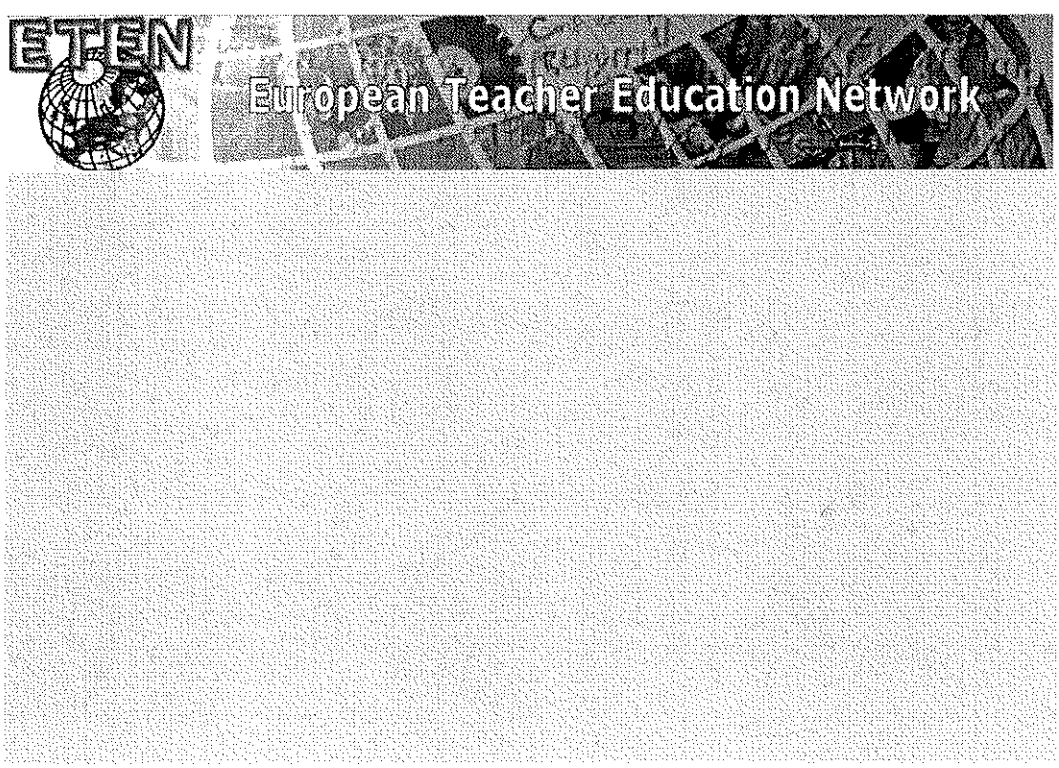
The Proceedings
of the 22nd Annual Conference
of the European Teacher Education Network

ETEN

EDITORS
Kristiina Kumpulainen & Auli Toom
University of Helsinki



ETEN 22
The Proceedings of the 22nd Annual Conference of the European Teacher Education Network

Editors:
Kristiina Kumpulainen & Auli Toom
University of Helsinki
Published jointly by ETEN (The European Teacher Education Network) and Department of Teacher Education, University of Helsinki.

ISBN 978–952–10–7862–0
ETEN Secretariat
Højvangseminariet International Office
Skolevej 6
DK 2600 Glostrup
Denmark

www.eten-online.org

Printed at the University of Helsinki,
Department of Teacher Education,
P.O. BOX 9 (Siltavuorenpenger 5A),
FI-00014 University of Helsinki
FINLAND

CONTENTS

I Mathematics Education

II Myths and Fairy Tales

III Reflective Practice

4. Informatic Tools, Isometries and Communication (in) Mathematics

Lúcia Matos and Isabel Cabrita
Basic School Castro Matoso; CIDTFF/ Dep. of Education, University of Aveiro

Abstract
In Portugal, the current Mathematics Program of Elementary Education and the document of the "Learning Goals" give the geometry the place it deserves. And advocate an approach of geometric topics, they also renovated, that gives space to the action of the student in the understanding of geometrical concepts, supported by dynamic environments of dynamic geometry. One topic that has undergone considerable changes was "geometric transformations". On the other hand, the "Patterns" have been gaining ground as an innovative way to contribute to the resolution of student's disinterest, lack of motivation and school failure. But there are few studies which are situated at the confluence of these dimensions. In this perspective, we developed a research that aimed to assess the impact of an approach of isometries through the study of patterns and using dynamic geometry environment, at the level of the development of transversal and specific skills in Geometry. We decided for a multiple case study, essentially qualitative, which took place in a context of action research. It involved a 9th grade class and, in particular, four pairs of students from an elementary school. We used inquiry, direct observation and document analysis, supported by several instruments, including logbook, questionnaires, informal conversations, tests, students' productions and internal documents of the school, as data collection techniques. For data processing, we used, mainly, content analysis. The study concluded that the approach of isometries centered on patterns, using the Geometer's Sketchpad (GSP), contributed, generally, to the development of geometrical knowledge; of mathematical communication; autonomy and to a more friendly relationship with geometry. In this communication, we highlighted the impact of the study at the level of interactions established during the mathematical activity.

Introduction

Following what has been happening internationally, in Portugal, the current Mathematics Program of Elementary Education – MPEE - (Ponte et al., 2007) gives Geometry a central role in the school curriculum, particularly with the introduction, since first grade, of a new way of facing and addressing the several geometric transformations. The isometries, in the Euclidian plan, can be explored from friezes and rosettes, establishing the foundation for a more thorough and deeper learning process as one advances in education.

Another vital aspect concerns the importance of the use of the computer and, in particular, the dynamic geometry environments (DGE). There are many authors and documents that have been stressing the importance and the advantages of using computational tools in the representation of geometric objects, while favoring an active role of the students in their leaning. (Candeias, 2005; Ribeiro, 2005; Ponte *et al.,* 2007; NCTM, 2000/2007; Hoyles & Lagrange, 2010; Breda *et al.*, 2011).

In learning Mathematics, the "Patterns" have also been gaining ground as a supportive and innovative context for students to engage in mathematical activities and develop mathematical communication. However, there are few studies which are situated at the confluence of these three dimensions.

The current article aims to present some results of a research work (Matos 2011) oriented by the following question – *To what extend can an approach of the rotations centered on the study of the patterns contribute to the development of mathematical transversal and specific skills*? It focused, therefore, on the students and aimed to assess the impact of an approach of the isometries, centered on the study of patterns and using Dynamic Geometry Environments, at the level of:

- Geometrical knowledge, specially related to the isometries;
- Communication;
- Autonomy;
- A more affective relationship towards Geometry,

In this article, it is underlined the impact at the level of the social interactions established.

Communication in mathematics

The communication in Mathematics has been, recently, a highly valued area by the results that the investigation has reached so far. In fact, studies like the ones of Cobb (1995), Voigt (1995), Wood (1998), Brendefur & Frykholm (2000), Lampert e Cobb (2003) and Martinho (2007) allow us to conclude that the communication can be a catalyst for learning mathematics, meaningful for the students. The *National Council of Teachers of Mathematics* (NCTM, 2000/2007) gives it a central role, too, highlighting the importance of: organizing and reinforcing mathematical thinking through communication; communicating the mathematical thinking consistently and clearly among students, teachers and others and using mathematical language to express mathematical ideas accurately.

So, it is not surprising that, in Portugal, in the current MPEE (Ponte *et al.*, 2007) the *mathematical communication* is highlighted, as a transversal capability to be developed throughout the entire academic life "both with the *problem solving* and the *mathematical thinking*". Emphasis is placed on: "the students should be able to *communicate* their ideas and read the others' ideas, organizing and clarifying their own mathematical thinking" (id: 5), and it is considered that the creation of opportunities for communication constitutes a crucial aspect while working, in particularly, within the classroom.

Although the importance of the written communication is emphasized as a way to "help the students consolidate their thinking, once it makes them think about the work and clarify their ideas in relation to the issues developed in class" (NCTM: 2007: 67), the oral communication is a fundamental part to the development of mathematical competence, both in terms of the (re)construction and disclosure of the knowledge, and of the development of other transversal and specific capabilities and attitudes. Alrø e Skovsmose (2006), for example, considers that the oral interactions while part of the dynamics of the communicative process constitute a privileged means for negotiation of meanings, in the Mathematics class. NCTM (2000/2007) also emphasizes the building of meanings, as well as the consolidation of ideas and their revelation. But, so that the interactions can gain added value, Boavida *et al.* (2008) reveal those which are created around the resolution of certain tasks, rather than the mere presentation focused on the product – "the interactions that occur in the course of the mathematical activity triggered by a task, create numerous learning opportunities that are unlikely to occur in a class of individualized work in which the interaction is, frequently, confined to the presentation, on the board, of the procedures used to get to the solution." (78).They stress, this way, its social dimension, an aspect investigated by Guerreiro (2012).

However, it is not enough that the students participate, explain their reasoning and listen to each other for learning to occur. The teacher has a crucial role in this process, by organizing the interactions that happen in the classroom, making them easier or inhibiting

them (Menezes, 1995). In fact, the teacher's practice can vary from "traditional" asking questions to a dialogue *praxis* in which the students play a more significant role, expressing their views and explaining their own way of thinking about the matters under discussion (Wood, 1998). In this case, the teacher creates favorable conditions so that Mathematics makes sense for the students and also so that students don't see it as a set of procedures, rules and processes that must be mechanized and stored for further use. Regardless of the paradigm that supports such actions, the questions assume a relevant role and those which request intervention, inviting students to participate, gain special importance (Menezes, 1995). But, above all, it is important to motivate and make room for the students to ask questions and reflect, key ingredients to an effective learning (Pedrosa *et al.,* 2005).

Method

We have opted for a multiple case study (Stake, 2007; Yin, 2005), essentially qualitative (Bodgan e Biklen, 1994). As suggested by these authors, the techniques for the data collection were very varied, considering the inquiry, the direct observation and the data analysis, backed up by a lot of tools, namely the logbook, the questionnaire, informal talks, the evaluation test, other students' works and internal documents of the school. For the data processing, we used statistical analysis and content analysis (Krippendorff & Bock, 2008) oriented by defined categories from the subjacent question to the study.

Due to some issues that have to do with the restrictions imposed upon this article, we will only refer to the results obtained by four pairs of students (G2, G4, G7 and G8) of a class of 21 students of the 9th form, related to the communication as social interaction, that emerges from a discursive practice, deriving from collective and individual processes of sharing meanings. The choice of the four pairs (made by the students according to their preferences) had to do, mainly, with their different school performance and also their easiness in communicating ideas, both orally and in writing.

We developed nine learning experiences within the unit "Circumference and Polygons. Rotations", and in its approach, by the researcher/teacher of the class, in Maths class, Supportive Study and Study Room, methodologies for collaborative work were privileged, using *Geometer's Sketchpad* (GSP).

We also had in mind to see, describe and interpret the learning process experienced by the students, in real-time and in the natural environment of the classroom which, together, integrated the individual work, the work in small groups and the large group discussion.

Results and discussion

The case G2

A2 and A14 showed great expectations in relation to the working sessions using GSP, once, in spite of using the computer to work and to do research at School and in Maths lessons, they had never used Geometry software. Both liked school but only A2 said that Maths was one of her favorite subjects. She was an attentive, organized and participative student, asking relevant questions and used to help her classmates whenever they needed. Her best friend was A14 and they used to work together in every school subject. Maths wasn't her favorite subject. She liked the new technologies, handling them well. Both of them showed commitment in the assigned tasks; however, A14 was more talkative and less hard-working. A2 liked to learn new things a lot more and showed great sense of responsibility in the work she performed.

Regarding interactions, A2, in many moments, assumed an "inquiring" role relatively to her workmate, questioning her about what they were going to do, which objects they were

going to need to do what they wanted. This attitude was taken in a natural way, maybe due to the fact that she was a good student. She helped her classmate to think, encouraged her in her learning process, helping her to think of it. Nevertheless, it was A14 who took the leadership of the mouse in the constructions although she recognized in her classmate someone to trust:

A2: *In this issue what do you need to know? The picture only shows rotational symmetries!*
A14: *So this belongs to the group of the rosettes...dihedral* (controlling the mouse).
A2: *And what do we have to do?*
A14: *We have to complete the rosette.*
A2: *Yes, but what do we have to find in the picture?*
A14: *I think it is the measurement of the amplitude of the rotation.*
A2: *Exactly. Do it.*
A14: *But, what points do I have to consider?*
A2: *Let's see... perhaps we have to consider these.* (pointing at the screen) *What do you think?*
A14: *I'll measure it to find out.*

A2 helped A14 in her learning process, making her understand what she was doing, not only through the questions she answered but also through the clues of resolution she was releasing.

Throughout the sessions, both students argued a lot, based on the feedback seen on the computer when manipulating the constructions:

A2: *Do you want to try different amplitudes, to see if they maintain the relationships?*
A14: *OK. I was thinking of making an angle range of 70° first and then a different one.*
A2: *Use the menu "Rotate" to see what is maintained. Don't select everything, just the hexagon.*
A14: *OK. I know how to do that, first I select the picture and only then I trigger the command.*
(A2 reads the question again)

Technology constituted, this way, a context for the discussion about the objects seen on the screen and about the effects of the several transformations that the software enabled. The use of the DGE also enabled to create, within the group, contexts of collaborative learning, which led them to clarify and structure their thinking – increasing the possibilities of a conceptual growth.

Thus, the context of the interaction set up a collaborative learning environment, since there was an active and shared involvement of the group in the resolution of the different issues, discussing ideas and situations. Indeed, the discussion generated around the records was constant and the horizontal interactions established within the pair were, in a symmetrical way, characterized by a mutual negotiation where it is acknowledged the contributions of both parts to carry out a final common conclusion. (Fernandes, 1997)

Concomitantly, dissipated vertical interactions were also verified in issues of focusing and short answers, in order to help the students overcome their doubts and truly engage in the task:

Researcher: *Observe the images you have built and take conclusions.*
A14: *It is a rotation with the same centre.*
Res.: *And what can you tell about the amplitude of the rotation angle?*
A14: *We have already measured; it equals the sum of the measurements of the other two.*
Res.: *Ok. Try with other amplitude measurements and see if the relationships are maintained.*

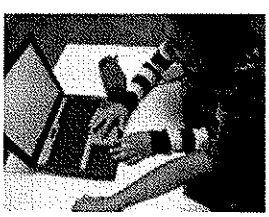
It was also observed, in the groups' exhibitions, that the researcher assumed an inquiring role, in an attempt to clarify and summarize the claims of the students. From the collected data, one can infer the importance of social interaction in learning Mathematics (Schoenfeld, 1992)

and, in particular, Geometry, for its decisive function for the construction of geometric knowledge. The idea that through the exchange of ideas knowledge is better understood by everyone was reinforced.

The case G4

A4 and A12 had a computer at home, which they used to play, to work and also to talk on Messenger. A4 could keep up, easily, the various issues and was responsible, even though, during classes, he seemed to be "distant". He didn't like going to the board, because he was afraid of failing in front of his classmates. A12, although rather reserved, was nice. She liked Maths but was not good at this subject. In class, she was attentive and hardworking, engaging herself in all activities, both by herself and within a group. However, she was more comfortable solving problems than in more open or complex tasks. She pointed Geometry as her favorite topic, because it was easier for her to understand.

The pair stood out by the way it mingled in the class. A significant contribution to this was the group work, without which the number of interactions would be smaller and, possibly, without much impact on the development of the relational competence.



Picture 1. Photographic record of A12 helping colleagues

The students worked, in a general way, collaboratively, both in the performance of geometrical constructions and in the preparation of responses, which allowed them to think about what each one of them was doing on the computer screen and to talk about the constructions. The fact that they worked well together was beneficial to their learning.

It was also clear that the fact that they had used the language of *menus* and had the possibility to point at the screen, with the finger or the mouse, helped them to think and understand what they were discussing, thereby, facilitating the development of arguments:

> A12: *You have to consider these points to the vector associated to the translation/slide.*
> A4: *Isn't it the one which has the opposite direction? This one* (pointing at the screen).
> A12: *Yes. But we have to consider the two vectors in the continuation of the frieze.*
> A4: *But it isn't correct!*
> A12: *You are right. We have to repeat the new motive formed by the two letters F.*

In general, A12 was the one who stood out during the experiment, both by the involvement in the achievement of the activities, and the presentation of the results to her peers, having placed pertinent and relevant issues for a better understanding of the general discussion:

> Researcher: *G4, can you explain how you built the frieze of the last question?*
> A4: We started by drawing a parallel line to the given vector. Then we determined the image of the crown by the flip. Afterwards, we applied the vector to the image and obtained the crown.
> A12: But now we need to repeat the new motive. What is the vector associated to this slide?

However, the ideas of A12 not always prevailed and, sometimes, they expected a recognition of validity. Most interactions were made equally, based on a hierarchical organization, according to its status, in particular, in the last sessions of intervention.

Thus, interactions confined collaborative learning contexts, since they influenced the way they shared geometric concepts and meanings. The exchange of ideas enabled a more active involvement in learning, as they became aware of their ideas in that dialogue. On the other hand, the discussions, moderated by the researcher, aimed to promote interactions within the group and other students in order to detect difficulties in understanding, help to reason and encourage participation and initiative.

The case G7

It was the first time that A15 and A19 worked together. A19 liked Mathematics and viewed it as one of his favorite subjects. He participated in class, did all the homework and enjoyed working in groups. He was good at Mathematics. A15 always had a lot of difficulty in Mathematics and always had bad grades at this subject. He did not participate orally; he wasn't attentive and did not solve tasks. He was nice and friendly but took advantage of every opportunity to get distracted. In general, he had poor school performance and showed a lack of motivation for studying. He was sorry that Mathematics meant nothing to him, only showing some interest when the issues did not require much mathematical knowledge.

A19 showed great enthusiasm for the proposed activities. His participation and hard work were constant. He showed a fairly good level of integration of knowledge and was quite sure when intervening to defend his views. He revealed great persistence in the discovery of conjectures and, in some cases, he continued investigating extra class. When the group was faced with an impasse, it was A19 who found a way to overcome it. There was a clearly asymmetry in the power relations in this group. Although his classmate told him what to do, A15 resisted participating in the task. He just wrote the answers thought by his classmate and did it without much personal involvement. Before the first difficulty, he used to ask him for help to write the requested replies:

> A19: *Now, build the rosette generated by successive rotations of 90°.*
> A15: *I've already understood that.*
> A15: *Do it. It's very easy!*
> A19: *Not now. I'll make the records. I know how to do that.*
> A19: *I'll see if you can answer the next question.*

It was notorious the evolution of A19's satisfaction in exploring this kind of tasks. He stood out from his classmate, by his involvement in doing the proposed tasks and by his role in sharing his ideas either with his classmates, or with the large group. The *Sketchpad* influenced, by far, the way people see Geometry and its learning. This aspect was not observed in A15. His participation consisted, basically, of following the path outlined by his classmate.

A19 was the one who, in the group, wanted the researcher's help more often, and who established verbal interaction with her more frequently, either by raising issues and putting questions, or by answering her questions. He was fully engaged in solving tasks, working with his colleague in a healthy way. They were usually the last ones to finish and could not always fully complete the activities. In spite of putting questions during their resolution, they showed some previous work in trying to overcome the problems which were emerging.

Indeed, the student-student interactions confined to a little collaborative learning, since there was not a joint cooperation for the work set. There was, by A15, a passive interest in building his own knowledge. His role was not decisive for the way how the exploitation of the tasks occurred. They played very different roles in the group, most probably due to their different academic performance. Therefore, the development of every task was always carried out by A19.

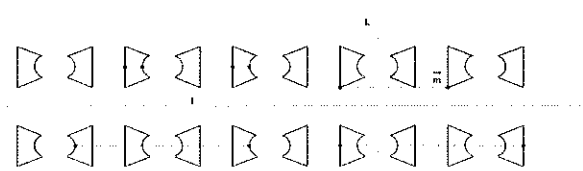
The case G8

Both A8 and A17 liked school, but A8 expressed a special preference for Mathematics and A17 for History. A8 was a very responsible student, and carried out her work very seriously, always trying to be the best. She was attentive and organized and could express herself, orally and in writing, very easily. She enjoyed going to the board and helping her classmates but when she needed help she wanted the teacher's help. On the contrary, A17 was very shy and introverted. She saw herself as an average student and didn't like the subject very much.

The dyad was pleased to work together and the A8's role was determinant to integrate her classmate in all activities, although this one didn't like this way of working very much. Especially in the beginning, A17 showed more difficulties than her classmate in taking initiative. Even though A8 took possession of the mouse more often, she was always concerned with sharing and discussing their ideas and make room for her classmate to participate. As the experience of the group increased, the role of A17, although not so crucial for taking initiative or solving problems, was still important. But it was mainly A8 who never gave up looking for new ways and tried not to ask for her researcher's support. On the other hand, in some situations, A17 was less comfortable in the exploitation of the tasks, maybe because she was a rather timid student. In the group, her participation consisted of, in most cases, following the path outlined by the classmate.

Picture 2. Photographic record of the group G8

In spite of working well together, who dictated the answers was A8 and it was not seen in the other student the need to change the situation or contradict her classmate. In this regard, let's see the solution presented by the pair on the third question:



Picture 3. Solution of the pair G8 on the task 8 – question 3

We made a reflection/flip of the figure associated with the line K, and then we flip the figures obtained through the axis l. Afterwards, we made successive translations associated with the vector m.

It was notorious the weak involvement of the group in the execution of the tasks, which resulted in a less active participation in the discussions, perhaps because they didn't like Geometry very much:

Researcher: *So, do you want to present your conclusions?*
A8: *We don't have anything to add.*
Res.: *But what were the conclusions you took from this question?*
A8: *It was the same, teacher. It's not worth it. Go on.*

For the interaction context, it was acknowledged a collaborative learning environment. However, the horizontal interactions established within the group, were characterized by an asymmetry in relation to the role that each one assumed in the exploitation of the tasks. Thus, the collaboration was based on an unequal relationship of power, in which A8 sought to dominate, not recognizing, sometimes, the other's contribution to the construction of meanings.

Regarding vertical interactions, the few that existed, as it sought to develop a more autonomous work, questions were directed to a focus point, in order to help the group overcome their doubts. The way they tried to respond to the different challenges that have been suggested allow us to assess that the work in small groups was important, since knowledge is best understood by each one, although not always had A17 profited from that interaction.

Conclusions

As Abrantes *et al.* (1999) recognized, it was also seen in this study that the geometric activities set excellent opportunities to develop mathematical communication and Geometry, in particular, appears to be a favorable field so that they can express their ideas and arguments. The fact that the tasks have been carried out in pairs involving a DGE enhanced the exchange of ideas, negotiation of meanings and development of arguments.

However, the difficulty in communicating ideas in written conditioned the way this teaching experience took place. On the other hand, it was clear that the open and difficult tasks are a challenge for which most students, especially these four cases, were not prepared. Indeed, the reaction of the pairs was very similar to the other dyads: a lot of levity in the way of facing the work of reflection on the tasks and the way of expressing and communicating that reflection, particularly in matters concerning the formulation of conjectures and the establishment of generalizations.

With respect to the way how the different groups interacted, the importance of the social interaction can be inferred for its decisive function to the construction of geometric knowledge. Although there were asymmetries in the involvement of the students in each pair, the interactions, in general, were always based on the work to be done.

On the other hand, the researcher's speech was, primarily, oriented to the making of questions that helped the students through their work assignment. Therefore, she assumed the role of moderator of the discussions, by managing the sequence of interventions and orientating, whenever necessary, the content. The questions were made to spot difficulties in the understanding of the geometric concepts and processes, helping them to think, motivating them to participate and also to see if they were following the work of the class.

Thus, the results suggest that the interactions established among the students in the groups G2 and G4 and the researcher, along with all collaborative work, point, clearly, to a positive evolution in their way of communicating, which was reflected in the work carried out throughout all tasks. The fact that the students felt more comfortable in small groups led to the confrontation of different understandings and to the emergence of new meanings. However, in the other two cases, in spite of interacting well together, the interactions established among them configured a collaborative learning environment but characterized by an asymmetry regarding the role that each one took on during the exploitation of the tasks. In fact, the cooperation was based on a disparity of power among them and the activities developed within the subgroups were always held under the initiative and guidance of A19 in the group G7 and of A8 in the group G8. Hence, although the verbal interactions were frequent,

there was little discussion around mathematical ideas among the members of the subgroups. A17 and A15 recognize some failure in the involvement in the work, making it clear that not everything went well, concerning the way they carried out the tasks within the group.

From the above, it can be concluded that interactions have taken a key role in the communication that occurred in the classroom and featured, in a decisive way, the nature of teaching and learning the subject, by establishing the difference between what can be done with others and all alone. In fact, the approach of rotations centered on the study of patterns, using DGE's and the work, mainly, in small and large groups constituted a good resource to promote communication in Mathematics (in most students).

References

Abrantes, P., Ponte, J. P., Fonseca, H., & Brunheira, L. (Eds.). (1999). *Investigações matemáticas na aula e no currículo.* Lisboa: APM e Projeto MPT.

Alrø, H. and Skovsmose, O. (2006). *Dialogue and learning in mathematics education.* Belo Horizonte (Brazil): Autêntica. (Translated into Portuguese by Orlando de Andrade Figueiredo.)

Boavida, A., Paiva, A., Cebola., G., Vale, I. & Pimentel, T. (2008). *A experiência matemática no ensino Básico.* Lisboa: DGIDC.

Bodgan, R. & Biklen, S. (1994). *Investigação qualitativa em educação: uma introdução à teoria e aos métodos.* Porto: Porto Editora.

Breda, A., Serrazina, L., Menezes, L., Sousa, Hélia & Oliveira, P. (2011). *Geometria e medida no ensino básico.* Lisboa: DGIDC.

Brendefur, J., & Frykholm, J. (2000). Promoting mathematical communication in the classroom: Two preservice teachers conceptions and practices. *Journal of Mathematics Teacher Education, 3*(2), 125-153.

Candeias, N. (2005). *Aprendizagem em ambientes de Geometria Dinâmica.* (Tese de Mestrado, Universidade de Lisboa).

Cobb, P. (1995). Mathematical learning and small-group interaction: Four case studies. In P. Cobb & H. Bauersfeld (Eds.), *The emergence of mathematical meaning: Interaction in classroom cultures* (pp. 25-129). Hillsdale, NJ: Lawrence Erlbaum Associates, Publishers.

Fernandes, E. (1997). O trabalho cooperativo num contexto de sala de aula. *Aná, Psicológica, 15*(4), 563-572. Disponível na Internet via WWW URL: http://www.scielo.oces.mctes.pt/pdf/aps/v15n4/v15n4a04.pdf. Arquivo capturado em 30 de Julho de 2009

Guerreiro, A. (2012). *Comunicação no ensino-aprendizagem da matemática: práticas no 1.º Ciclo.* (Tese de Doutoramento, Universidade de Lisboa).

Hoyles, C. and J.-B. Lagrange (eds.) (2010): *Mathematics Education and Technology—Rethinking the Terrain.* The 17th ICMI Study. Springer: New York.

Lampert, M., & Cobb. P. (2003). Communication and language. In J. Kilpatrick, W. G. Martin & D. Shifter (Eds.), *A research companion to Principles and standards for school mathematics* (237-249). Reston, VA: NCTM.

Martinho, H. (2007). *A comunicação na aula de Matemática: Um projecto colaborativo com três professoras do ensino básico.* (Tese de Doutoramento, Universidade de Lisboa).

Matos, L. (2011). *Abordagem das rotações centrada nos padrões – um estudo de caso com alunos do 9.º ano.* (Dissertação de Mestrado, Universidade de Aveiro).

Menezes, L. (1995). *Concepções e práticas de professores de matemática: Contributos para o estudo da pergunta.* (Dissertação de Mestrado, Universidade de Lisboa).

NCTM. (2000). *Principles and standards for school mathematics.* Reston, VA: Author. (translated in 2007 by APM: Lisboa).

Pedrosa de Jesus, M., Neri de Souza, F., Teixeira-Dias, J., & Watts, M. (2005). Organising the chemistry of question-based learning: a case study. *Research in Science & Technological Education, 23*(2), 179-193.

Ponte, J., Serrazina, L., Guimarães, H., Breda, A., Guimarães, F., Sousa, H., Menezes, L., Martins, G. & Oliveira, P. (2007). *Programa de Matemática do Ensino Básico*. Lisboa: Ministério da Educação, Direcção Geral de Inovação e Desenvolvimento Curricular.

Ribeiro, A. (2005). *O Cabri-Géomètre e a construção de uma nova cultura matemática*. (Tese de doutoramento, Universidade de Aveiro).

Schoenfeld, A. (1992). Learning to think mathematically: problem solving, metacognition, and sense making in mathematics. In: Grouws, D.A. (Ed.), *Handbook of Research on Mathematics Teaching* (pp. 334-370). New York: MacMillan Publishing.

Stake, R. (2007). *A arte da investigação com Estudos de Caso*. Lisboa: Fundação Calouste Gulbenkian.

Vale e A. Barbosa (Org.) (2009). Patterns – Multiple Perspectives and Contexts in Mathematics Education. ESEVC. ISBN: 978-989-95980-4-1

Voigt, J. (1995). Thematic patterns of interaction and sociomathematical norms. In P. Cobb & H. Bauersfeld (Eds.), *The emergence of mathematical meaning: Interaction in classroom cultures* (163-202). Hillsdale, NJ: Lawrence Erlbaum Associates, Publishers.

Wood, T. (1998). Alternative patterns of communication in mathematics classes: Funneling or focusing? In H. Steinbring; M. Bartolini Bussi & A. Sierpinska (Eds.), *Language and communication in the mathematics classroom* (pp. 167-178). Reston, VA: NCTM.

Yin, R. (2005). *Estudo do caso – planejamento e métodos*. Porto Alegre: BooKman.