had four open questions – In your opinion what does it mean to be creative?; In which disciplines is it possible to be creative?; Can the teacher be creative in mathematics? How? Give an example that shows the teacher's creativity; Can the student be creative in mathematics? How? Give an example that shows the student's creativity. The fifth question presented thirteen statements concerning creativity (e.g *I consider myself creative*; *School* restricts *students' creativity*; *Students' creativity can be assessed; In mathematics, everything is created, we don't create anything new*.). In the 6th and final question, resolutions of a task of visual counts were presented and students were requested to indicate the most creative and justify their choice.

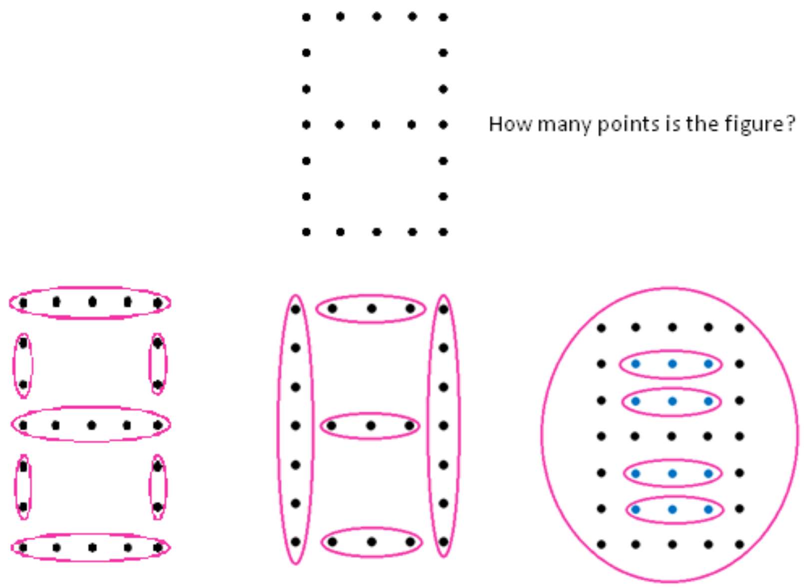



**Fig. 1.** Last question of the Questionnaire

Then, the teacher/researcher passed a pre-test, previously validated by students from another 8th grade class, from the same school, similar to the one that took part in our study. This validation procedure showed that it was not necessary to change anything in the test, nor in the conditions of its application.

The test included six questions. The first one presented three sequences: two figurative ones (one with currency symbols and the other with squares of different sizes) and a numeric one (the Fibonacci sequence). Students were asked to identify the two following terms for each of them. In the second question, they were invited to explore different ways of counting symbols present in a figure and write down the corresponding numeric expressions. The third question presented a situation opposite to this one. Students were given a figure and a numeric expression and they had to draw a way of "seeing" corresponding to the expression presented. The fourth question concerned the recognition of a repeating figurative pattern - ABCCD, ABCCD, ABCCD … - and the fifth question, a growth figurative pattern. Both required that students wrote an algebraic expression referring to a distant element. In the last question, students were asked to create a sequence of figures, using a certain formation law.

Then, the teacher/researcher implemented the didactic intervention, supported by a sequence of seven tasks previously validated and presented (Vale et al., 2011). A detailed description of this sequence can be found in (Tavares, 2012 - http://hdl.handle.net/10773/9929).

The first two tasks (solved in one session – 90min) were based on the idea that visual arrangement plays an important role in finding calculation strategies in a simpler and more intuitive way (Vale et al., 2011) and were related to visual counting. First of all, the teacher/researcher presented visual arrangements and asked the students: i) to explore different ways of counting the symbols included in those visual arrangements; and ii) to write the corresponding numeric expressions. Still, within these tasks, students should find a way of *seeing* a visual arrangement using the corresponding numeric expression as a start and other ways of seeing it. Multiple copies of the figures were made available to pairs of students. At the end of the lesson, the teacher/researcher collected the various productions and analyzed them.

At the beginning of session 2, students were given a document presenting scans of various resolutions of tasks 1 and 2, different from each other and presented by other pairs of students, and a scale of creativity (see Figure 2). Each resolution was identified with a letter. They were asked to match the letters with the position that seemed most appropriate, according to the scale (with 10 points) and justify their choices. The application of that scale would serve to detect any changes in opinions about creativity during the empirical study.

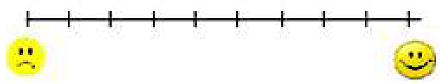


**Fig. 2.** Scale of creativity

Then, five tasks were distributed to pairs of students which focused on: the identification and description of repeating patterns and growth patterns and their continuation; the identification of the position of certain elements of the module (the repeating unit in the pattern); using functional reasoning to determine distant elements; writing the algebraic expression that allowed to determine the position of a certain element of the module; creating visual representations of a given sequence.

After the fourth task of this second group students were given another document presenting the solutions some of them had proposed and were invited to assess them using the scale of creativity.

During the six sessions (90min each), students were organized in pairs (although there was a group of three elements), because they had been working like this since the beginning of the school year. They had to present their solutions to the teacher and to the other students and the strategies underlying their answers were discussed in the class so that everybody could reflect on the work done by each pair. The main conclusions were co-constructed by all, registered on the blackboard by a student or by the teacher/researcher and recorded in the students' notebooks.

At the end of each class the teacher/researcher collected the students' productions. The field notes were analyzed as soon as possible and they were used to improve the logbook. All these documents and the audio records were analyzed before the following session, so that the plan could be changed, if necessary.

Two months after the didactic intervention, the teacher/researcher passed the post-test (similar to the pre-test) and the same questionnaire students had answered in the beginning of the study. This repetition of the questionnaire was intended to collect data allowing us to determine if there had been any changes in their beliefs and/or conceptions on creativity. The pre-test and the post-test had double aims: the initial one gave us an idea of students' knowledge and competences before the didactic intervention and helped us to readapt it to them, and the final one allowed us to assess what they had learned concerning sequences and regularities.

All the data collected were the object of content analysis using categories related to: i) some dimensions of creativity – fluency, flexibility and originality. The elaboration was not particularly considered in this study –; ii) beliefs and/or conceptions on creativity – novelty, originality, simplicity and others found during the didactic intervention and iii) reasoning – functional and nonfunctional.

We also tried to analyze the main strategies used to *see* the patterns – constructive and deconstructive. We equally took into account aspects such as the reading direction – horizontal, vertical, oblique and mixed –, the form – rectangular, square, triangular, … – and the existence or absence of symmetry. All these aspects can be taken into account in the analysis of creativity.

**Results**

Taking into account the analysis of the answers from the first questionnaire, regarding the beliefs and/or conceptions on creativity, we concluded that the concept of creativity presented by Gonçalo, Joana, António, Margarida and Daniela was related to the idea of generating something new, original and different from the usual: *"For me being creative means making something that does not exist yet, i.e. creating something original or even completely new."* and *"For me being creative means being imaginative, to create something unusual."*

The analysis of the answers given by those same students in the second questionnaire revealed that they were associating creativity to complexity, an idea emphasized by Meissner (2011). This idea was already evident when the students met the scale of creativity - they considered the most complex resolutions (e.g. the ones that involve adding elements and, then, subtracting them) as the most creative ones.

However, Manuel, whose representation remained the same from the beginning until the end of the study, related creativity to simplicity - *"The C is the most creative. For me, being creative is being capable of solving everyday problems in a simple and fast though effective way*." (see next figure), which goes against the idea expressed in the literature on this topic (Meissner, 2011).

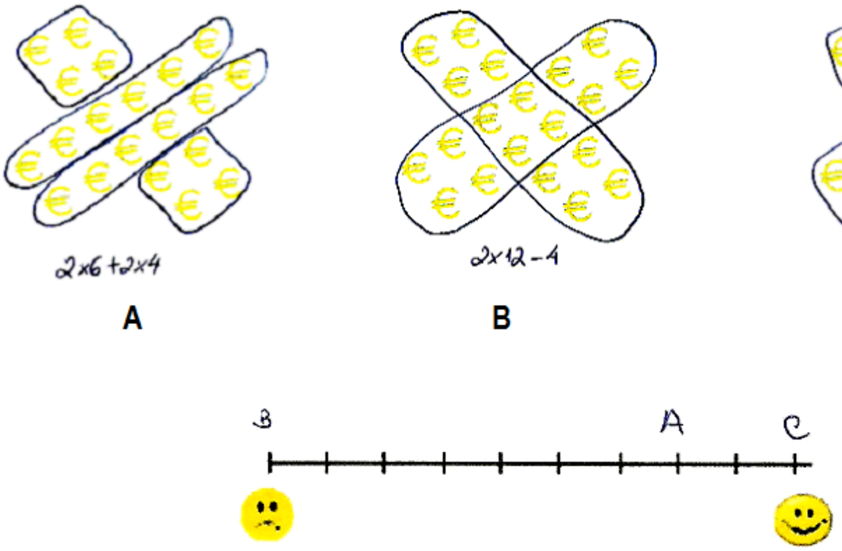



**Fig. 3.** Manuel response to the first scale of creativity

In the beginning, only Gonçalo and Margarida considered that one could be creative in every subject. In their answers to the final questionnaire, all the other students revealed they were aware of this fact: Joana and Margarida mentioned all the subjects, António excluded the mother tongue and Daniela included Mathematics.

The six students kept considering that a teacher could be creative in Mathematics, but Joana and António thought this was not possible for the students. Their answers to the final questionnaire have shown that their opinion had changed.

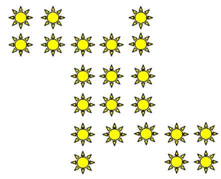
Thanks to this study, these students got aware of the fact that creativity can be a result of cooperative work (Levenson, 2011). We must emphasize that only Joana did not share this opinion in her answers to the first questionnaire.

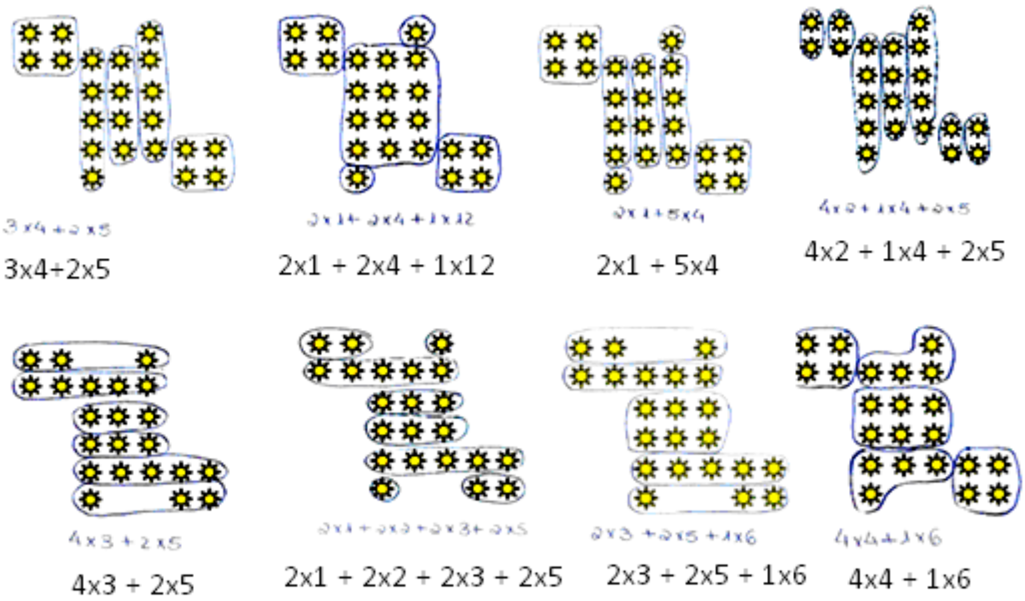
Most of these students believed from the beginning that creativity may be developed at school, but they thought this institution could be responsible for its underdevelopment (cf. Robinson and Aronica, 2009). Only Gonçalo disagreed with the idea that school did not allow the development of creativity in his answer to the final questionnaire.

There were different opinions in what concerned the possibility of assessing students' creativity: Manuel, Margarida and Daniela thought this was possible, but the others disagreed, according to their answers to both questionnaires.

This happened also for the following statements: "In Mathematics, everything is created, no one can create anything new.", "In Mathematics, you cannot be creative: there is only one answer.", "Mathematics is a creative subject, such as music and arts.", "Creativity must be present in Mathematics classes, so that the students can learn better." In their answers to the final questionnaire, all these students disagreed with the two first statements and agreed with the other two. Nevertheless, in the initial questionnaire, Joana and António had agreed with the two first statements, Daniela with the second and Gonçalo and Margarida with the third. For more details, see Tavares (2012).

In what concerns the dimensions of creativity, the analysis of the answers given to the pre-test and the post-test revealed that all students improved in terms of fluency: they presented more and more different ways of *seeing* (cf. figures 4 and 5) - taking advantage of the reading direction, symmetry and different geometric configurations. The task asked – *"Observe the following figure. How many suns compose the figure? Discover different ways to count and write the respective numeric expressions."*





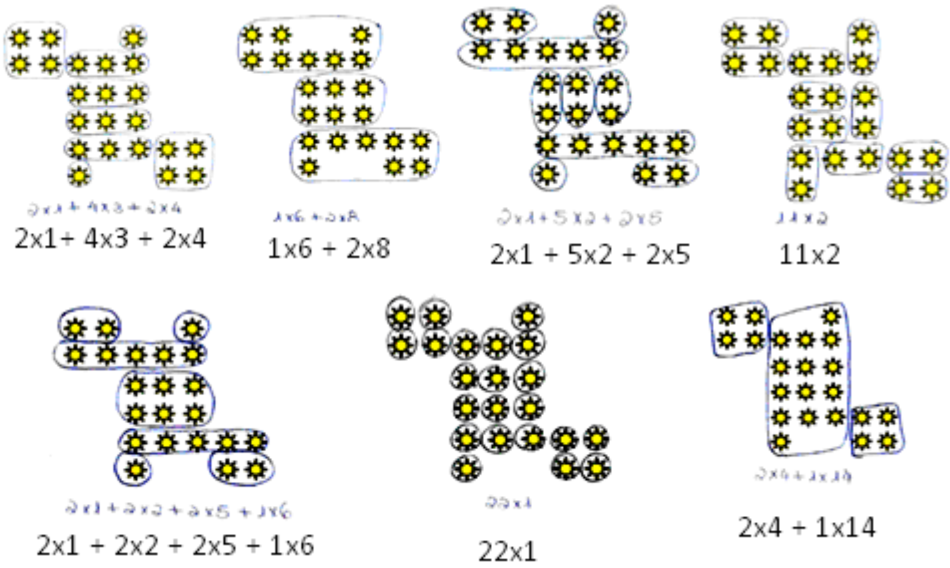



**Fig. 4.** Ways of counting and corresponding numeric expressions presented by Manuel in his answer to the second question in the pre-test

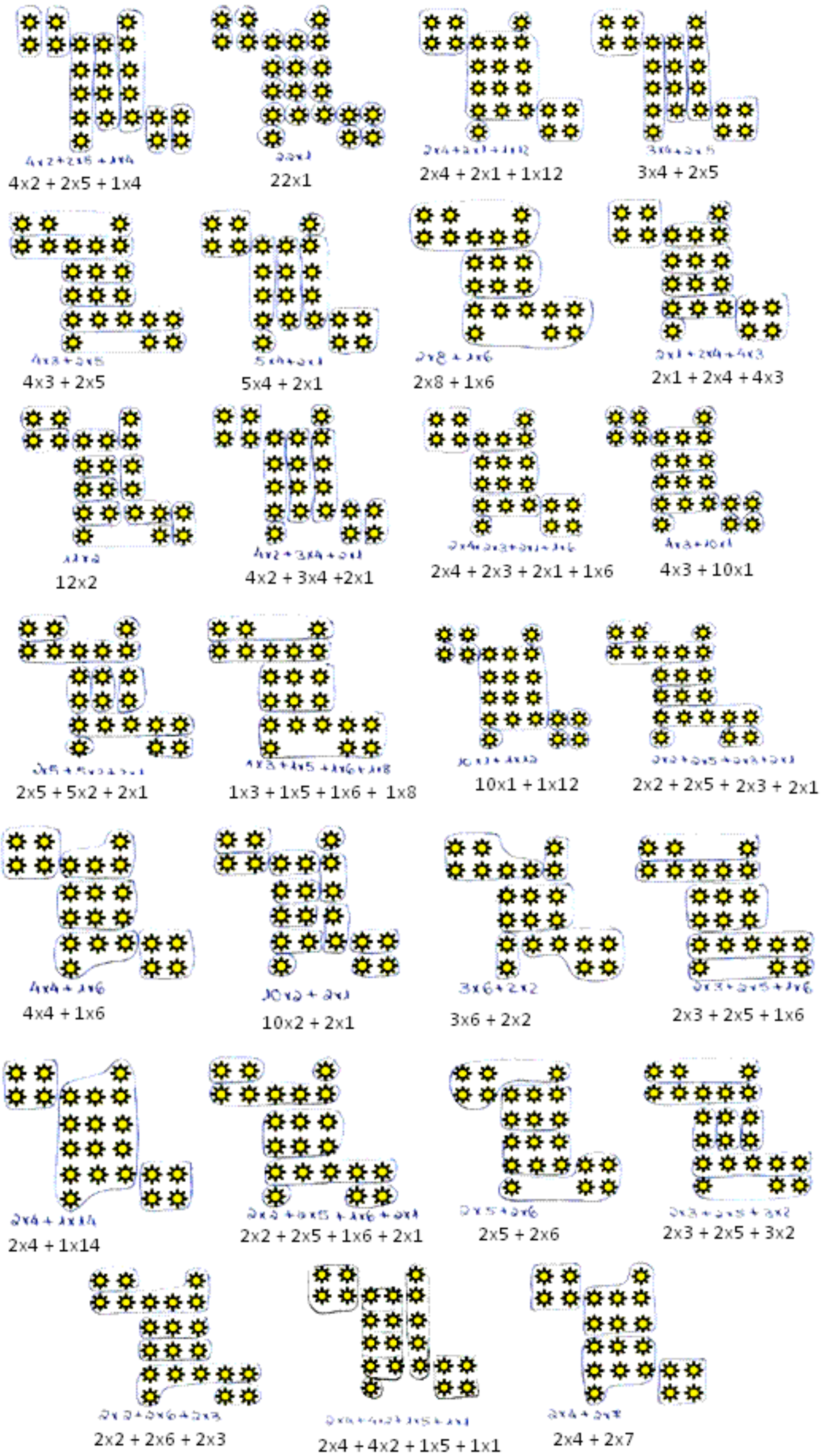



**Fig. 5.** Ways of counting and corresponding numeric expressions presented by Manuel in his answer to the second question in the post-test

Apart from Gonçalo, António and Margarida revealed a better performance in terms of flexibility than they had been using during the study, adding elements to the figure, which were then subtracted in the post-test (cf. figure 6).

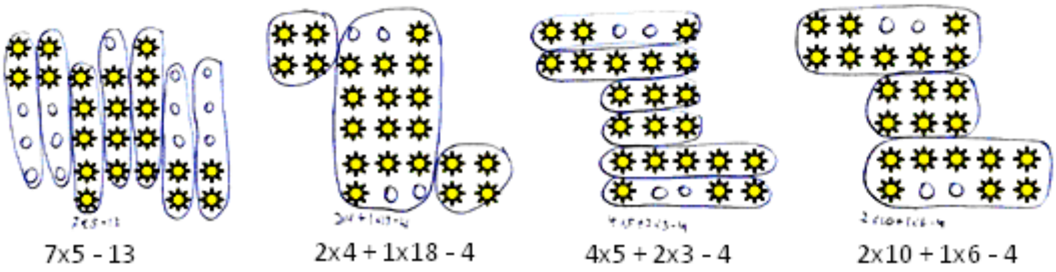



**Fig. 6.** Examples of ways of counting and corresponding numeric expressions presented by Margarida in her answer to the second question in the post-test.

As for originality, Gonçalo, Joana and António presented ways of counting that were used by very few students in the whole class. Joana's ways of counting were exclusive (cf. figure 7).

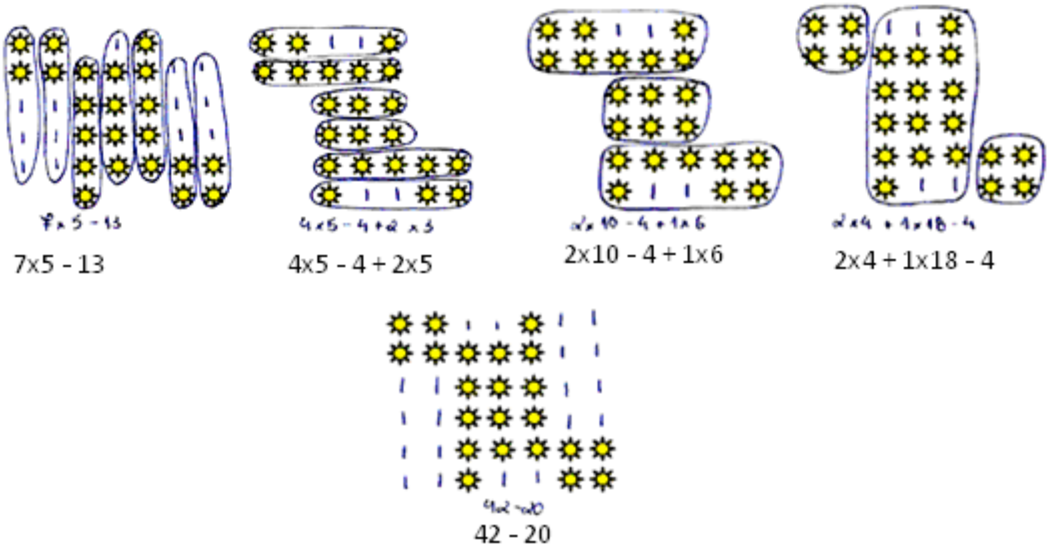



**Fig. 7.** Examples of ways of counting and corresponding numeric expressions presented by Joana in her answer to the second question in the post-test

Most visual presentations included symmetry (mainly horizontal or vertical axis of reflection), different reading direction, and rectangular, quadrangular and hexagonal geometrical forms. For more details about this subject, see Tavares (2012).

Students also evolved in the way they were able to figuratively represent a particular sequence. An example is provided in the following figure, which shows a response to the question "*Create a sequence of drawings that correspond to consecutive terms of a pattern whose formation law is* $2n + 3$ ".

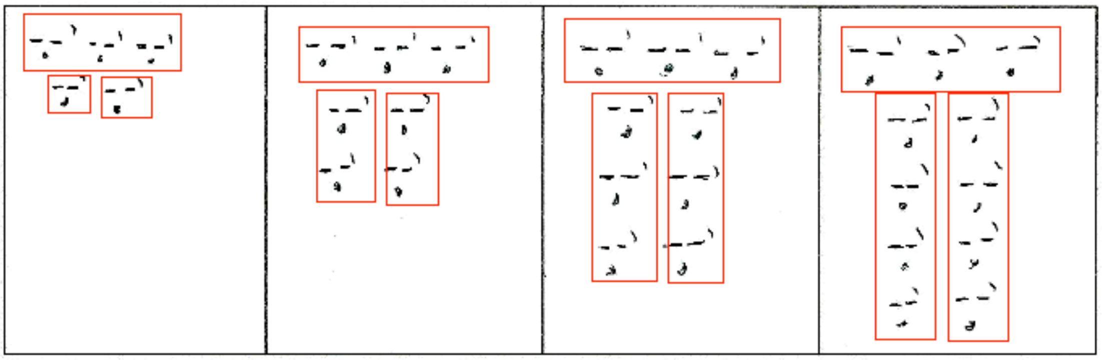
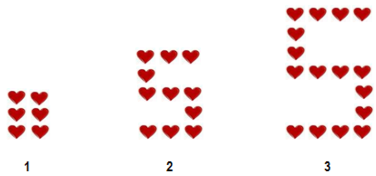


**Fig. 8.** Answer to question 2 Task 6 by Margarida e Daniela

In terms of reasoning, the analysis of the answers given to the three questions concerning this aspect in the pre-test and the post-test revealed that there was a positive evolution – from recursive reasoning to functional reasoning – regarding Manuel and Daniela and especially António. Thus, evolution occurred gradually, as the different tasks proposed were solved. For example, in the resolution of the third task, Joana/António and Margarida/Daniela used alternatively recursive and functional reasoning, while in the fourth task all of them used functional reasoning only.

Figure 9 presents two examples of the answers given to this question:

*"Francisco and Madalena met five years ago... and it was love at first sight. On Valentine's Day, both drew hearts. (...) In figures 2 and 3, you must draw different ways of seeing the forms they have drawn. Please explain how you can obtain the number of hearts featured in the seventh figure without drawing it, just using these ways of 'seeing'"*



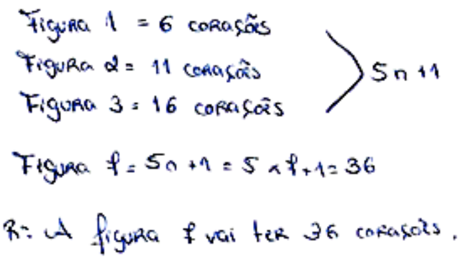



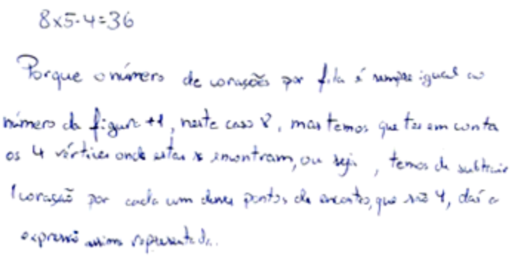
*Figure 1 = 6 hearts*

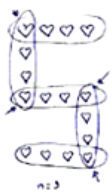
*Figure 2 = 11 hearts* — *5n + 11*

*Figure 3 = 16 hearts*

*Figure 7 = 5n + 1 = 5 x 7 + 1 = 36*

*A: Figure 7 will have 36 hearts.*





*8 x 5 = 36*

*Because the number of hearts per line (horizontal and vertical) equals the number of figure +1 (8). However, we must consider the vertices in which they converge (4). We need to subtract 4 hearts, which result in the expression presented.*

**Fig. 9.** Examples of answers given to the fourth question in task 4

For more details about this subject, see Tavares (2012).

## Final remarks

The didactical intervention, supported by previously studied (Vale et al, 2011) challenging, well sequenced, but accessible tasks (that the students can 'catch'), were effectively solved by the pairs of students. Then, different answers to the tasks were presented and discussed with the whole class and, finally, the main aspects were highlighted (as defended by several authors – e.g. Ponte et al, 2007; Vale et al 2012). The whole process influenced the development of transversal and specific math skills.

Indeed, our study revealed considerable improvement in what concerned: i) the different dimensions of creativity – fluency, flexibility and originality –and ii) the use of functional reasoning. Results challenge several beliefs and/or conceptions about creativity. In fact, students were allowed to use more and different counting strategies taking advantage of the direction of reading, the symmetry, the geometrical forms and more complex constructive or even deconstructive strategies (Rivera & Becker, 2008). They were also able to translate these ways of *seeing* using mathematical language. Inversely, they were able to represent a mathematical expression figuratively, making sense of numerical expressions relating them to the visual representation. Students were also able to use these skills to achieve a far generalization using functional and non-recursive reasoning, as concluded by Barbosa (2010), Vale et al (2011, 2012).

They gained more accurate and favorable opinions about creativity, e.g., that one could be creative in every subject; that creativity may be developed at school; that creativity can be a result of cooperative work; that a teacher or a student could be creative in Mathematics that Mathematics is a creative subject, just like music and arts; that creativity must be present in Mathematics classes, so that students can learn better (Levenson, 2011; Robinson and Aronica, 2009).

These more favorable views about creativity can influence its development and even contribute to a more positive vision of Education, mathematics, teachers and even students, determining factors to educational success.

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