Polarization Scattering Property of Cascaded Polarization Controllers

Nelson J. Muga, Mário F. Ferreira, and Armando Nolasco Pinto

ABSTRACT—The relation between the allowed range of variation of polarization controller wave-plates angles and the respective polarization scattering properties is investigated. It is demonstrated that a nearly uniform polarization scattering over the Poincaré sphere is obtained using a concatenation of three polarization controllers with angles randomly changed between -π/4 and π/4. It is also shown that an improvement of the scattering properties is obtained if the configuration angles are allowed to change between -π/2 and π/2.

Keywords—Polarization controller, polarization scattering, wave-plates angles range.

I. Introduction

The advances obtained with the introduction of optical amplification and chromatic dispersion compensation techniques allow the deployment of optical fiber transmission systems operating up to 40 Gbps per optical channel, reaching thousands of kilometers. These systems are limited essentially by polarization dependent effects, mainly, polarization mode dispersion (PMD). Therefore, the study of PMD is a key factor for the advance of ultra-high speed optical transmissions systems. Polarization dependent effects are difficult to analyze due to their intrinsic statistical behavior. PMD emulators are essential in the development and assessment of PMD compensation devices [1].

A device or a concatenation of devices, with the ability to produce a uniform polarization scattering over the Poincaré sphere can be useful in the development of PMD emulators. Indeed, some authors have proposed PMD emulators based on pieces of polarization maintaining fibers, interconnected with uniform polarization scattering devices [1]-[4]. In this context, fiber-coil-based polarization controllers (PCs) [2] can be used to scatter the state of polarization (SOP) over the Poincaré sphere [1], [2]. In [2], we presented a detailed study of polarization scattering using this kind of PC. It was shown that using a concatenation of three PCs, and changing the wave-plates of each PC following a uniform distribution between -π and π, a good uniform polarization scattering is achieved. In that work, only the influence of the number of concatenated PCs on the scattering properties was analyzed. In this letter, we extend that analysis by also considering the influence of the wave-plate angle range. The dependence of the scattering properties on both the number of concatenated PCs and the wave-plate angle range is analyzed. Therefore, the obtained analytical expressions for the variance of the Stokes vector parameters are explicitly functions of these two parameters.

II. Polarization Scattering

The PC is used to scatter the SOP results from the concatenation of two quarter-wave-plates (QWP) and one half-wave-plate (HWP), with the last one placed between the other two. The input and output SOPs can be respectively represented in the 3D Stokes space by the following two general Stokes vectors:

$$\hat{s}_i = [(s_1)_i, (s_2)_i, (s_3)_i]^T , \quad (1)$$

$$\hat{s}_o = [(s_1)_o, (s_2)_o, (s_3)_o]^T , \quad (2)$$

where T indicates the transpose. The QWP and HWP are also

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Nelson J. Muga (phone: + 351 234 377 900, email: muga@av.it.pt) and Armando Nolasco Pinto (email: anp@det.ua.pt) are with the Instituto de Telecomunicações, Universidade de Aveiro, Aveiro, Portugal.

Mário F. Ferreira (email: mferreira@fis.ua.pt) is with the Physics Department, Universidade de Aveiro, Aveiro, Portugal.

represented in the Stokes space by their respective Muller matrices [2].

1. Theory

Previous results show that just one PC is clearly not enough to provide uniform SOP scatterings [2]. In order to enhance the SOP scattering, a concatenation of several PCs must be considered, as shown in Fig. 1. The SOP at the n-th PC output, $\hat{s}_{n+1}$, is related to the SOP at the input of the same PC, $\hat{s}_n$, by the following expression,

$$\hat{s}_{n+1} = \boldsymbol{F_n}\, \hat{s}_n , \tag{3}$$

where the matrix $\boldsymbol{F_n}(\theta_{1,n}, \theta_{2,n}, \theta_{3,n})$ represents the n-th PC ($\theta_{1,n}$, $\theta_{2,n}$, and $\theta_{3,n}$ are the QWP, HWP, and QWP angles, respectively).

We assume that the PC angles are randomly changed, following a uniform distribution between $-m\pi/4$ and $m\pi/4$, where m is a positive integer number. We also assume that the angles are independently changed. Therefore, the average value of the $\boldsymbol{F}$-matrix elements, f_{ij}, will be equal for all the n PCs and will be hereafter designated by $<f_{ij}>$.

The average values of the matrix elements are calculated through the evaluation of a triple integral, as shown in [2]. In the present case, the limits of integration are $-m\pi/4$ to $m\pi/4$, and the probability density function for each wave-plate angle is $2/(m\pi)$. After this, we obtain

$$\left\langle f_{ij} \right\rangle_{\theta_i \in [-m\frac{\pi}{4}, m\frac{\pi}{4}]} = \delta_{2i}\ \delta_{2j}\ \delta_{1(-1)^{m-1}} \left(\frac{2}{m\pi} \right)^2 , \tag{4}$$

where δ_{ij} is the Kronecker delta. In contrast with the other five matrix elements, $<f_{22}>$ only vanishes for particular wave-plates angles ranges, that is, if m is an even number. For odd values of m, $<f_{22}>$ does not vanish, but converges to zero as m increases. In order to analyse the statistics of the system resulting from the concatenation of several PCs, we use (4) into (3), which is subsequently applied iteratively for an arbitrary initial SOP, $\hat{s}_1 = [(s_1)_1, (s_2)_1, (s_3)_1]^T$. Then, the mean values of the Stokes vector components at the end of the n-th PC, $<\hat{s}_{n+1}>$, are obtained:

$$\begin{bmatrix} \langle (s_1)_{n+1} \rangle \\ \langle (s_2)_{n+1} \rangle \\ \langle (s_3)_{n+1} \rangle \end{bmatrix}_{\theta_i \in [-m\frac{\pi}{4}, m\frac{\pi}{4}]}$$
$$= \delta_{1(-1)^{m-1}} (s_2)_1 \frac{1}{\left(\frac{(m\pi)^2}{4} \right)^n} \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} + \delta_{n0} (s_2)_1 \left(1 - \delta_{1(-1)^{m-1}} \frac{4}{(m\pi)^2} \right) \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}. \tag{5}$$

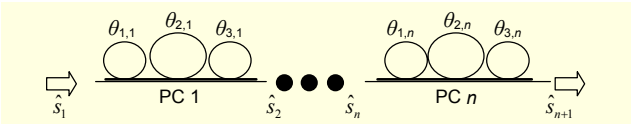

Fig. 1. Schematic representation of concatenation of n PCs and respective input and output SOPs.

In order to determine the variance of the three Stokes vector components we have to calculate their respective mean square values. Using the integration limit $-m\pi/4$ to $m\pi/4$, and a probability density function equal to $2/(m\pi)$, we obtain

$$\begin{bmatrix} \langle (s_1)^2_{n+1} \rangle \\ \langle (s_2)^2_{n+1} \rangle \\ \langle (s_3)^2_{n+1} \rangle \end{bmatrix} = \frac{1}{3} \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} + \frac{a}{4^n} \begin{bmatrix} 1 \\ 1 \\ -2 \end{bmatrix} + \delta_{n0} b \begin{bmatrix} 1 \\ -1 \\ 0 \end{bmatrix}, \tag{6}$$

where $a = 1/2\left(1/3 - (s_3)_1^2\right)$ and $b = 1/2\left((s_1)_1^2 - (s_2)_1^2\right)$.

In contrast with the mean value case, the mean square values of the three Stokes vector components are no more dependent on the wave-plate angle range. From (6), we verify that the mean square values converge to 1/3 with a factor of $a/4^n$.

The variances of each Stokes vector component at the end of the n-th PC are given by

$$\sigma^2(s_i) = <(s_i)^2_{n+1}> - <(s_i)_{n+1}>^2 . \tag{7}$$

To compare the distributions resulting from the SOP scattering with a theoretical uniform distribution of points, a normalized variance function can be defined as

$$\sigma^2_N(s_i) = [\sigma^2(s_i) - \sigma^2_T] / \sigma^2_T , \tag{8}$$

where $\sigma^2_T = 1/3$ is the value for a uniform distribution. This is the figure of merit that we use to evaluate our results.

2. Simulation Results

We start by simulating SOP scatterings for two different wave-plate angle ranges $-\pi/4$ to $\pi/4$ and $-\pi/2$ to $\pi/2$, corresponding to m=1 and m=2, respectively. The σ^2_N values, calculated for each distribution as a function of the number of concatenated PCs, are represented in Fig. 2. The initial SOP, $\hat{s}_1 = [0,1,0]^T$, was considered in all simulations. A good agreement is observed between analytical and simulated results. The obtained results show that in both cases, σ^2_N converges quickly to zero. We observe that if only one PC is used, the normalized variance for the second component of the Stokes vector, $\sigma^2_N(s_2)$, is better for the distribution obtained with m=2. For m=2 the normalized variances $\sigma^2_N(s_2)$ and $\sigma^2_N(s_1)$ are degenerated. From (5) and (6), in conjugation with the results presented in Fig. 2, we can conclude that scattering with

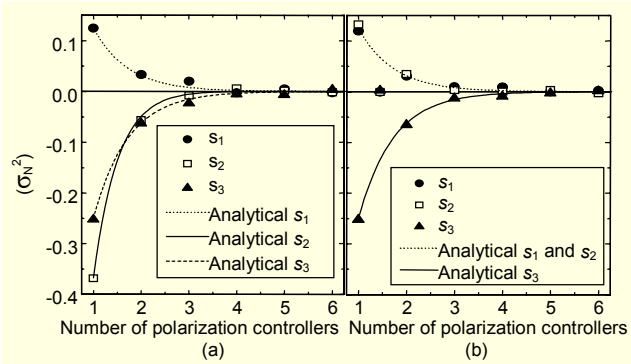

Fig. 2. Normalized variances evolution as a function of the number of concatenated PCs for (a) m=1 and (b) m=2.

m=2 provides better results than with m=1, when the number of PCs is small (less than 10).

If we have a uniform distribution of points over the Poincaré sphere two conditions are satisfied. The projection of points over the axes s_1, s_2, and s_3 has a uniform distribution between -1 and 1, and the variances of the distribution over the axes have the ratio of 1/3. The previous results show that the distributions obtained with our system clearly satisfy the second condition. In the following, we show that the first condition is also satisfied.

In order to quantify the influence of the number of PCs and the wave-plate angle range parameter on the uniformity of the scattering we calculate, for each Stokes vector component distribution, $p_{(s_i)}$, a normalized deviation factor (NDF) [2], [3]. Figure 3(b) presents the sum of the three Stokes vector component distribution NDFs as a function of the number of concatenated PCs. The results show that for a small number of PCs (up to six) a significant NDF improvement is observed if the wave-plate angle range m=2 is used instead of m=1. For six or more concatenated PCs the wave-plate angle range becomes no more relevant since the uniformity degree of the obtained distributions is similar. The points depicted in Fig. 3(a) represent the 50,000 SOPs scattered from a system comprising a concatenation of three PCs, with wave-plate angles randomly changed between -π/4 to π/4, and an initial SOP $\hat{s}_1 = [0,1,0]^T$. The figure shows a good uniform distribution of points over the sphere surface.

III. Conclusion

We have derived a general expression for the Stokes vector component variances, including both the number of concatenated PCs and the wave-plate angle range dependence. We showed that it is possible to obtain a uniform polarization scattering over the Poincaré sphere by a concatenating at least three PCs devices and randomly changing the configuration

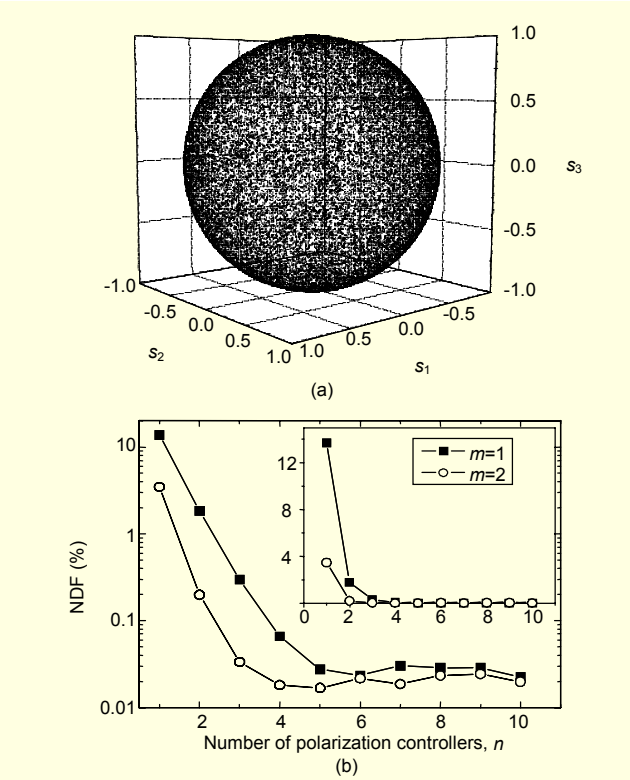

Fig. 3. (a) Poincaré sphere representation of the output SOPs and (b) sum of Stokes vector component distribution NDFs as a function of the number of PCs. Inset shows the plot in a normal scale.

of the wave-plates PC angles between -π/4 and π/4. We also have shown that improvement of the scattering properties is achieved if the configuration angles are allowed to change between -π/2 and π/2.

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