universidade de aveiro theoria poiesis praxis

1st International Conference on Math Education and Technology (ICMET 2023)

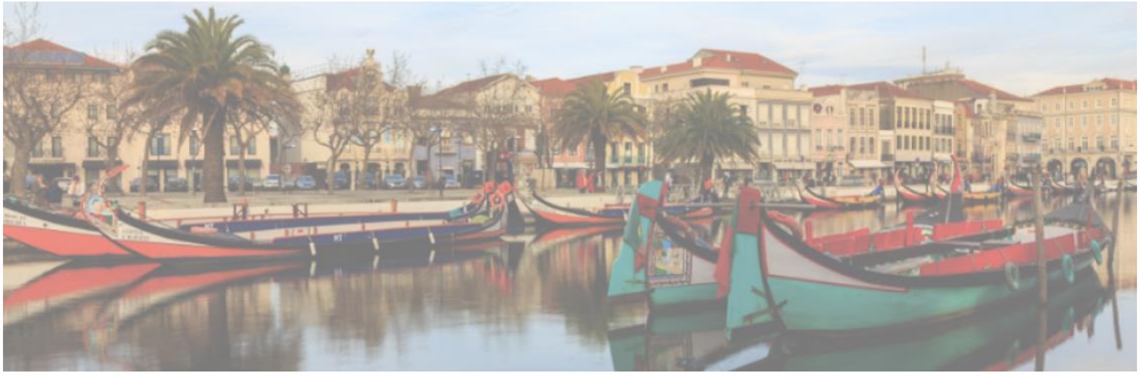
Book of Abstracts

Editors

Vanda Santos

Isabel Cabrita | Luís Descalço | Margarida M. Pinheiro

Nuno Bastos | Paula Carvalho | Paula Oliveira | Teresa B. Neto



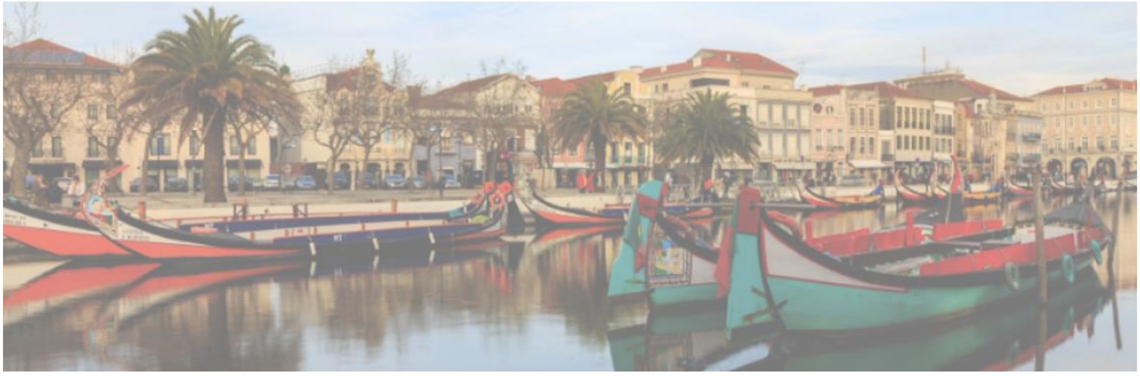
1st International Conference on Math Education and Technology 2023 (ICMET2023) – Book of Abstracts

Organizing Committee

Vanda Santos, University of Aveiro, Portugal
Isabel Cabrita, University of Aveiro, Portugal
Luís Descalço, University of Aveiro, Portugal
Margarida M. Pinheiro, University of Aveiro, Portugal
Nuno Bastos, Polytechnic Institute of Viseu, Portugal
Paula Carvalho, University of Aveiro, Portugal
Paula Oliveira, University of Aveiro, Portugal
Teresa B. Neto, University of Aveiro, Portugal

Scientífic Committee

Augusto Rasga, Higher Institute of Education Sciences, Angola
Álvaro Nolla de Celis, Autonomous University of Madrid, Spain
Ana Paula Aires, University of Trás-os-Montes and Alto Douro, Portugal
António Domingos, NOVA School of Science and Technology of Lisbon, Portugal
António Ribeiro, Polytechnic Institute of Viseu, Portugal
Bernardo Filipe Matias, Higher Institute of Education Sciences, Angola
Carlos Albuquerque, University of Lisbon, Portugal
Cristina Caridade, Polytechnic Institute of Coimbra, Portugal
Dina dos Santos Tavares, Polytechnic Institute of Leiria, Portugal
Fátima Regina Jorge, Polytechnic Institute of Castelo Branco, Portugal
Francisco Botana, University of Vigo, Spain
Gilles Aldon, French Institute of Education, France
Helena Campos, University of Trás-os-Montes and Alto Douro, Portugal
Isabel Cabrita, University of Aveiro, Portugal
Isabel Vale, Polytechnic Institute of Viana do Castelo, Portugal
Jaime Carvalho e Silva, University of Coimbra, Portugal
Ján Gunčaga, Faculty of Education, Comenius University in Bratislava, Slovakia
Joana Teles, University of Coimbra, Portugal
Luís Descalço ,University of Aveiro, Portugal
Margarida M. Pinheiro, University of Aveiro, Portugal
M. Pilar Vélez, Universidad Nebrija, Spain
Maria Cecília Costa, University of Trás-os-Montes and Alto Douro, Portugal
Maria Manuel Nascimento, University of Trás-os-Montes and Alto Douro, Portugal
Mariotti Maria Alessandra, University of Siena, Italy
Mirosława Sajka, Pedagogical University of Krakow, Poland
Nuno Bastos, Polytechnic Institute of Viseu, Portugal
Patrícia Silva, Federal University of Rio Grande do Sul, Brasil
Paula Carvalho, University of Aveiro, Portugal



Paula Oliveira, University of Aveiro, Portugal
Pedro Manuel Baptista Palhares, University of Minho, Portugal
Pedro Quaresma, University of Coimbra, Portugal
Philippe Richard, University of Montréal, Canada
Thierry Dana-Picard, Jerusalem College of Technology, Israel
Tiago Emanuel Klübler, State University of Western Paraná, Brasil
Vanda Santos, University of Aveiro, Portugal
Zoltán Kovács, Eszterházy Károly Catholic University, Hungary
Zoltán Kovács, Johannes Kepler University, Austria

Cover

Joana Pereira

Publisher

UA Editora: University of Aveiro
Library, Documental Information and Museology Services

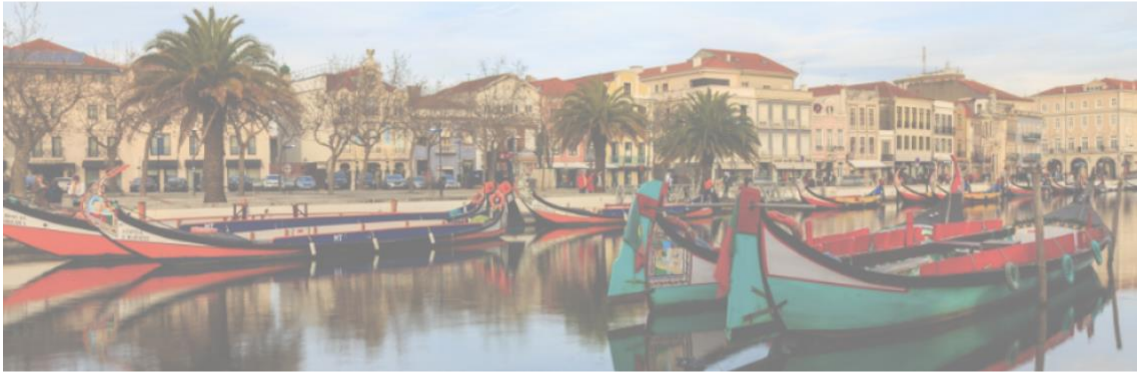
1st edition – March 2024

ISBN: 978-972-789-908-1

DOI: https://doi.org/10.48528/ss41-df14

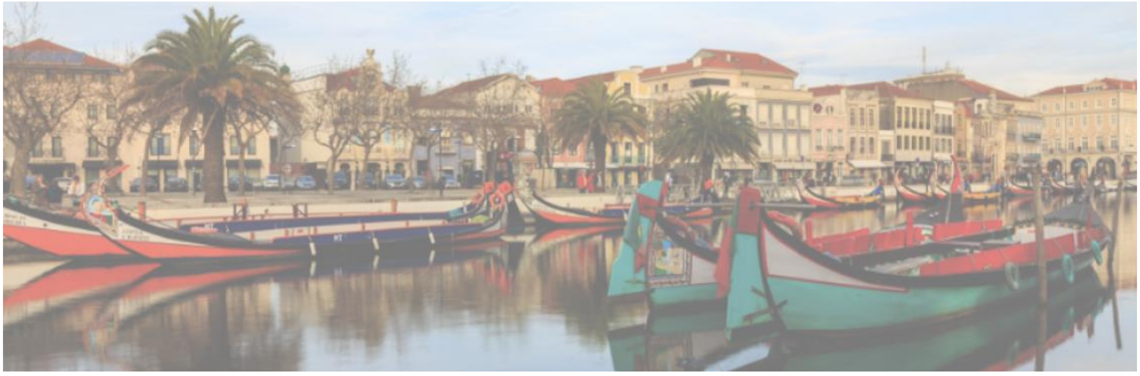
Sponsors

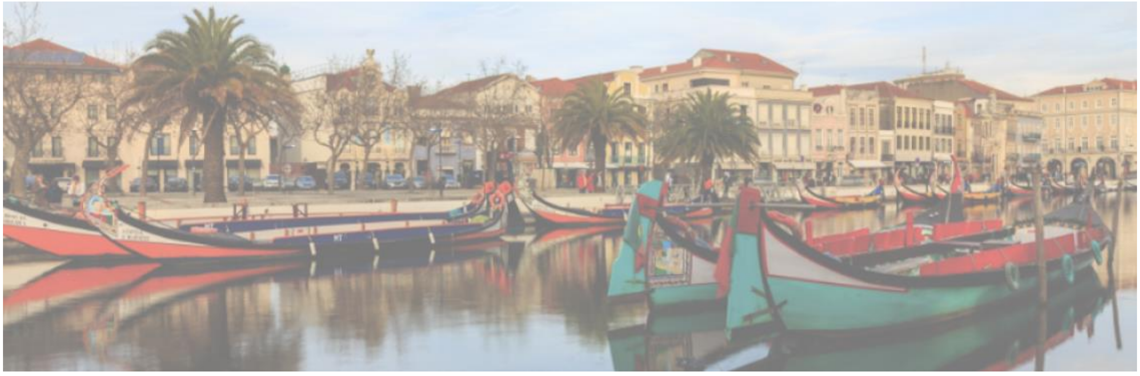
The Conference, the book of abstracts of which are now published, was financed by National Funds through FCT – Foundation for Science and Technology, I.P., within the scope of the project UIDB/00194/2020 and UIDB/04106/2020 and had the support of the Department of Education and Psychology and of the Department of Mathematics of University of Aveiro.



1st International Conference on Math Education and Technology 2023 (ICMET2023) – Book of Abstracts

TABLE OF CONTENTS





1st International Conference on Math Education and Technology 2023 (ICMET2023) – Book of Abstracts

PREFACE

The Department of Education and Psychology, in collaboration with the Department of Mathematics of the University of Aveiro, Portugal, organized the 1st International Conference on Math Education and Technology 2023 (**ICMET 2023**), held from October 2nd to 4th, 2023.

The main objectives of the **ICMET 2023** were to bring together experts from around the world, providing opportunities for networking and reflection in the fields of Mathematics, Education and Technology. Beyond exploring digital tools in Mathematics education, the conference delved into broader topics such as STE(A)M – Science, Technology, Engineering (Arts), and Mathematics.

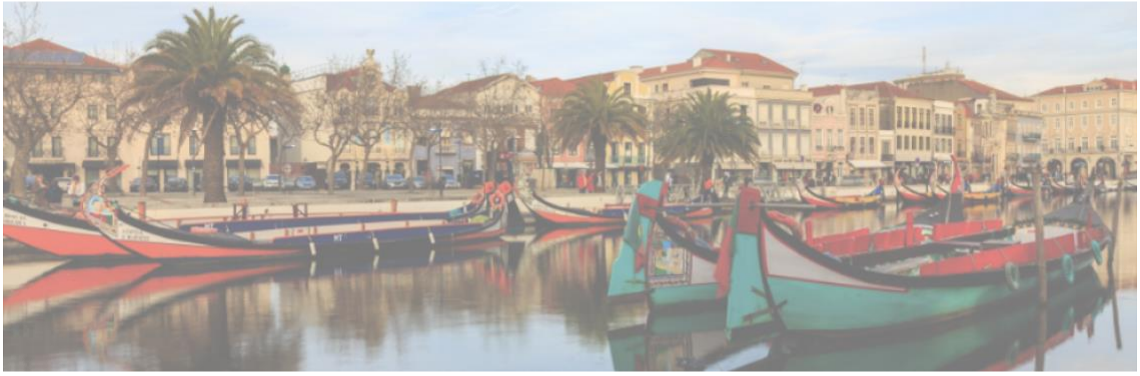
This international conference brought together researchers from diverse higher education institutions and secondary school teachers associated with STE(A)M areas across three continents.

The synergy of Mathematics, Education, and Technology emerged as a catalyst for innovation and societal progress. The integration of Mathematics with technological tools not only launches scientific discoveries and technological advancements but also facilitates data-driven decision-making.

This integration prepares individuals for careers in STE(A)M fields, contributing to innovation and global competitiveness. **ICMET 2023** served as a platform for collaboration, knowledge exchange, and the advancement of interdisciplinary approaches to education and technology in the ever-evolving environment of Mathematics.

This book of abstracts had the contribution of distinguished plenary speakers.

Wei-Chi Yang explored the expansion of exam-based university problems through dynamic geometric approaches. This innovative perspective involves integrating a computer algebra system with dynamic geometry, enhancing not only mathematical exploration but also providing a robust methodology for addressing real-life problems.



Andreia Hall addressed the meaning of numbers in our daily lives and the way they influence our understanding from early childhood. The discussion begins with rationales and extends to the characteristics of decimals, deducing their traits from the fractions representing them, employing basic number theory. An example illustrates the transformation of decimals into visual images.

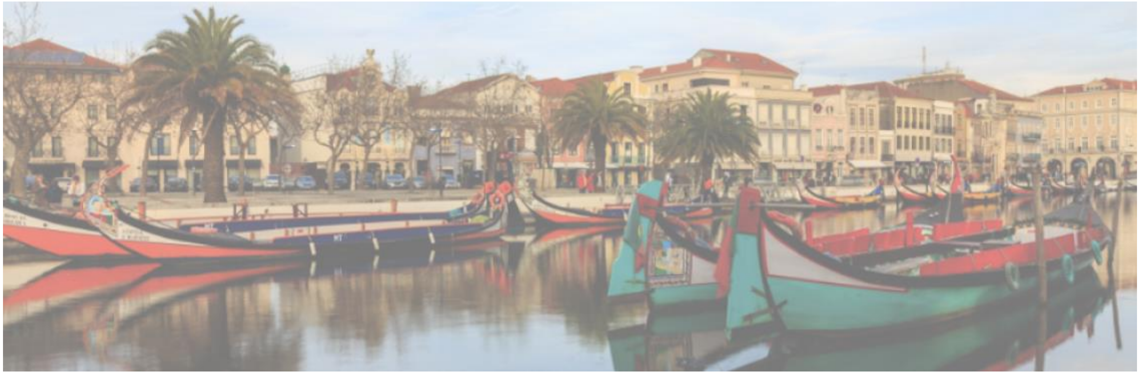
Tomás Recio discussed the Automated Reasoning Tools developed in GeoGebra Discovery. Through various examples, Recio underscored the performance of these features, emphasizing the capabilities of the “automated geometer.” The presentation proposed an algorithmic approach to assess the relevance of a geometric theorem, allowing the “automated geometer” to highlight results aligned with human expectations. The potential educational impact of these technological advancements was also explored.

This proceeding encompasses discussions across various domains, including Math Curriculum/ Curriculum Design, Teacher Education, History of Mathematics, STE(A)M, Creativity and Assessment. These topics served as focal points, engaging participants in insightful conversations that span the diverse and interconnected facets of mathematics education.

The Math Curriculum stands as the backbone of fostering numerical literacy, geometry, algebra, statistics, and critical thinking skills. Curriculum Design plays a pivotal role in shaping how mathematical concepts are communicated, ensuring a comprehensive and engaging learning experience for students.
The workshop of Dennis J. DeBay and Josie Smith was about the focus on creating active and participatory learning environments in asynchronous educational settings. The poster of Margarida M. Marques, Alvaro Nolla, Teresa B. Neto and Lúcia Pombo suggested an examination or study that involves the application of geometric principles to the monuments located in the city of Aveiro.

Teacher Education forms the cornerstone for effective mathematics instruction. Providing educators with innovative teaching strategies and a profound understanding of pedagogy is essential to cultivate a mathematically trained generation.
The poster of Ana Barbosa and Isabel Vale indicated a methodology to problem-solving process that involves the application of engineering principles and practices.

Diving into the annals of academia, the History of Mathematics unveils the evolution of numerical, geometric, algebraic and statistic principles, showing the rich topics that underly the subject.



Understanding this historical context can inspire students and educators alike, fostering a deeper appreciation for the beauty of mathematical concepts.

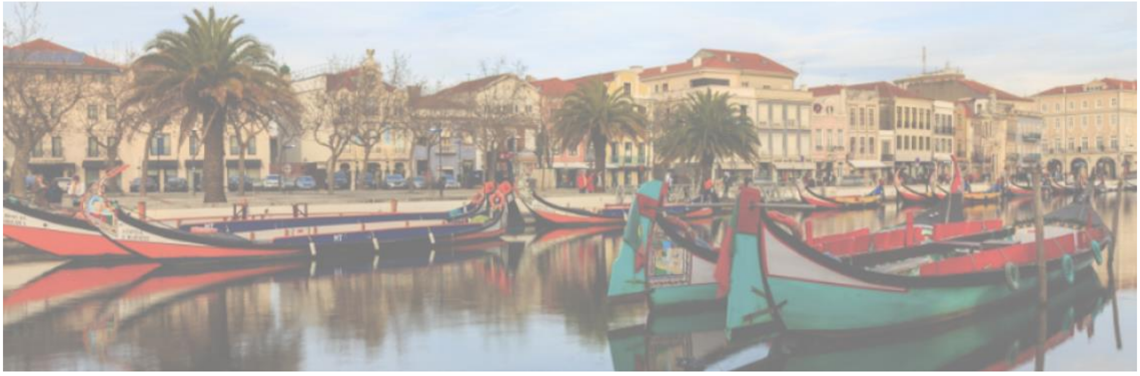
The integration of Science, Technology, Engineering, Arts, and Mathematics (STE(A)M) represents a holistic approach to education. It encourages interdisciplinary connections, promoting creativity and problem-solving skills essential for navigating the complexities of the modern world.
The poster of Carlotta Vielmo was about a deliberate planning or design of an exploratory data analysis task using RStudio, and the subsequent part of the statement, "an evaluation a posteriori," indicates that an assessment or evaluation of the activity is performed after it has been carried out.

Creativity is a driving force in mathematical exploration. Encouraging students to think out of the box fostering a deeper understanding of mathematical concepts and their real-world applications.
The poster of Teodora Cătinaş is about a research or study topic that explores interpolation techniques specifically applied to triangles where the sides are not straight but curved.

At last, assessment is a crucial aspect of the educational journey. Evaluating students ensures that learning objectives are met and provides valuable insights into refining teaching methodologies. A well-structured assessment system promotes continuous improvement and student success.

The triumvirate of Mathematics, Education, and Technology forms a dynamic and interconnected ecosystem. This powerful combination equips individuals with the intellectual tools required to navigate a constantly evolving world, fostering innovation, progress, and a collective pursuit of knowledge that transcends geographical borders and societal boundaries. Recognizing the significance of this trinity is crucial for shaping a future where education is inclusive, technology is transformative, and mathematical thinking stands as a cornerstone of societal development.

The editors aspire that the compiled texts here offer a chance for readers to explore further and contemplate the mentioned themes!



Plenary Speakers

Inspiring creative, innovative, and computational thinking in STEM through technology

Wei-Chi Yang[1]

[1]Radford University, Virginia, USA

Abstract

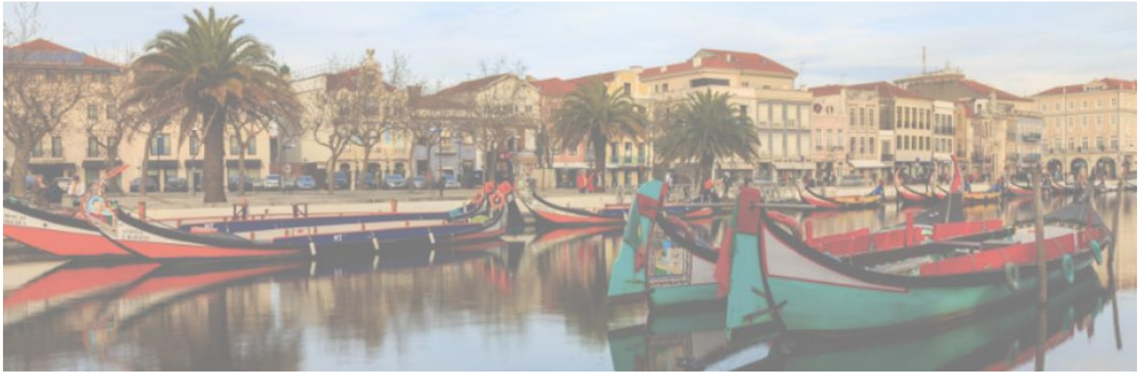
In this talk, I demonstrate how some rote types of exam-based university problems can be expanded to problems for undergraduates, graduates, and even researchers for further investigation. We shall see how dynamic geometric approaches can provide critical intuition and motivation to learners and make challenging problems more accessible to more students. Integration of the computer algebra system with the dynamic geometry system will not only allow us to make conjectures and discover more mathematics but also provide us with an excellent methodology to deal with many real-life problems.

We need to cultivate large collections of innovative and creative activities in STEM, that can only be resolved using human brains. Otherwise, AI will train our future math teachers and do some mathematics for us automatically. My own experiences as a learner, teacher, and researcher are outlined as follows:

- Search for the sources of exploratory activities:
- Exploration with technological tools, you will make many unexpected discoveries and exploratory activities.
- Adopt dynamic geometry software for making conjectures.
- Testing with many scenarios before forming conjectures.
- Prove it theoretically with the help of a CAS.
- Such a process indeed requires Computational Thinking.

As a mathematics and technology enthusiast, practitioner, and learner, I started with a rote type of examination problem, and the 5-year journey of exploratory activities led me to Calculus, Linear Algebra, Multivariable Calculus, Projective Geometry, Differential Geometry, and Topology. See articles in the References section.

We need to make math accessible to more students, which will be good for society in general. Many students are intimidated by algebraic manipulation-intensive problems from an early age. We are losing potential students as a result. Teaching to the test can never promote creative, innovative, and computational thinking skills. We need to recognize the importance of implementing a curriculum that involves the exploration of mathematics and its applications through the timely use of technological tools.



Make Math to be **F**un **A**ccessible **C**hallenging and **T**heoretical. Only when mathematics is made more accessible to more students will more students be inspired to investigate problems ranging from the simple to the challenging.

Keywords: computational thinking; technology.

Acknowledgments

I would like to express my sincere gratitude toward the organizing committee of ICMET 2023.

References

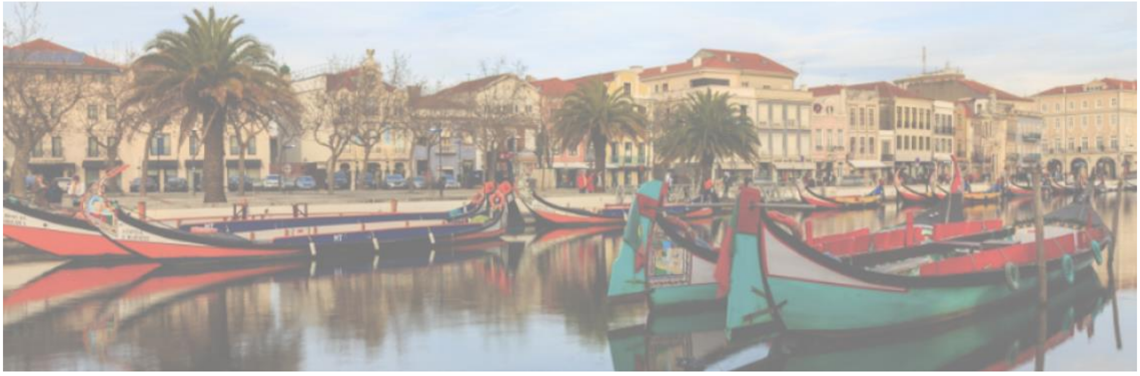
Yang, W.-C. (2020) Exploring Locus Surfaces Involving Pseudo Antipodal Points, at the Proceedings (refereed) of the 25th Asian Technology Conference in Mathematics, 2020, Mathematics and Technology, LLC, Scopus indexed, ISSN 1940-4204 (online version).

Yang, W.-C., & Morante, A. (2021). 3D Locus Problems of Lines Passing Through A Fixed Point. *The Electronic Journal of Mathematics and Technology*, 15(1), ISSN 1933-2823, pp1-22.

Yang, W.-C., & Morante, A. (2022). Locus Surfaces and Linear Transformations when a Fixed Point is at an Infinity. *The Electronic Journal of Mathematics and Technology*, 16(1), ISSN 1933-2823, pp1-24.

Yang, W.-C. (2022) Ellipsoid is Tangent to its Locus under a Linear Transformation, Isometries, and Sheared Maps, The Proceedings of the 27th Asian Technology Conference in Mathematics, ISSN 1940-4204, https://atcm.mathandtech.org/EP2022/invited/21949.pdf.

Yang, W.-C, Dávila, D., & Ho, W. K. (2023) Another Topological View of Curves and Surfaces Inspired by 2D and 3D Locus Problems. *The Electronic Journal of Mathematics and Technology*, 17(2), ISSN 1933-2823.



Dancing along to decimals of rational numbers

Andreia Hall[1], Nuno Bastos[2]
[1]CIDMA – Center for Research and Development in Mathematics and Applications, DMat, University of Aveiro
[2]Polythechnic Institute of Viseu, CIDMA – Center for Research and Development in Mathematics and Applications

Abstract

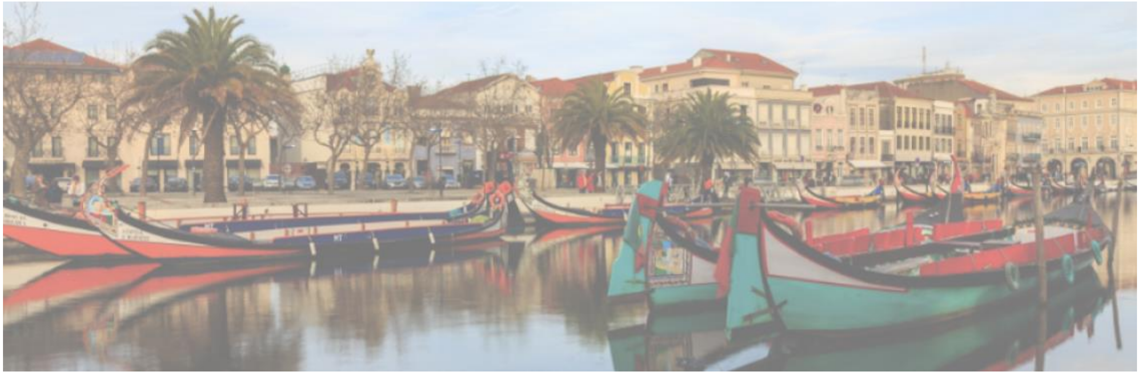
Numbers play a fundamental role in our daily lives. Since early childhood, we learn what they are and how to represent them. Despite our familiarity with numbers, there are numerous details that often go unnoticed, much like the unnoticed details of the street where we live and pass by every day.

Rational numbers are typically represented as decimals or fractions. In decimal form, rational numbers are either terminating or infinite repeating. The characteristics of these decimals can be deduced from the fractions that represent them, using basic number theory. For example, by examining a fraction, we can determine whether the corresponding decimal is terminating or non-terminating (Hardy and Wright, 1979; Pires, 2013). This only depends on the prime factor decomposition of the denominator, provided that the fraction is irreducible. It will be terminating if there are no prime factors other than two and five.

It is often said that a picture is worth a thousand words. What if we were to convert decimals into images? By dividing a circle into ten equal parts numbered from 0 to 9, decimals can be transformed into visual paths defined by the sequence of digits (to the right of the decimal point).

In this talk we will show some trajectories created by decimals of rational numbers. The resulting images are surprisingly diversified and beautiful.

Figures 1 shows some examples, namely the trajectories obtained by the decimals of all irreducible fractions with denominators 3, 11 and 13. Irreducible fractions with denominator 3 have two possible decimal expansions, $.\overline{3}$ or $.\overline{6}$, which have distinct repetends, both of length one. These trajectories degenerate into isolated points. Irreducible fractions with denominator 11 have ten possible decimal expansions, all of which strictly repeating, which reduce to five distinct repetends of length two: $\overline{09}$, $\overline{18}$, $\overline{27}$, $\overline{36}$, and $\overline{45}$. These paths translate into line segments. Fractions with denominator 13 have twelve possible decimal expansions which reduce to two distinct repetends of length six: $\overline{076923}$ and $\overline{153846}$. The corresponding paths are self-intersecting hexagons.



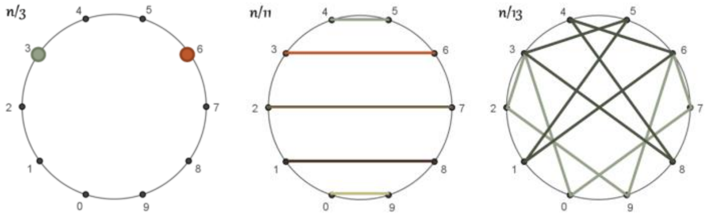


Figure 1: Decimal trajectories of all irreducible fractions with denominators 3, 11 and 13.

Animations for the trajectories given in Figure 1 can be accessed here$_3$, here$_{11}$ and here$_{13}$.

Figure 2 shows the trajectories obtained from fractions with denominators 10, 18 and 20. In all cases, whenever the decimal is terminating, the trajectories traced are those corresponding to the repeating versions of these decimals. For instance 1/10=0,1 =0,100000…=0,099999…..

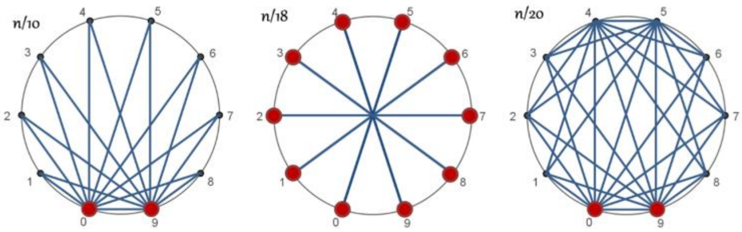


Figure 2: Decimal trajectories of all fractions with denominators 10, 18 and 20.

Animations of the trajectories for denominators 10 and 18 may be seen here$_{10}$ and here$_{18}$.

Keywords: rational numbers; number visualization; repeating decimals; fractions.

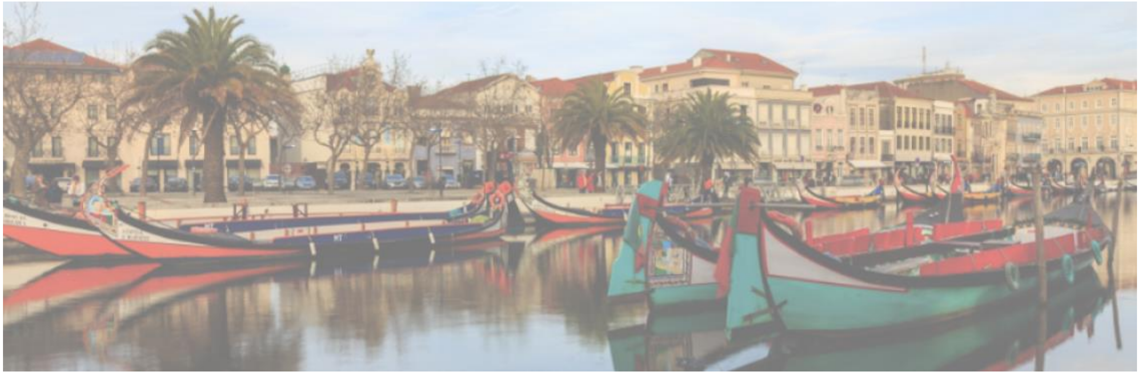
Acknowledgements

This work was supported by CIDMA and is funded by the Fundação para a Ciência e a Tecnologia, I.P. (FCT, Funder ID = 50110000187) under Grants UIDB/04106/2020 and UIDP/04106/2020.

References

Hardy, H., & Wright, E. (1979). *An Introduction to the Theory of Numbers* (5th ed.). Clarendon Press.

Pires, F. S. T. (2013). *Representação dos números na forma decimal e generalização a outras bases* [Master's thesis, University of Aveiro]. RIA Repositório Institucional. http://hdl.handle.net/10773/13317



Looking for interesting theorems in geometry

Tomás Recio[1]

[1]Universidad Antonio de Nebrija

Abstract

In our talk we will describe some on-going improvements concerning the Automated Reasoning Tools developed in GeoGebra Discovery (see Kovács et al. (2022)) providing different examples of the performance of these features.

In particular, we will consider the behavior of our “automated geometer”, capable to automatically discover a large amount of mathematically rigorous results holding between the elements of a given geometric figure. But the output of the “automated geometer” mixes, without pointing out any difference, relevant and trivial (from a human point of view) statements.

Thus, our current research interest focuses on the proposal of an algorithmic way to evaluate the relevance of a geometric theorem, allowing the “automated geometer” to highlight results that could meet human expectative (according to our criteria).

The proposed algorithm considers as input the “algebraic” translation of the give statement, so “geometrically” equivalent results might be considered as “different” by our approach (ie. It is important the way we do a geometric construction of the input of a statement, as different constructions of the same figure are for our algorithm, different statements...).

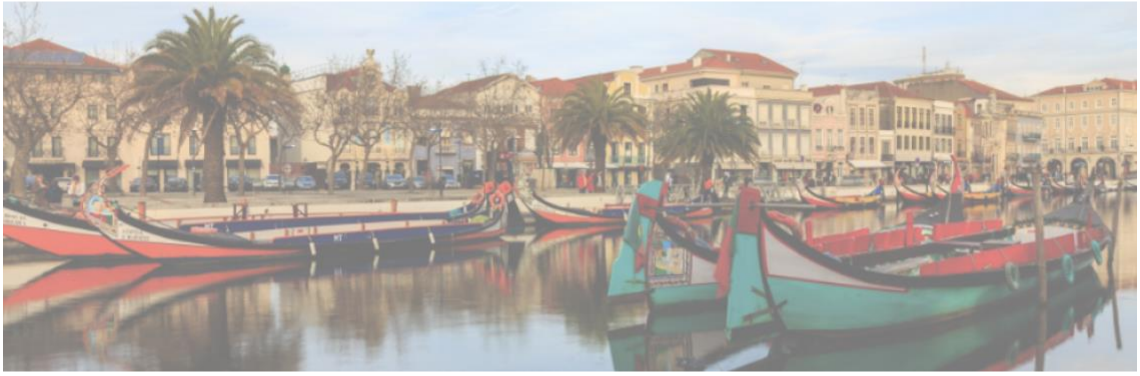
Then we will define, as our ranking measure, the degree of the syzygies (see Cox et al. (1992) for an introduction to this concept and to its computation) describing the thesis as a combination of the hypotheses.

We will test our algorithm in different statements, some of elementary character (typical school Euclidean geometry theorems) or more involved, such as Problem 3 in Quaresma and Santos, 2022, or some of the EGMO (European Girls Mathematics Olympiad) problems, see Ariño-Morera (2022).

Our work is in an initial stage, so we need to test our proposal on a large number of tests, analyzing, from different points of view, the results.

Finally, the possible educational impact of these new technological developments, will be discussed.

Keywords: GeoGebra Discovery; dynamic geometry; GeoGebra; automated reasoning; mathematics education.



Acknowledgements

Partially supported by a grant PID2020-113192GB-I00 (Mathematical Visualization: Foundations, Algorithms and Applications) from the Spanish MICINN.

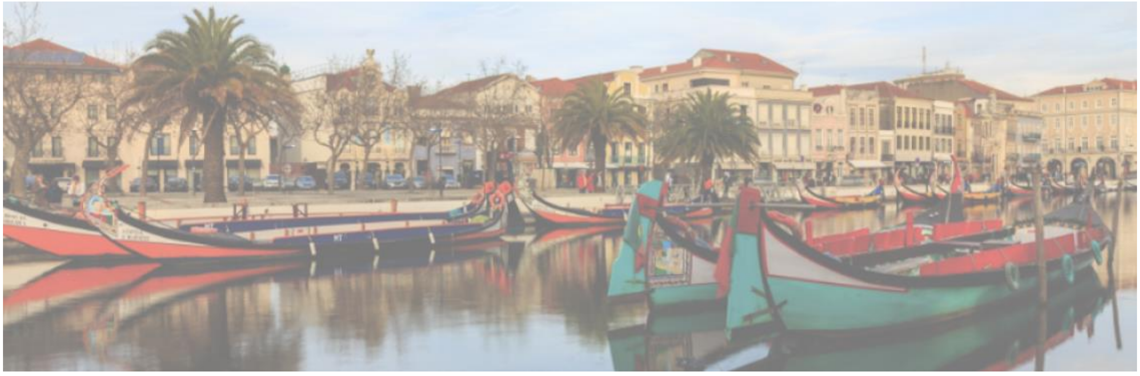
References

Ariño-Morera, B. (2022). *GeoGebra discovery at EGMO 2022. Revista do Instituto GeoGebra internacional de São Paulo,* 11(2), pp. 5-16. https://**doi**.org/10.23925/2237-9657.2022.**v11i2p005**-016.

Cox, D., Little, J., & O'Shea, D. (1992): *Ideals, Varieties, and Algorithms: An Introduction to Computational Algebraic Geometry and Commutative Algebra*, Undergraduate Texts in Mathematics, NY: Springer.

Kovács, Z., Recio, T., & Vélez, M.P. (2022). Automated Reasoning Tools with GeoGebra: What are they? What are they good for*?* In Richard, P.R., Vélez, M.P. and Van Vaerenbergh, S. (Eds.). *Mathematics Education in the Age of Artificial Intelligence*. Series: Mathematics Education in the Digital Era; Springer Nature Switzerland AG, pages 23-44. https://doi.org/10.1007/978-3-030-86909-0_2

Quaresma, P., & Santos, V. (2022). Four Geometry Problems to Introduce Automated Deduc4on in Secondary Schools. In Marcos, J., Neuper W. and Quaresma, P. (Eds.): Theorem Proving Components for Educational Software 2021 (ThEdu'21). EPTCS 354, 2022, pp. 27–42, https://doi.org/10.4204/EPTCS.354.3



Difficulties experienced by 6th-grade students in maths tasks based on exploratory teaching

Alexandra Mendes[1], Ana Oliveira[2], Dina Tavares[2,3] & Hugo Menino[2]
[1] Education and Social Sciences Higher School, Polytechnic of Leiria
[2] CI&DEI, Education and Social Sciences Higher School, Polytechnic of Leiria
[3] CIDMA, Department of Mathematics, University of Aveiro

Abstract

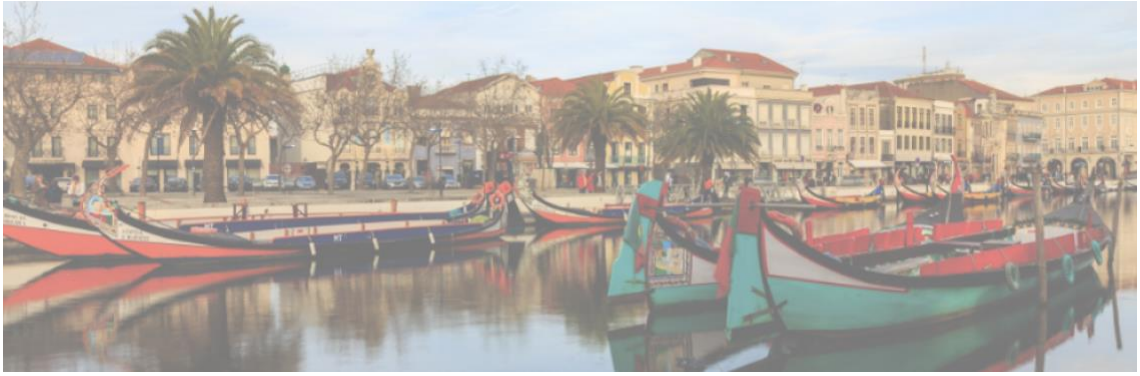
The idea that learning maths is hard and not enjoyable is the result of a social stigma, passed down from generation to generation. This concept is rooted in society's discourse and can lead to learning difficulties and less positive attitudes on the part of students. Considering this perspective, it is up to the teacher to demystify this paradoxical duality that maths is useful but complex (Santos, 2008).

Ponte (2005) says the difficulties students experience in learning maths may be related to the teaching and learning methodologies adopted by teachers. In this sense, the exploratory teaching methodology can help to overcome students' difficulties insofar as it enables more meaningful learning. It involves encouraging students to explain their own ways of thinking, but it also encourages them to make an effort to understand the explanations of other classmates, thus enhancing mathematical communication (Mestre & Oliveira, 2014). Canavarro (2011) states that the aim of this methodology is not to focus on the discovery of mathematical ideas or the invention of concepts and procedures by the students.

Based on these assumptions, one might think that there are several difficulties experienced by the students that justify why teachers don't often adopt this method. Therefore, this study seeks to identify and reflect on the difficulties experienced by a class of 6th-grade students in carrying out mathematical tasks based on exploratory teaching.

To this end, a qualitative methodology was carried out since it aims to find an intention through its practice, in a direct source of data in a natural environment (Bogdan & Biklen, 1994). Four tasks on sequences and regularities were designed and implemented and data collection focused on direct participant observation and document analysis of the students' productions.

The results show that the students had three main difficulties: i) interpreting the written statements, ii) presenting their solutions and discussing the strategies they had implemented, and iii) working in groups. According to Sim-Sim (1998), difficulties in interpreting written statements are related to the acquisition of grammatical structures of the language. Vale and Pimentel (2004) corroborate this by stating that



"based on the assumption that in order to understand it is essential to relate", the interpretation of statements "is an extremely important stage in the teaching of problem-solving" (p. 16).

This difficulty may be related to the difficulty identified in ii). In this way, exploratory teaching can prove to be an important strategy since "talking, drawing or writing about mathematical reasoning offers opportunities to justify thoughts, synthesise ideas and become aware of intuitions" (Boavida et al., 2008, p. 68).

Finally, students' difficulties in group work may centre on positive interdependence, since not everyone in the group worked equally, and on face-to-face interaction, which seems to be related to the fact that not all students were encouraged to fulfil the group's own goal (Freitas & Freitas, 2002).

In conclusion, this study was important because, since the benefits of exploratory teaching are recognised, it is essential to identify some challenges that may be experienced by the students. Identifying these challenges can be an important starting point and a way of raising awareness of the need to adopt strategies that can not only minimise (or even eliminate) these difficulties.

Keywords: exploratory teaching; sequences and regularities; maths tasks; difficulties.

Acknowledgements

Ana Oliveira was supported by Portuguese funds through the Centre for Studies in Education and Innovation (CI&DEI), and Foundation for Science and Technology, within the scope of the project Refª UIDB/05507/2020. Dina Tavares was supported by Portuguese funds through the CIDMA – Center for Research and Development in Mathematics and Applications, and the Foundation for Science and Technology, within project UIDB/04106/2020. The authors would also like to thank the Polytechnic of Leiria for their support.

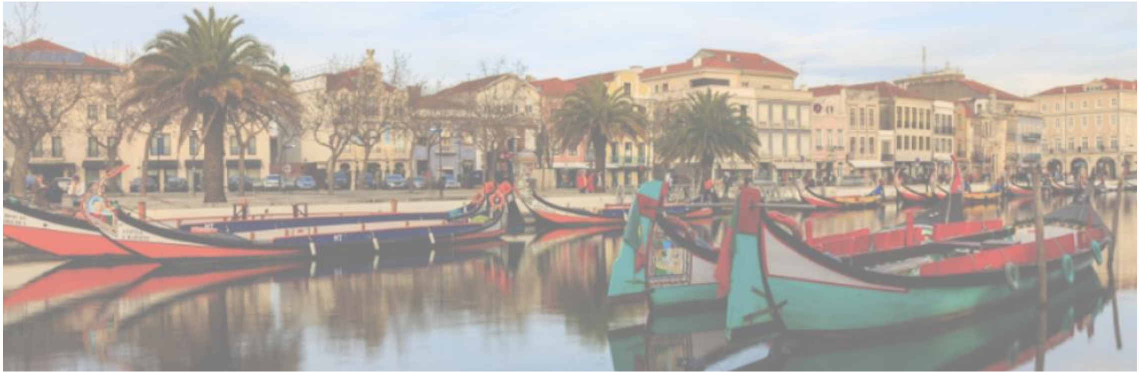
References

Boavida, A. M., Paiva, A. L., Cebola, G., Vale, I., & Pimentel, T. (2008). A Experiência Matemática no Ensino Básico-Programa de Formação Contínua em Matemática para Professores dos 1.º e 2.º Ciclos do Ensino Básico. Direcção-Geral de Inovação e de Desenvolvimento Curricular.

Bogdan, R.C. & Biklen, S. K. (1994). Investigação qualitativa em educação - uma introdução à teorias e aos métodos. Porto Editora.

Canavarro, A. P. (2011). Ensino exploratório da Matemática: Práticas e desafios. Educação e Matemática, (115), 11-17. http://hdl.handle.net/10174/4265

Freitas, M. & Freitas, C. (2002). Aprendizagem Cooperativa. ASA.



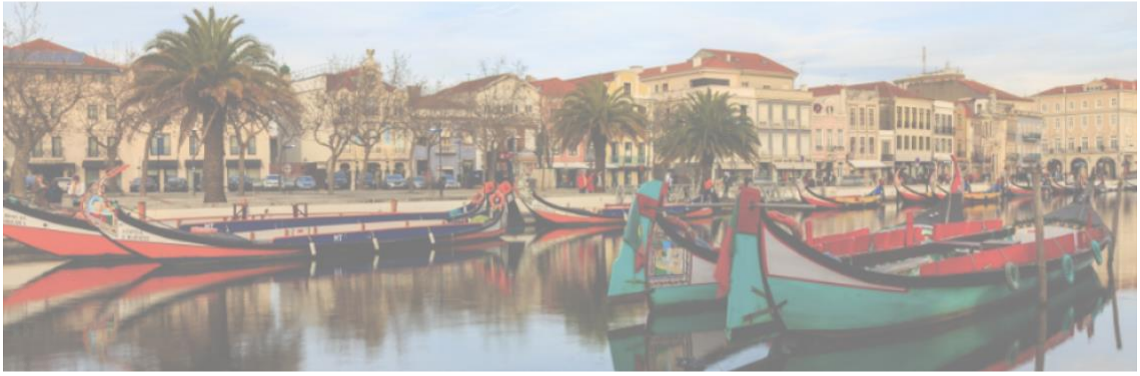
Mestre, C., & Oliveira, H. (2014). A construção coletiva da generalização num contexto de ensino exploratório com alunos do 4. º ano. Práticas profissionais dos professores de matemática, 283- 308.

Ponte, J. (2005). Gestão curricular em Matemática. IN GTI (ed.), O professor e o desenvolvimento curricular (pp. 11-34). APM.

Santos, V. M. (2008). A Matemática escolar, o aluno e o professor paradoxos aparentes e polarizações em discussão. Campinas, 28 (74), 25-38. https://www.scielo.br/j/ccedes/a/8CJ4rMnNFCNwnLPhQZYWJXs/?format=pdf&lang=pt

Sim-Sim, I. (1998). Desenvolvimento da Linguagem. Universidade Aberta.

Vale, I., & Pimentel, T. (2004). Resolução de problemas. In Palhares, P. (coord.), Elementos de Matemática para professores do Ensino Básico. (pp. 7-51). Lidel.



A continuum space is the infinitely great

Qing Li[1]

[1] Shijiazhuang Traditional Chinese Hospital, Shijiazhuang ,China

Abstract

In modern higher mathematics education courses, calculus is the most important content, and its emergence is to solve the problem of how to calculate the length of line segments and surface area. Regardless of how modern mathematics develops, it is impossible to calculate exact values (infinite values) with existing calculus because calculus are approximate. For example, for a given line segment or high-dimensional surface, we define it as a manifold that can be infinitely and arbitrarily divided into smaller parts. Here a certain line segment or surface is understood as a set of innumerable infinitesimals and can be calculated by calculus. From the new definition below, it can be seen that this above is not a fact.

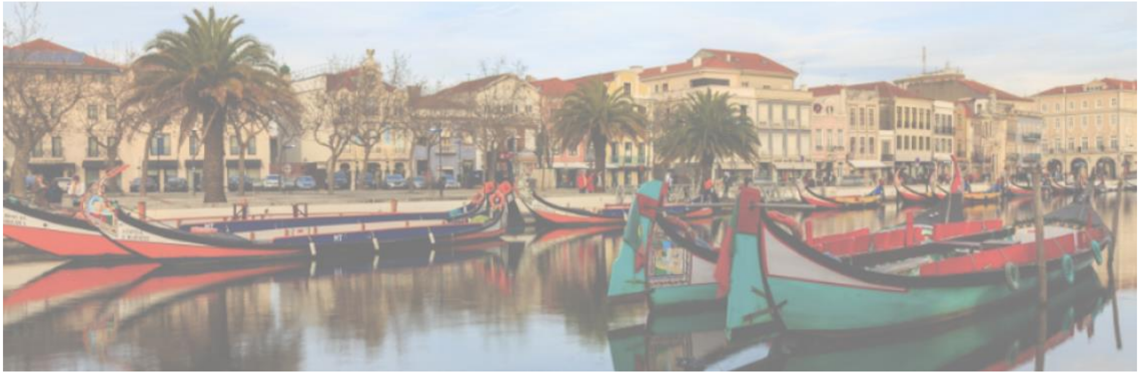
In this paper an infinitely small quantity is defined as a one-dimensional quantity of finite length without sizes of space, while an infinitely great quantity is reached by the superposition or accumulation of infinitely many finite quantities, by the way of the change in direction. The change in direction indicates that there is a jump from a finite quantity to infinitely many finite quantities (infinitely great). The form of the manifestation of the infinitely great is one quantitative continuum that cannot be operated by any algorithms, such as the operations of addition, subtraction, multiplication, and division. All parts of space we see is this one quantitative continuum that cannot be talked about anything outside of it and can compresses any quantities outside of it to nothing. The change in direction suggests that the infinitesimal are not parts of infinite great and it does not truly exist (it is only reference quantities that define the infinitely great) due to the infinitely great is only one quantitative continuum where there is only one quantity to exist. Therefore, any line segment or high-dimensional manifold we see in daily life are this one quantitative continuum .

As a result, from this new definition, the infinite exact value of a circumferential length (π) has been obtained here.

Key words: calculus; infinitely great; change in direction; one quantitative continuum.

References

Li, Q.(2021). A geomerty consisting of singularities containing only integers. Research Square, http://doi.org/10.21203/rs.3.rs-219046/v1

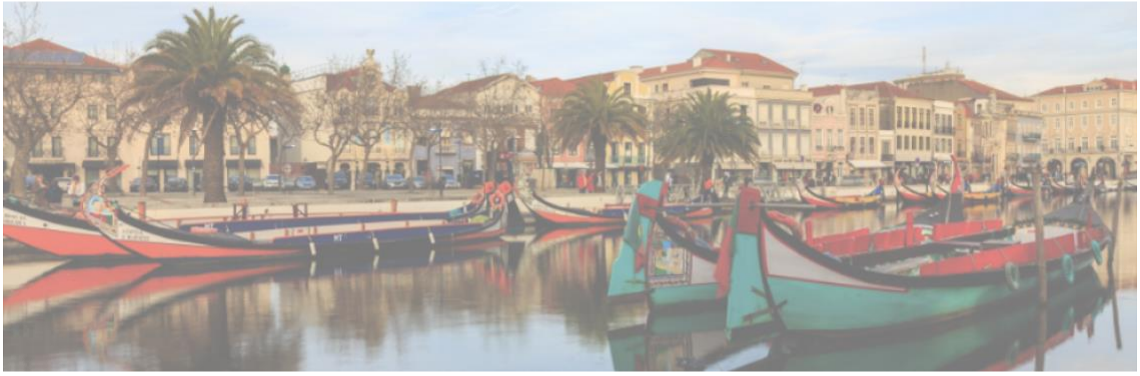


Wiles, A.(1995). Modular elliptic curves and Fermat's Last Theorem. *Annals of Mathematics*,141(3), 443-552. https://doi.org/10.2307/2118559

Kahn, J., & Markovic, V. (2012). Immersing almost geodesic surfaces in a closed hyperbolic three manifold. *Annals of Mathematics*, 175(3),1127-1190. https://doi.org/10.4007/annals.2012.175.3.4

Böhm ,C., & Wilking, B. (2008). Manifolds with positive curvature operators are space forms. *Annals of Mathematics* ,167(3), 1079-1097. https://doi.org/10.4007/annals.2008.167.1079

Cheeger ,J., & Naber, A.(2015). Regularity of Einstein manifolds and the codimension 4 conjecture. *Annals of Mathematics* ,182(3), 1093-1165.https://doi.org/10.4007/annals.2015.182.3.5



Fostering engaged learning communities in the asynchronish classroom: Leveraging agile collaboration through Slack

Dennis J. DeBay[1], Josie Smith2[2]

[1]University of Colorado Denver

[2]University of Colorado Colorado Springs

Abstract

Accelerated by the COVID pandemic and lower course enrollments, mathematics education is rapidly transforming. Traditional in-person classes now shift online, posing challenges for communication and community-building in mathematics education courses. How can instructors foster meaningful engagement and authentic discourse in a non-formal virtual community? (Rourke et al., 1999) These communities of inquiry enable an interplay of synchronous and asynchronous mathematics education supported by digital technology.

Regarding math courses, asynchronish learning models are driving current changes. Asynchronish design blends synchronous elements into asynchronous online spaces. While students access materials at their convenience, limited synchronous interactions hinder engagement. Asynchronish course design seeks self-pacing while fostering collaboration and community. Balancing challenges and opportunities, creating authentic online environments remains crucial (Garrison et al., 1999). This proposal merges agile learning communities and transformative potential, using Slack to have students engage in mathematics education courses.

Workshop Overview:

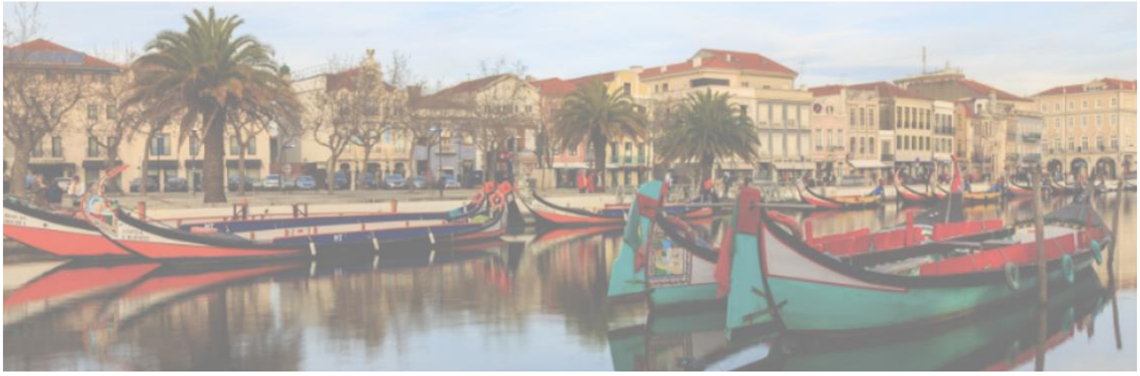
"Fostering Engaged Learning Communities in the Asynchronish Classroom: Leveraging Agile Collaboration through Slack," a 60-minute session, addresses authentic discourse, communication, and community-building in asynchronish education. Merging insights from diverse workshops, it offers a comprehensive understanding of agile collaboration tools, focusing on Slack, to nurture engagement and dialogues.

Session Structure:

The workshop systematically guides participants through agile learning communities and technologies like Slack for mathematics and mathematics education courses.

Understanding the Imperative:

Explore the shift to asynchronish learning, emphasizing authentic virtual environments. Grasp challenges educators face, and the role of discourse, communication, and community-building in student success.



The Agile Learning Landscape:

Delve into agile learning communities connecting educational elements. Introduce foundational principles of collaboration technology, reshaping education.

Empowering Learning with Online Learning Communities:

Highlight Slack's potential to emulate social media interactions. Showcase how conversation streams foster synchronous communication, supported by case studies. Slack's integration in education elevates engagement and community.

Applying Agile Principles:

Explore integrating agile principles into educational practices, harmonizing with tech-centric philosophies. Discuss organizational structures, open mindsets, and informal communication's role in nurturing engaged learning. Uncover the link between these principles and Slack's user experience.

Interactive Engagement:

Engage participants with hands-on Slack activities, immersing them in real-time discussions. Experience Slack's potential, fostering a nuanced understanding.

Conclusion:

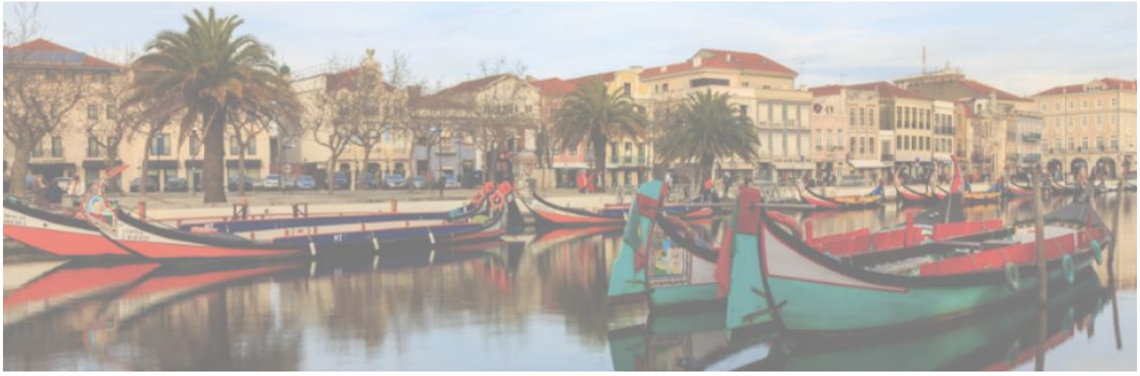
In conclusion, "Fostering Engaged Learning Communities in the Asynchronish Classroom: Leveraging Agile Collaboration through Slack" equips educators with tools to enrich online teaching. Depart with actionable strategies, infusing authenticity and engagement, fueled by agile collaboration principles and Slack's dynamics. Amid uncharted asynchronish education, embrace agile learning communities for modern education. (Garrison et al., 1999)

Keywords: stem education; building community; online learning; asynchron-ish.

References:

Garrison, D. R., Anderson, T., & Archer, W. (1999). Critical Inquiry in a Text-Based Environment: Computer Conferencing in Higher Education. The Internet and Higher Education, 2(2), 87–105.

Rourke, L., Anderson, T., Garrison, D. R., & Archer, W. (1999). Assessing social presence in asynchronous text-based computer conferencing. The Journal of Distance Education/Revue de L'ducation Distance, 14(2), 50–71.



A geometric exploration of monuments in the city of Aveiro

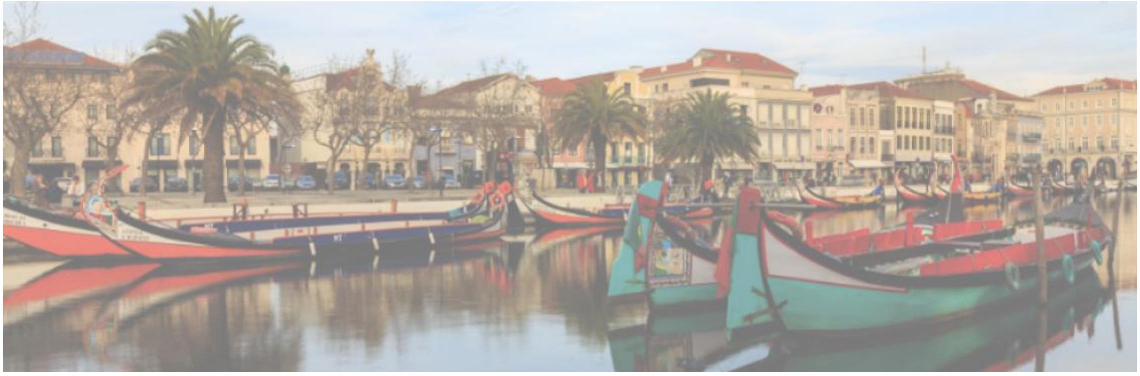
Margarida M. Marques[1], Alvaro Nolla[2], Teresa B. Neto[1], Lúcia Pombo[1]
[1]CIDTFF – Centro de Investigação em Didática e Tecnologia na Formação de Formadores/ Universidade de Aveiro
[2]Facultad de Formación del Profesorado y Educación/Universidad Autónoma de Madrid

Abstract

Geometry allows the development of several skills, namely, geometric thinking, problem solving and argumentation. Additionally, the development of student spatial visualization capabilities is essential for their learning of space. Research involving digital technologies has been shown to contribute to a favourable view of geometry (Arzarello, Ferrara & Robutti, 2012, Clark-Wilson, Robutti & Thomas, 2020). For example, the inclusion of mobile technologies in the so-called Math Trails facilitates the teaching and learning process by creating collaborative and meaningful outdoor mathematical experiences, as well as enhancing the support of modelling activities (Cahyono & Ludwig, 2018, Jablonski, Barlovits & Ludwig, 2023). Digital tools allow teachers to design and implement geolocated routes that include contextualized problems that require students to visualize and manipulate geometric models of real-world objects to solve them, connecting mathematics to everyday life.

Mobile devices, augmented reality (AR) and digital educational games in outdoor city environments can be mobilized to promote mathematics education to different target groups (Neto, Pombo, 2020). The EduCITY project is based on the exploration of the urban territory as an experimental living laboratory, bringing education into real contexts. It creates a smart learning city environment, comprising: i) a mobile app with active AR challenging interdisciplinary games; and ii) an open and easy-to-use web-platform that supports a participatory dynamic where the citizens co-create these educational games and multimedia resources. Citizens can also explore AR games through extension activities on strategic paths in the city, towards sustainable development education (Pombo & Marques, 2023).

In the scope of EduCITY, several geometrical challenges have been developed in strategic points of the city, such as the “Coreto” [Bandstand] of the city green park and the monument of an important figure of the city "Egas Salgeiro". In addition to their socio-cultural dimension, these challenges and their associated multimedia resources involve mathematical concepts and processes, namely the similarity of figures and proportional reasoning. They have been integrated into a location game through tourist sites in Aveiro, tested by about 80 doctoral students in Education under the Summer School "The European Educational Research Association (EERA) Summer School 2023". Volunteers analyzed and evaluated these challenges through a questionnaire in terms of their interest, clarity, scientific soundness, and potential as role model for others to develop new challenges. International PhD students revealed positive perceptions of the



geometric challenges. The "Egas Salgeiro" challenge was better rated than the "Coreto" challenge, particularly in terms of the interest and clarity of the question. The latter challenge required more mathematical calculations, which could be an explanatory factor given the respondents' non-mathematical background. This hypothesis needs further research.

This work supports the evaluation of the geometric challenges and the consideration of possible revisions to increase their suitability for different target players.

Keywords: geometry; mobile learning; game-based learning; higher education.

Acknowledgements

EduCITY is funded by Portuguese funds through FCT - Foundation for Science and Technology within the framework of the PTDC/CED-EDG/0197/2021 project. The work of the first author is funded by national funds through FCT – Fundação para a Ciência e a Tecnologia, I.P., under the Scientific Employment Stimulus - Individual Call – [2022.02153.CEECIND].

References

Arzarello, F., Ferrara, F., & Robutti, O. (2012). Mathematical modelling with technology: The role of dynamic representations. *Teaching Mathematics and Its Applications*, *31*(1), 20-30. https://doi.org/10.1093/teamat/hrr027

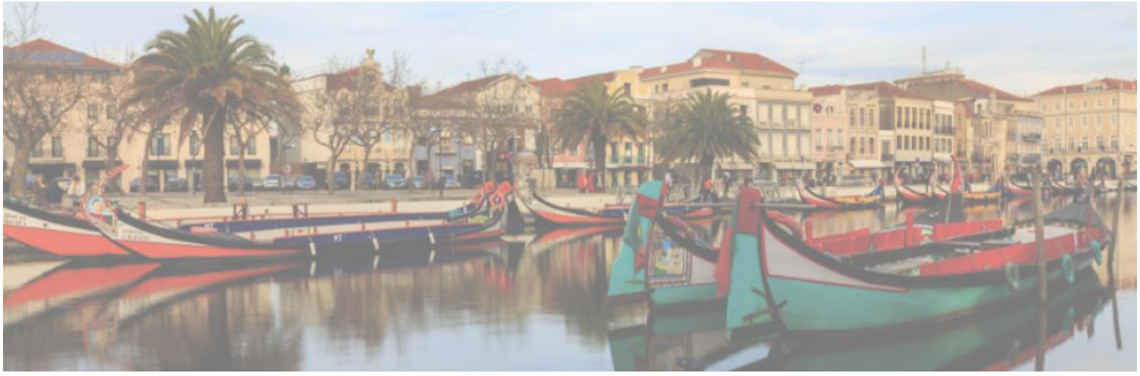
Cahyono, A. N., & Ludwig, M. (2018). Teaching and learning mathematics around the City Supported by the Use of Digital Technology. *EURASIA Journal of Mathematics, Science and Technology Education*, *15*(1), em1654. https://doi.org/10.29333/ejmste/99514

Clark-Wilson, A., Robutti, O. & Thomas, M. (2020). Teaching with digital technology. *ZDM Mathematics Education,* 52, 1223-1242. https://doi.org/10.1007/s11858-020-01196-0

Jablonski, S., Barlovits S., & Ludwig, M. (2023). How digital tools support the validation of outdoor modelling results. *Frontiers in Education*, 8, 1145588. https://doi.org/10.3389/feduc.2023.1145588

Neto, T., & Pombo, L. (2020). A formação inicial de professores para uma Educação interdisciplinar – o exemplo do projeto EduPARK. *Revista Saber & Educar*, 28, 1-12. http://dx.doi.org/10.17346/se.vol28.389

Pombo, L., & Marques, M. M. (2023). EduCITY as a smart learning city environment towards education for sustainability - work in progress. In T. Bastiaens (Ed.), *EdMedia + Innovate Learning, jul 10, 2023* (p. 133-139). AACE. https://www.learntechlib.org/primary/p/222493/



Teacher Education

Using GeoGebra Discovery in the context of Plane Geometry

Celina Aparecida Almeida Pereira Abar[1], Daniel Mendes Inácio de Souza[2]

[1, 2]Pontifical Catholic University of São Paulo Brazil

Abstract

Currently, modern Dynamic Geometry Systems have come to include features for automated reasoning that allow the discovery, automatic and mathematically rigorous verification of geometric theorems (Kovács et al., 2022).

A collection of features and commands, Automatic Reasoning Tools (ART), which allow a mathematical verification and the automatic discovery of general propositions about figures of Euclidean Geometry constructed by the user, has been incorporated into GeoGebra, in an experimental version called GeoGebra Discovery.

These tools and commands allow the user to automatically conjecture, discover and verify statements displayed by the software about different elements of a given geometric construction.

The GeoGebra Discovery ART set, so far, is composed of some resources: *Relation*, *LocusEquation*, *Prove*, *ProveDetails* and *Discover* (Kovács et al., 2022).

Some basic automated reasoning features are available in Classic GeoGebra, and yet other ART commands can be found in GeoGebra Discovery online.

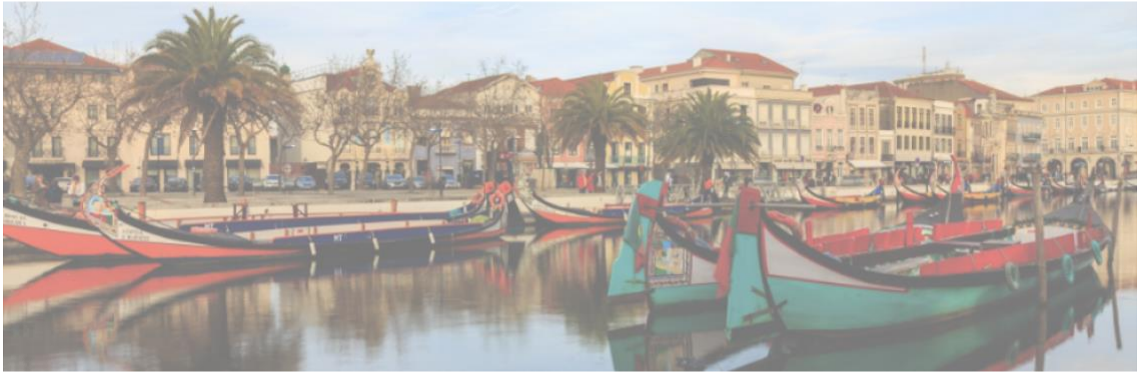
This study explores the relevant points observed for two months course taught online by the Teams platform in 2023 for two teachers of basic and higher course of the discipline of Mathematics.

The objective of the course was to investigate the potentialities of the use of GeoGebra Discovery in the study of some properties of Plane Geometry, observing the course and the discussions promoted by participants about manipulating the software.

Design Research was used as a methodology to conduct the studies, because as Collins et al. (2004) maintain, this approach progressive refinement throughout the investigative process.

The analysis considers the theory of the Diffusion of Innovation of Rogers (2003), since GeoGebra Discovery is an experimental tool and, thus, individuals had to become aware of this innovation, explore it and go through the decision-making process of adopting or rejecting it as a tool for your teaching practice, understanding its potentialities in the study of Geometry.

The theory of the Diffusion of Innovation (Rogers, 2003) allows to analyze the development of the work



with teachers and to understand their situation when faced with a new idea.

The participants had interest and previous knowledge about the GeoGebra tool, reinforcing the decision to participate in the course. Thus, it is perceived that the stage of knowledge Rogers (2003) was in force, since the members wanted to know, understand, and explore how to use that innovation (GeoGebra Discovery) especially in the classroom.

The results showed that the participants built a positive attitude about the new idea and started the process of how to include such a tool in the curricular planning.

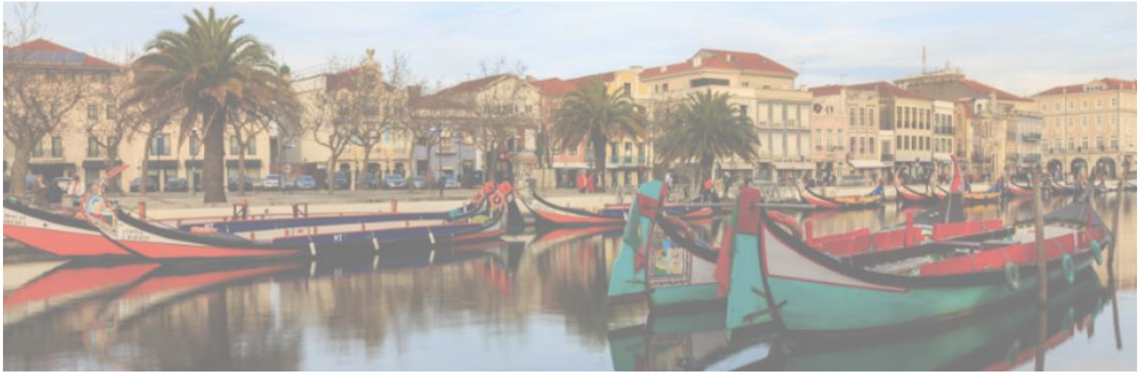
Keywords: GeoGebra Discovery; plane geometry; diffusion of innovation.

References

Collins, A., Joseph, D., & Bielaczyc, K. (2004). Design research: Theoretical and methodological issues. *Journal of the Learning Sciences*, 13(1), 15–42.

Kovács, Z., Recio, T. & Vélez, M.P. (2022). Automated Reasoning Tools with GeoGebra: What Are They? What Are They Good For? In: Richard, P.R., Vélez, M.P., Van Vaerenbergh, S. (eds) *Mathematics Education in the Age of Artificial Intelligence*. Mathematics Education in the Digital Era, 17. Springer, Cham.

Rogers, E. M. (2003). *Diffusion of innovations* (5a ed.). Free Press.



Development of TPACK in an environment of continuous teacher training in mathematics teaching using GeoGebra

Pedro Pimenta[1], António Domingos[2], Maria Cristina Costa[3]
[1]Agrupamento de Escolas do Monte de Caparica, CICS.NOVA
[2]Universidade Nova de Lisboa, CICS.NOVA
[3]Instituto Politécnico de Tomar, Ci2, CICS.NOVA

Abstract

The use of technology is seen as an asset to teaching. Specifically for mathematics teaching, we highlight GeoGebra, a tool that can contribute to improve pedagogical practices and promote meaningful learning (Weinhald et. al, 2020).

For technology to be integrated into pedagogical practices, it is necessary to implement professional development programmes (PDP) for in-service teachers (Zehetmeier & Krainer, 2011), to develop their knowledge and skills for effective teaching with technology, which we frame under Technological Pedagogical and Content Knowledge (TPACK) (Koehler et. al, 2013).

In this research, using a qualitative methodology (content analysis), with an interpretative paradigm (Cohen et. al, 2018; Creswell, 2012) we analyze the development of teachers' TPACK within the framework of a continuous training course implemented with middle and secondary school math teachers. We begin by presenting and characterizing the tasks proposed to the trainees, intentionally designed to promote the development of teachers' knowledge to teach mathematics using technology (Pimenta et. al, 2022). Then we analyze teachers' productions resulting from these proposals and also the tasks they autonomously created to implement with students, for learning mathematics using GeoGebra, in order to illustrate the knowledge mobilized in the construction of the tasks.

Based on data analysis during the training programmes, we verified that teachers mobilized different domains of TPACK, namely Content Knowledge (CK), Pedagogical Knowledge (PK) and Technological Knowledge (TK), including the integration of these domains, resulting in the TPACK.

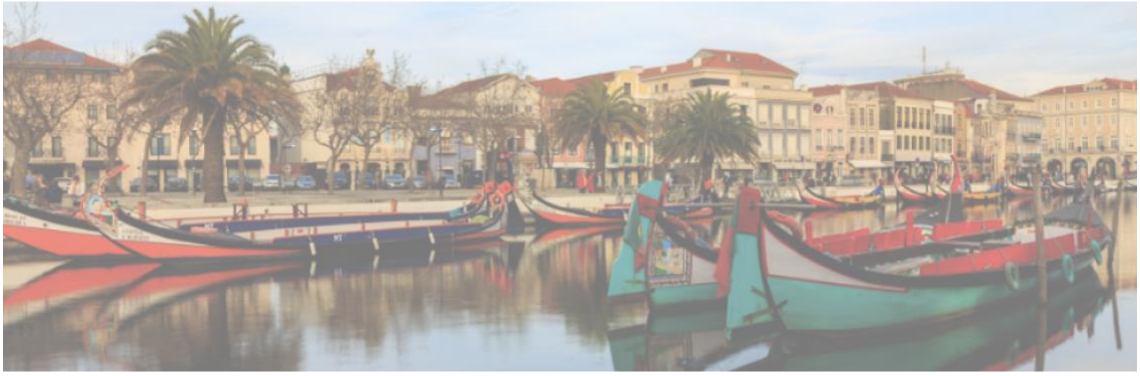
Teachers' participation in the study was voluntary and, in addition, their anonymity was guaranteed.

Keywords: teachers TPACK; GeoGebra; mathematics teaching; teacher training.

References

Cohen, L., Manion, L., & Morrison, K. (2018). *Research Methods in Education*. Routledge

Creswell, J. H. (2012). *Educational Research*. Pearson

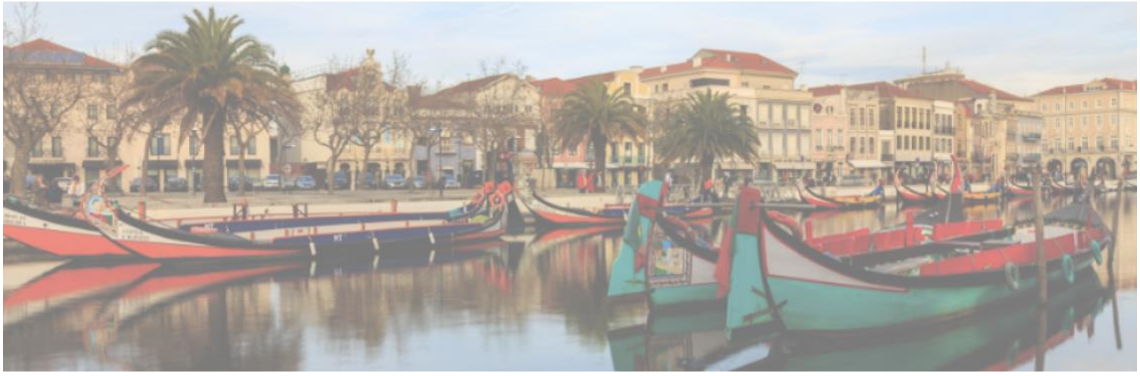


Koehler, M. J., Mishra, P., & Cain, W. (2013). What is technological pedagogical content knowledge (TPACK)? *Journal of Education*, *193*(3), 13–19. https://doi.org/10.1177/002205741319300303

Pimenta, P., Domingos, A., & Costa, C. (2022). Formação de professores em ensino de matemática com recurso à tecnologia – do currículo prescrito ao currículo em ação. In Rodrigues, A., Domingos, A., & Serrazina, L. (Eds.) (2022). *Livro de atas do Encontro de Investigação em Educação Matemática* (*EIEM22*). FCT NOVA

Weinhandl, R., Lavicza, Z., Hohenwarter, M., & Schallert, S. (2020). Enhancing flipped mathematics education by utilising GeoGebra. *International Journal of Education in Mathematics, Science and Technology*, *8*(1), 1–15.

Zehetmeier, S., & Krainer, K. (2011). Ways of promoting the sustainability of mathematics teachers' professional development. *ZDM - International Journal on Mathematics Education*, 43(6), 875–887. Scopus. https://doi.org/10.1007/s11858-011-0358-x



Creativity for all: the power of multiple (re)solution problems

Isabel Vale[1], Ana Barbosa[2]

[1] Instituto Politécnico de Viana do Castelo, Escola Superior de Educação, Viana do Castelo & CIEC, Instituto de Educação, Universidade do Minho, Braga, Portugal

[2] Instituto Politécnico de Viana do Castelo, Escola Superior de Educação, Viana do Castelo; CIEC, Instituto de Educação, Universidade do Minho, Braga; & inED, Escola Superior de Educação, Instituto Politécnico do Porto, Porto, Portugal

Abstract

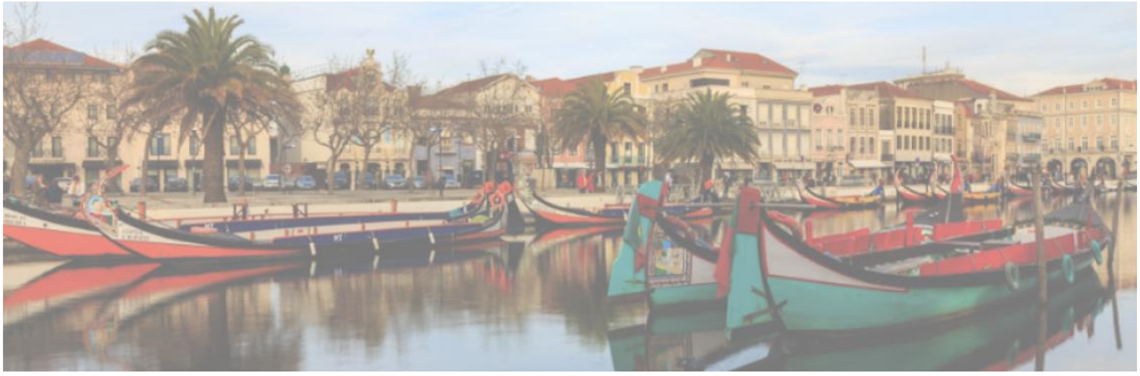
In all areas of knowledge, the need to have more creative people, capable of providing innovative solutions in problem solving has been emphasized. Creativity begins with curiosity and involves students in exploration and experimentation supported by their imagination and originality and plays an important role in mathematics education, being a dynamic characteristic that students can develop (Leikin, 2014; Vale et al., 2018). Thus, teachers should seek to develop students' mathematical understanding posing challenging tasks that contemplate multiple-solutions, involving a variety of representations, connections and motivate them to work with each other using their personal different thinking processes. Multiple (re)solution tasks allow students to choose the most convenient strategy to achieve the solution, giving them the chance to think outside the box (Leikin, 2014; Vale & Barbosa, 2023). In this context, we are particularly interested in visual solutions because they can help students overcome some difficulties with mathematical concepts and procedures, allowing them to successfully solve a given problem situation and challenge them to come up with original ways of finding a solution (e.g. Vale & Barbosa, 2023).

In this talk, we will focus on the importance of challenging tasks with multiple (re)solutions and representations in mathematics learning and in the development of students' creativity and mathematical comprehension. This discussion will be illustrated with several examples of students' productions, focused on different mathematical themes, and their multiple solutions and representations. Particularly we will focus on different qualitative and interpretative studies carried out with elementary pre-service teachers (children aged 6–12), where we analyze the types of strategies and representations used, allowing us to identify their creative potential through three of its dimensions (fluency, flexibility, originality). We will show that the use of multiple solution tasks provokes the creative potential of learners.

Keywords: problem solving; creativity; multiple-solution tasks; pre-service teachers.

References

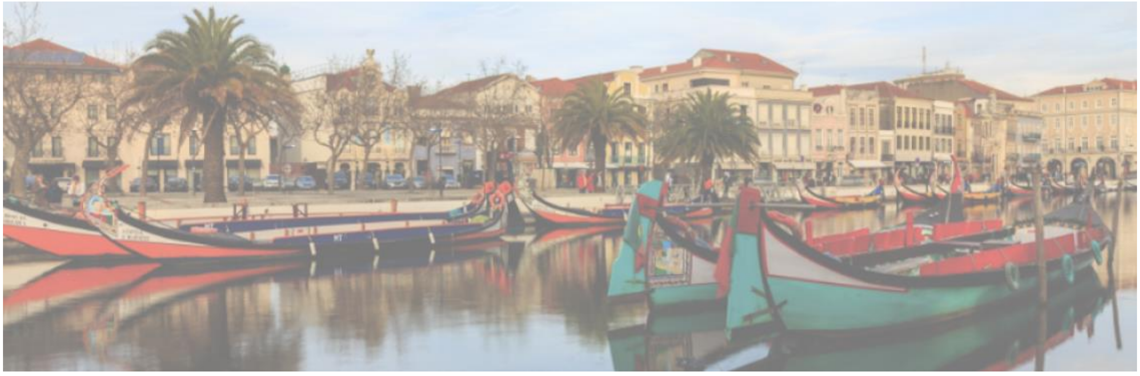
Leikin, R. (2014). Challenging mathematics with multiple solution tasks and mathematical investigations



in geometry. In Y. Li, E. A. Silver, & S. Li (Eds.), *Transforming mathematics instruction: Multiple approaches and practices* (pp. 59–80). Springer.

Vale, I., & Barbosa, A. (2023). Visualization: A Pathway to Mathematical Challenging Tasks. In R. Leikin (Ed.), *Mathematical Challenges For All. Research in Mathematics Education* (pp.283-306). Springer.

Vale, I., Pimentel, T., & Barbosa, A. (2018). The power of seeing in problem solving and creativity: An issue under discussion. In S. Carreira, N. Amado, & K. Jones (Eds.), *Broadening the scope of research on mathematical problem solving: A focus on technology, creativity and affect* (pp. 243–272). Springer.



Integrating Art in the STEAM Methodology to Enhance Motivation in Teaching Geometry to Prospective Educators

Lilla Korenova[1], Angelika Schmid[2], Minju Jeong[3]
[1]Faculty of Education, Comenius University in Bratislava, Slovakia
[2]Faculty of Education, University of Ostrava, Czech Republic
[3]Linz School of Education, Johannes Kepler University, Austria

Abstract

The aspect of motivating individuals to teach geometry represents significant importance in the training of aspiring mathematics teachers. We focus on finding ways to enhance intrinsic motivation to deepen the knowledge and skills necessary for their future teaching practice. Integrating the arts into STEM education (creating STEAM) fosters skills in creative thinking that may increase student engagement. STEAM education, an approach that integrates science, technology, engineering, arts, and mathematics, is based on the principles of constructivism.

In this context, a novel approach has been developed involving activities that combine artistic elements with congruent representations, such as the discussion of the importance of lines and shapes for achieving ideal composition in abstract art, exploring the golden ratio, finding the Fibonacci sequence in nature, and discovering connections with identical representations in geometry. Such an innovative approach aims to bolster motivation, enhance visualization skills, and facilitate a deeper comprehension of congruent representations within geometry. Alongside the integration of art, students' aesthetic perceptions were also harnessed within the GeoGebra platform by creating 3D models that represent real objects. Moreover, the application of 3D printing within the GeoGebra environment was leveraged to craft sculptures that productively showcased the pragmatic utilization of congruent representations in spatial contexts.

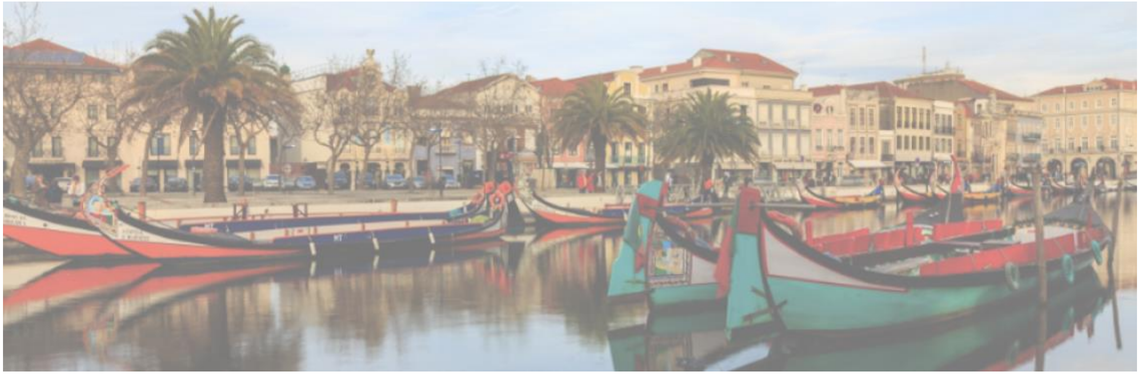
The paper intends to present a curated selection of these practical activities, accompanied by the findings from a preliminary research study conducted as part of the pedagogical training of future mathematics teachers.

Keywords: STEAM education; motivation; teacher training; GeoGebra.

Acknowledgements

The research has been supported by projects:

SGS16/PdF/2023: “Mobile Technologies and Augmented Reality in Mathematics and STEAM Education”



KEGA 026UK-4/2022 “The Concept of Constructionism and Augmented Reality in STEM Education”

Erasmus+, 2021-1-LU01-KA220-SCH-000034433 project for Co-creating Transdisciplinary “STEM-to-STEAM Pedagogical Innovations through Connecting International Learning Communities”

References

Anđić, B., Ulbrich, E., Dana-Picard, T. N., & Laviza, Z. (2022, February). Usability of 3D modelling and printing in STEAM education: primary school teachers perspective. In *Twelfth Congress of the European Society of Research in Mathematics Education (CERME12)*.

Conradty, C., & Bogner, F. X. (2018). From STEM to STEAM: How to monitor creativity. *Creativity Research Journal*, *30*(3), 233-240.

Deci, E. L., Vallerand, R. J., Pelletier, L. G., & Ryan, R. M. (1991). Motivation and education: The self-determination perspective. *Educational psychologist*, *26*(3-4), 325-346.

Dürer, A. (1535) *Principles of Geometry*, Paris.

El Bedewy, S. (2023). Fostering transdisciplinary steam education practices through architectural modelling inspired by technological, cultural, and historical perspectives/Author Shereen El Bedewy.

Ghyka, M. C. (1977). *The geometry of art and life*. Courier Corporation.

Henriksen, D. (2016). The seven transdisciplinary habits of mind of creative teachers: An exploratory study of award winning teachers. *Thinking Skills and Creativity*, *22*, 212-232.

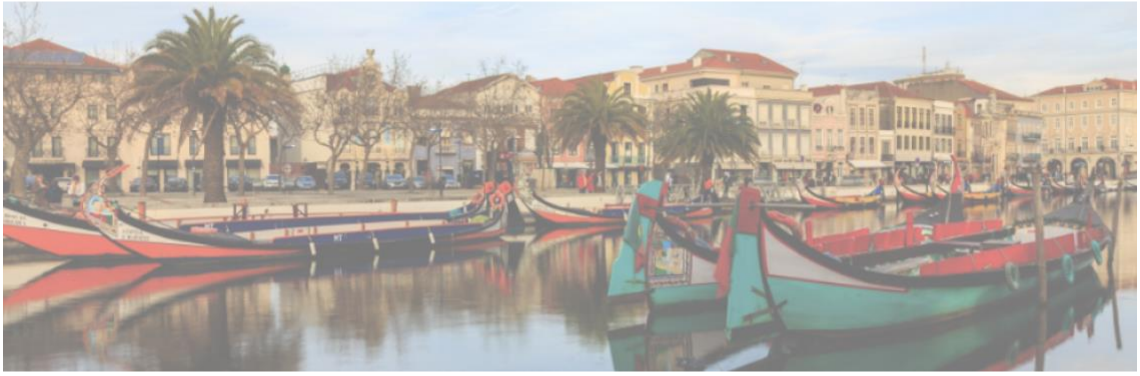
Kostrub, D. (2022). Učiteľ-výskumník. Profesia založená na výskume. *[Teacher-researcher. A research-based profession] Bratislava: Univerzita Komenského v Bratislave*. *[Bratislava: Comenius University in Bratislava]*

Matos, A., Santos, V., & B. Neto, T. (2023). Poly-Universe Resource for Solving Geometric Tasks by Portuguese Basic Education Students. *Open Education Studies*, *5*(1), 20220181.

Pytlik, M., & Kostolányová, K. (2019, December). Tasks to develop spatial imagination and creativity with 3D graphics. In *AIP Conference Proceedings* (Vol. 2186, No. 1). AIP Publishing.

Santos, V., Vaz-Rebelo, P., Bidarra, G., Stettner, E., Guncaga, J., & Korenova, L. (2022, August). Teacher Training in the Fields of STEAM: From Physical to Digital Tools. In *International Conference on Technology and Innovation in Learning, Teaching and Education* (pp. 171-177). Cham: Springer Nature Switzerland.
Quigley, C. F., & Herro, D. (2019). *An educator's guide to steam: Engaging students using real-world problems*. Teachers College Press.

Žilková, K., Partová, E., Kopáčová, J., Tkačik, Š., Mokriš, M., Budínová, I., & Gunčaga, J. (2018). *Young Children's Concepts of Geometric Shapes*. Harlow: Pearson.



Solving authentic problems through engineering design: a steaM approach

Ana Barbosa[1], Isabel Vale[2]

[1]Instituto Politécnico de Viana do Castelo, Escola Superior de Educação, Viana do Castelo; CIEC, Instituto de Educação, Universidade do Minho, Braga; & inED, Escola Superior de Educação, Instituto Politécnico do Porto, Porto, Portugal

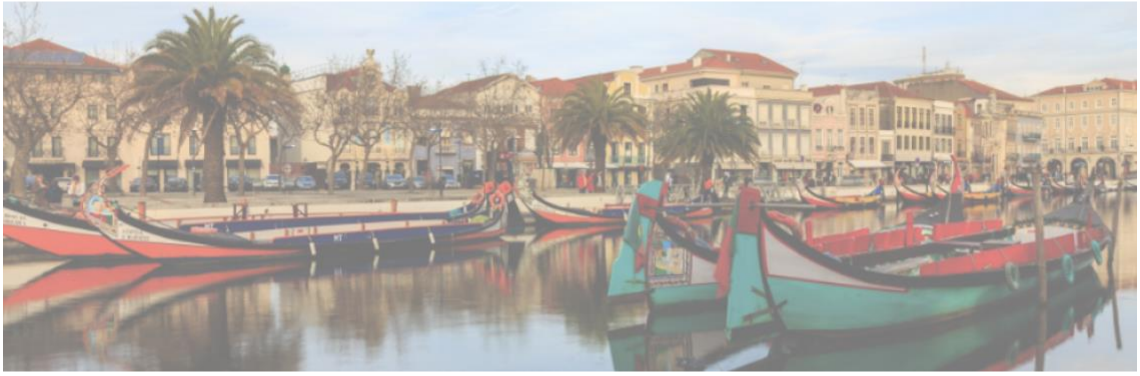
[2] Instituto Politécnico de Viana do Castelo, Escola Superior de Educação, Viana do Castelo & CIEC, Instituto de Educação, Universidade do Minho, Braga, Portugal

Abstract

There is a growing emphasis in a meaningful learning that encourages students to be involved in individual and collaborative experiences, that promote the development of skills, as being creative, think critically, solve problems, communicate and collaborate. So, the use of challenging tasks with multiple-solutions, that require a hands-on/minds-on activity, leading students to active learning fits those ideas. Hence, the disciplinary integration through Science, Technology, Engineering, Arts and Mathematics (STEAM) education may be a contribution, as it is a teaching and learning approach where students build and demonstrate understanding using a design process in which STEAM areas work together. One way to approach it is solving authentic problems (AP) through the use of Engineering Design (ED) process, that we adapted, composed of seven-steps (problem; imagine; design the solution; (re)build; (re)test and evaluate; redesign; and solution) (English & King, 2019; Vale et al., 2022). In our perspective, AP are situations that represent types of problems found in a real context, which facilitate the establishment of connections with reality (Sevin et al., 2018; Vale et al., 2022)

Based on these ideas, we report an ongoing study, with elementary pre-service teachers (PST) (children aged 6–12) in which we intend to understand and characterize the performance underlying the use of ED, in solving AP, identifying the main difficulties and contents mobilized by the participants, as well as their engagement. We followed an exploratory qualitative and interpretative methodology. Data was collected in a holistic, descriptive and interpretative manner, through: participant observation, observational notes, reactions and interactions; written documents; artefacts; and photos. Preliminary results indicated that this experience allowed the PST to recognize ED is tied to mathematics and physics, in particular with problem solving, giving them an opportunity to solve a real-life problem with simple materials. We identified some difficulties in the mobilization of adequate scientific language in the argumentation of their decisions; in the use and interpretation of the ED cycle and in the models' 3D sketches. Students were engaged in this experience and said that they would like to use this approach with other problems.

Keywords: engineering design; authentic problems; active learning; elementary preservice teachers.

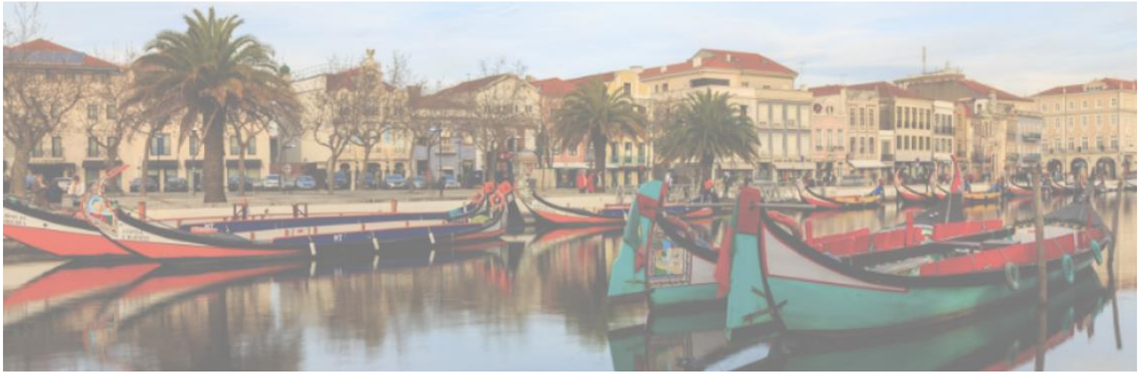


References

English, L.D., & King, D. (2019). STEM integration in sixth grade: Designing and constructing paper bridges. *IJSME, 17*, 863–884. https://doi.org/10.1007/s10763-018-9912-0

Sevinc, S., & Lesh, R. (2018). Training mathematics teachers for realistic math problems: a case of modeling-based teacher education courses. *ZDM, 50*, 301–314. https://doi.org/10.1007/s11858-017-0898-9

Vale, I., Barbosa, A., Peixoto, A., & Fernandes, F. (2022). Solving problems through engineering design: An exploratory study with pre-service teachers. *Education Sciences, 12*(12), 889. https://doi.org/10.3390/educsci12120889



History of Mathematics

Exploring the Rich History of Japanese Mathematics (Wasan) in the Mathematics Classroom

Vanda Santos[1], Jaime Carvalho e Silva [2]

[11]CIDTFF – Centro de Investigação em Didática e Tecnologia na Formação de Formadores/ Universidade de Aveiro

[2]Deopartamento de Matemática, Universidade de Coimbra

Abstract

Japanese mathematics, known as Wasan, holds fascinating historical significance and offers valuable insights for mathematics education. During the Edo period in Japan, the country experienced a phase of national isolation where connections with the Western world and Western mathematics were severed. The Sangaku tablets, intricately crafted with mathematical, historical, artistic, and religious significance, stand as remarkable artefacts from that era.

In this context, the historical perspective of Mathematics can be a valuable means of awakening students' interest in Mathematics. The inclusion of the History of Mathematics in the teaching of Basic Education has been emphasized by specialists in Mathematics Education, and interdisciplinary practices can provide this enriching interaction, even if gradually in the classroom.

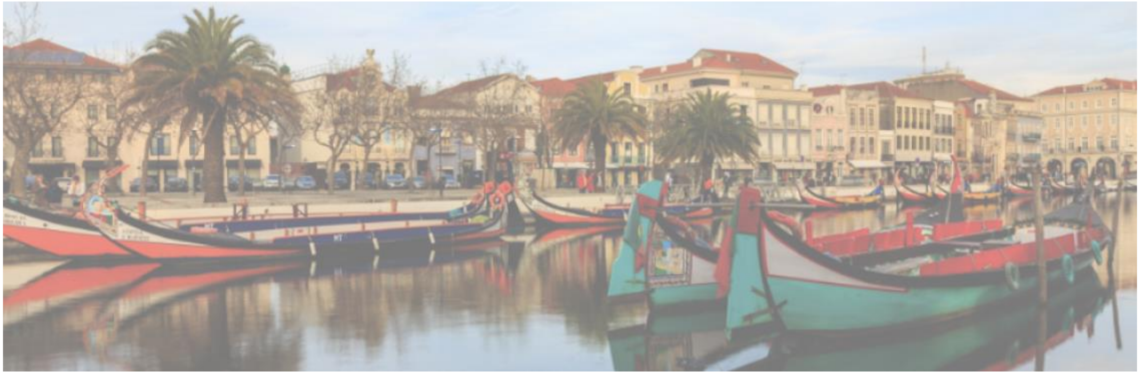
This work intends to describe how the use of interdisciplinary and ludic-pedagogical activities, based on the History of Mathematics, can improve the learning of Mathematics. By learning mathematical content together with its history, students are encouraged to look at mathematics in a different and more engaging way. Historical materials also help build the concept of mathematics, making the learning process more meaningful.

An interdisciplinary activity carried out with 7th grade students from a Portuguese Basic Education School, involving the theme of Sacred Geometry, will be described. We observed the interest aroused in the students by this theme and the work developed by them, showing the enriching potential of the use of the History of Mathematics in the pedagogical practices.

Keywords: math history; geometry; math curriculum; sacred geometry.

Acknowledgements

The work of the first author is funded by national funds through FCT – Foundation for Science and

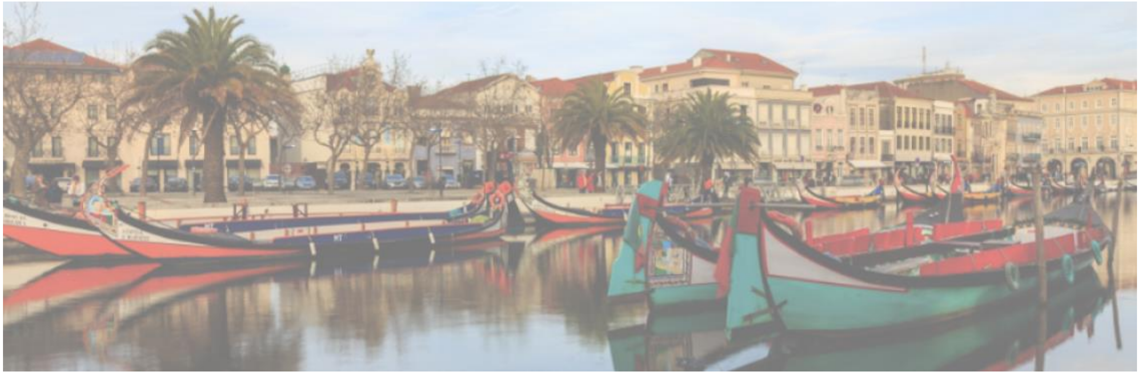


Technology, I.P., within the scope of the Project UIDB/00194/2020 and in the scope of the framework contract foreseen in the numbers 4, 5 and 6 of the article 23, of the Decree-Law 57/2016, of August 29, changed by Law 57/2017, of July 19.

References

Ahuja, M., Uegaki, W., & Matsushita, K. (2004). Japanese theorem: A little known theorem with many proofs-part I. *Missouri Journal of Mathematical Sciences*, 16(2), 72-81.

Hidetoshi, F. & Pedoe, P. (1989). Japanese temple geometry problems. Sangaku. Winnipeg: Charles Babbage. ISBN 9780919611214; OCLC 474564475.



The history of Mathematics in the light of artificial intelligence: some observations for classroom work

Eugenio Díaz Barriga Arceo[1], J. Alexandre dos Santos Vaz Martins [2]
[1]Universidad Autónoma del Estado de México
[2]Instituto Politécnico da Guarda

Abstract

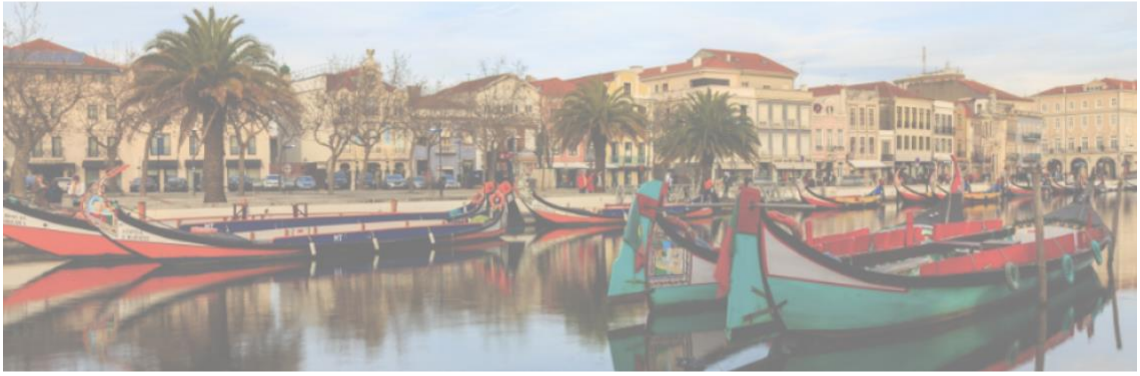
Now a days there is great interest in incorporating the use of Artificial Intelligence (AI) into day-to-day academic school work (Dykema, 2023; Gadanidis, 2017; Melchor et al., 2023). It has been used, for example, in the search for information and in the treatment of large volumes of data to carry out tests and calculations. In the classroom, the bustle is increasing due to the incorporation of AI, which is a source for students to carry out documented essays, develop creative projects, collect information, transfer arduous calculations to the machine, build algorithms for solving of various problems, among other applications. Among the teachers, the atmosphere is heated by the debate on whether its use as a component in the evaluation of learning should be allowed or sanctioned.

An inexhaustible source of teachings and learnings for all ages is the History of Mathematics (HM). The knowledge construction process is a great collaborative task that has been carried out by all cultures, with multiple resources. Frequently, new pieces of the historical puzzle come to enrich the panorama of ancient knowledge, offering increasingly precise ideas of what was known in previous times (IREM Historie des Mathématiques, 1998).

However, in this work we report some risks when we explore with AI some topics that involve classic HM problems. So, we propose a non-exhaustive list of considerations for HM with AI classroom work, as well as some development paths in this context. We will try to make it also a starting point for a more in-depth reflection on the use of AI in mathematics classes (Dikema, 2023; Green-Harper, 2023).

In the present, AI enables searches in vast databases, from many digitized libraries. Despite this, HM appears to be too large and elusive library to abridge. In this paper, we will explore the fusion and interactions between AI and HM, taking the following 4 examples: pythagorean triples; the Basel problem; Lagrange multipliers; the construction of the cycloid **(**Boyer, 1968; IREM Historie des Mathématiques, 1998).

In the interaction with ChatGPT, version 3.5, these problems showed that the AI gives preference to searching in the network for the information raised in order to solve mathematical problems, giving preference to numerical and symbolic calculations, and it is even possible to create program codes.



However, in terms of the search for finer information, it does not distinguish between historical stages, even falling into significant inconsistencies. In addition, geometric modeling is set aside, perhaps due to a lack of bibliographic material in this direction (Díaz-Barriga, 2009; Martins, Roca & Nascimento, 2014).

Keywords: history of mathematics; artificial intelligence; classroom work.

Acknowledgements

We acknowledge the UDI of Instituto Politécnico da Guarda, and also the Universidad Autónoma del Estado de México.

References

Belda, I. (2011). *Mentes, máquinas y matemáticas. La Inteligencia Artificial y sus retos*. Editorial RBA. Navarra, España.

Boyer, C. B. (1968). *A History of Mathematics*. John Wiley and Sons.

Díaz Barriga, E. (2009). *Geometría dinámica con Cabri-Géomètre*. Toluca, México. Editorial Kali.
Melchor. P.M., Lomibao, L.S. Parcutilo, J. (2023). Exploring the Potential of Integrating AI into Mathematics Education for Generation Alpha: Approaches, Challenges, and Readiness of Philippine Tertiary Classrooms: A Review of Literature. *Journal of Innovation in Teaching and Learning*, Vol. 3.
Green-Harper, R. (2023). *The Pros and Cons of Using AI in Learning: Does ChatGPT Help or Hinder Learning Outcomes?*. eLearning Industry.

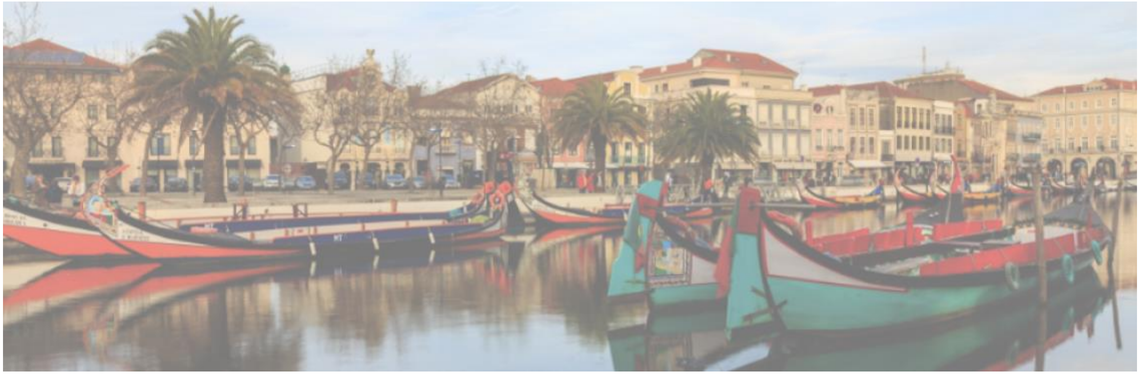
Dykema, K. (2023). *Mathematics and Artificial Intelligence*. NCTM.

Gadanidis, G. (2017). Artificial Intelligence, Computational Thinking and Mathematics Education. *International Journal of Information and Learning Technology*, 34(2), 133-139.

IREM Historie des Mathématiques (1998). Aux origines du Calcul Infinitésimal. Comprendre les mathématiques par les textes historiques. Cercle D'Histoire des sciences. *IREM de Basse Normandie*. Ellipses.

Martins, J. A., Estrada, A. & Nascimento, M. M. (2014). Do you need to see it to believe it? Let's see statistics and geometry dynamically together!. *European Journal of Science and Mathematics Education*, 2(1), 39-52.

Weisstein, E. (1998). CRC Concise Encyclopedia of Mathematics. pp. 1467.



STE(A)M

Mathematical trails with MathCityMap and GeoGebra Discovery

Belén Ariño-Morera[1], Angélica Martínez-Zarzuelo[2], Tomás Recio[3]
[1]Universidad Rey Juan Carlos/Universidad Complutense de Madrid
[2]Universidad Complutense de Madrid
[3]Universidad Antonio de Nebrija

Abstract

MathCityMap[1] is a technological tool (app and web portal), developed since 2012 at the Goethe University of Frankfurt, and freely available over computers and smartphones. It aims to facilitate and improve, both in the educational context and for public use, the creation and development of outdoor mathematics trails. See Ludwig and Jesberg (2015) for an early introduction, and Taranto et al. (2021) for a more recent description in the context of teacher training.

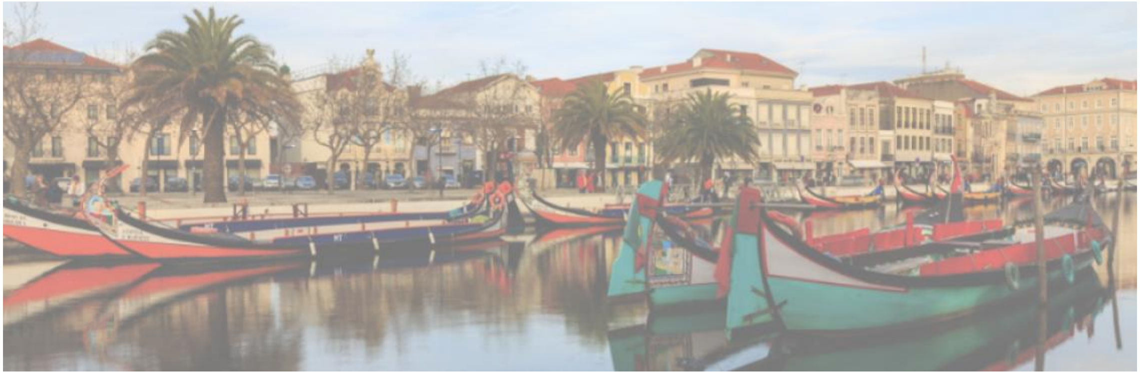
Through the web portal users (for example, teachers) can create their own mathematical routes, or to view and follow trails built by others. The portal helps the trail author both by automatizing some steps concerning the edition of the created mathematical tasks associated to concrete objects along a trail, as well as enhancing the inspiration of the trail author by presenting a large repository of tasks and trails for a potential reuse.

On the other hand, the MathCityMap app guides final users (students, hikers, tourists...) along a created math trail they can select from those close to their location and available in the MathCityMap repository. Then hikers have two options: one, to walk along the trial without internet connection, by downloading the trail data to the smartphone. Two, with connection, the MathCityMap app guides the user towards the position of the different tasks and provides hints (if required) and feedback about the mathematical solution provided by the walker.

GeoGebra Discovery[2] is a free, experimental version of GeoGebra[3], dealing with automated reasoning in elementary geometry. Among other improvements, GeoGebra Discovery includes a collection of Automated Reasoning Tools for elementary geometry, through a set of commands like *Discover, StepwiseDiscovery, ProveDetails, Prove, LocusEquation, Relation*, etc. See Kóvacs et al. (2022) for a detailed description of these tools.

---

[1] https://mathcitymap.eu/en/
[2] https://kovzol.github.io/geogebra-discovery/
[3] https://www.geogebra.org



Numerous studies already exist on learning with the support of GeoGebra, or with MathCityMap, and more recently, regarding the exploration, beyond the classroom, of the mathematics involved in everyday situations, with the cooperation of MathCityMap and GeoGebra 3D (augmented reality, 3D printing). See, for example, Botana et al. (2020).

In our talk we address a new way of merging MathCityMap and GeoGebra for educational purposes, showing, through some examples, our on-going project, that aims to explore the potential cooperation of the automated reasoning tools in GeoGebra Discovery and MathCityMap, describing a scenario in which both technologies complement each other, using GeoGebra Discovery mechanical intelligence to help students visualize, conjecture, and discover new, hidden, geometric features on real world objects, artworks (see, for a recent example in this context, Ariño-Morera et al. (2023)), monuments, etc. along MathCityMap trails.

The idea is to design suitable MathCityMap tasks by capturing real world images through a smartphone, placing them on the window of GeoGebra Discovery, building over them a mathematical model, and then allowing GeoGebra Discovery to automatically output information on the geometric properties of the considered object.

Finally, the educational impact of this cooperation, and its future development as an educational experience with students, will be analyzed.

Keywords: MathCityMap; GeoGebra Discovery; outdoor mathematics; mathematics trails; dynamic geometry; GeoGebra; mathematics education.

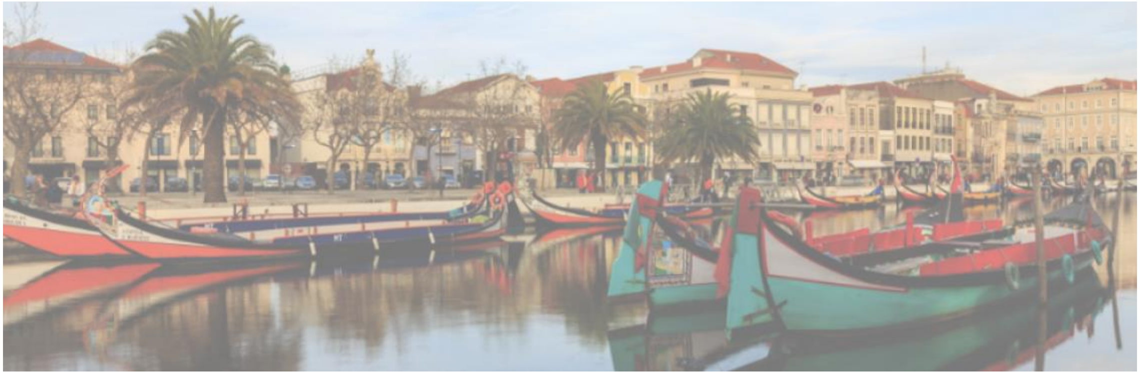
Acknowledgements

Third author partially supported by a grant PID2020-113192GB-I00 (Mathematical Visualization: Foundations, Algorithms and Applications) from the Spanish MICINN.

References

Ariño-Morera, B, Benito, A., Nolla, A., Recio, T., & Seoane, E. (2023). Looking at Okuda's artwork through GeoGebra: A Citizen Science experience. *AIMS-Mathematics*, Volume 8, Issue 8, pages 17433-17447. https://doi.org/10.3934/math.2023890

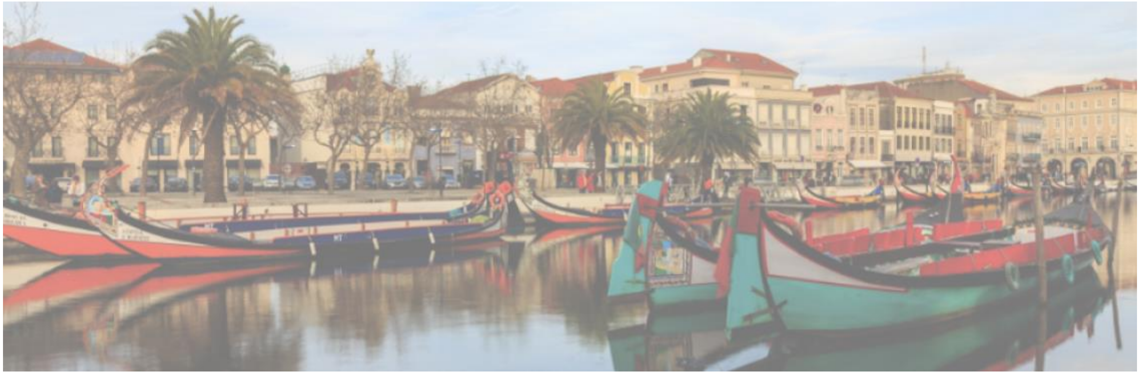
Botana F., Kovács Z., & Recio T. (2020). Automatically Augmented Reality for Outdoor Mathematics. In Ludwig, M., Jablonski, S., Caldeira A., & Moura A. (Eds.). *Research on Outdoor STEM Education in the digiTal Age. Proceedings of the ROSETA Online Conference in June 2020.* WTM -- Verlag für wissenschaftliche Texte und Medien, Münster 2020. Conference Proceedings in Mathematics Education (6), pages 71-78. https://doi.org/10.37626/GA9783959871440.0



Kovács, Z., Recio, T., & Vélez, M.P. (2022). Automated Reasoning Tools with GeoGebra: What are they? What are they good for? In Richard, P.R., Vélez, M.P. and Van Vaerenbergh, S. (Eds.). *Mathematics Education in the Age of Artificial Intelligence*. Series: Mathematics Education in the Digital Era; Springer Nature Switzerland AG, pages 23-44. https://doi.org/10.1007/978-3-030-86909-0_2

Ludwig, M., & Jesberg, J. (2015). Using Mobile Technology to Provide Outdoor Modelling Tasks—The MathCityMap-Project. *Procedia—Soc. Behav. Sci.* 191, pages 2776–2781. https://doi.org/10.1016/j.sbspro.2015.04.517

Taranto, E., Jablonski, S., Recio, T., Mercat, C., Cunha, E., Lázaro, C., Ludwig, M., & Mammana, M.F. (2021). Professional Development in Mathematics Education—Evaluation of a MOOC on Outdoor Mathematics, *Mathematics* 2021, Volume 9, Issue 22, 2975. https://doi.org/10.3390/math9222975



From the Steam Engine to STEAM Education: An Experience with Pre-Service Mathematics Teachers

Ángel C. Herrero[1], Tomás Recio[1], Piedad Tolmos[2], M. Pilar Vélez[1]
[1]Universidad Antonio de Nebrija
[2]Universidad Rey Juan Carlos

Abstract

This contribution summarizes and disseminates work by the authors that has been recently published in an international journal, in this year 2023, see Herrero et al. (2023).

In our talk we describe an educational experience in the context of the master's degree that is compulsory in Spain to become a secondary education mathematics teacher. Master's students from two universities in Madrid (Spain) attended lectures that addressed—emphasizing the concourse of GeoGebra (https://www.geogebra.org) a dynamic geometry software package—some historical, didactic and mathematical issues related to linkage mechanisms, such as those arising in the 18th and 19th centuries during the development of the steam engine.

Afterwards, participants were asked to provide three different kinds of feedback, all within the context of the received lectures' topic:

(i) working on an assigned group task,

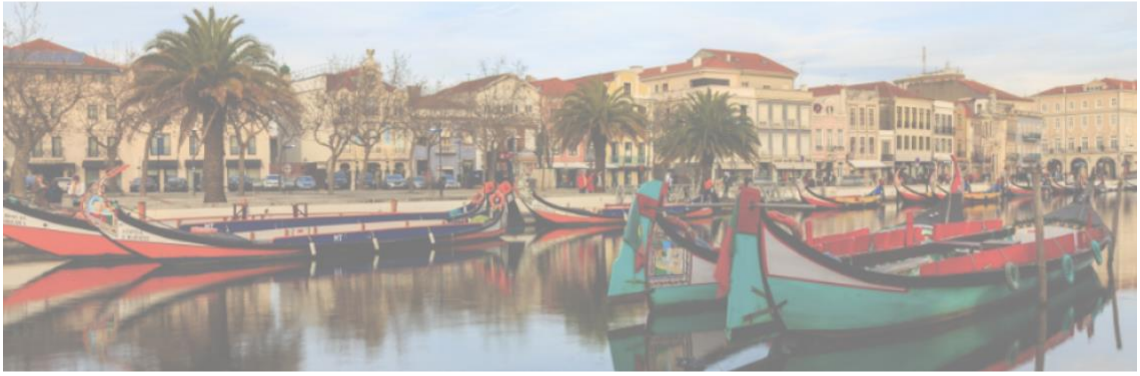
(ii) individually answering a questionnaire, and

(iii) proposing some classroom activity addressed to their prospective pupils.

In our paper, in the framework of Mason's reflective discourse analysis (Mason, (2002)), the information supplied by the participants has been analyzed, focusing on what they have (or consider having) learned from the experience and on what could be their perception of the potential interest of using linkages as a methodological tool for their future professional activity as teachers.

Finally, in our talk, after a brief description of the development and contents of the mentioned experience, we present and discuss the obtained results and conclusions.

Keywords: dynamic geometry; GeoGebra; STEAM education; mathematics education; mathematics teacher initial training; reflective discourse analysis.



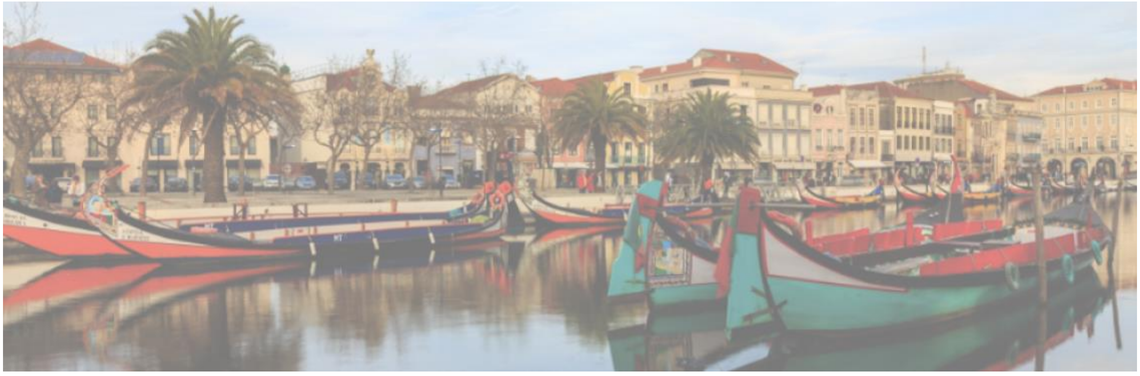
Acknowledgements

Second and fourth authors partially supported by a grant PID2020-113192GB-I00 (Mathematical Visualization: Foundations, Algorithms and Applications) from the Spanish MICINN.

References

Herrero, A. C., Recio, T., Tolmos, P., & Vélez, M.P. (2023). From the Steam Engine to STEAM Education: An Experience with Pre-Service Mathematics Teachers. *Mathematics*, Volume 11, Issue 2, 473, 16, Jan. 2023. https://doi.org/10.3390/math11020473

Mason, J. (2002). *Researching your own practice: The discipline of noticing*, Nueva York, Routledge.



Teaching Intriguing Geometric Loci with GeoGebra Discovery

Belén Ariño-Morera[1], Tomás Recio[2], Piedad Tolmos[3]
[1]Universidad Rey Juan Carlos /Universidad Complutense de Madrid
[2]Universidad Antonio de Nebrija
[3]Universidad Rey Juan Carlos

Abstract

The title of our contribution refers to a similar one from a chapter by Ferrarello et al. in the book "Mathematics and Technology. Advances in Mathematics Education" (see Ferrarello et al., 2017).

In that chapter the authors describe an experience, involving over 200 secondary education students from Sicily, where the students had to construct and to explore different geometric loci, with the help of a Dynamic Geometry System (GeoGebra). The authors analyze in detail the performance of students in three particular loci, and the impact of GeoGebra on the whole experimental activity.

On our contribution we deal with the same three loci, but now focusing on the analysis of the potential differences (pros and cons) and consequences (need of methodological changes) that could take place in a hypothetical, repeated version of the same experience, assuming the students would be trained and allowed to use automated reasoning tools, with GeoGebra Discovery (http://autgeo.online/geogebra-discovery/).
Thus, we will show how in some of these three loci, the previously considered educational strategy could turn out to be almost irrelevant, while in other of these loci, the educational impact of the exploratory activity could still be a very relevant one, but we consider it would have now to be developed with a different protocol, focusing students' loci investigation on issues --that we will detail in our communication-- not fully considered in the previous experience.

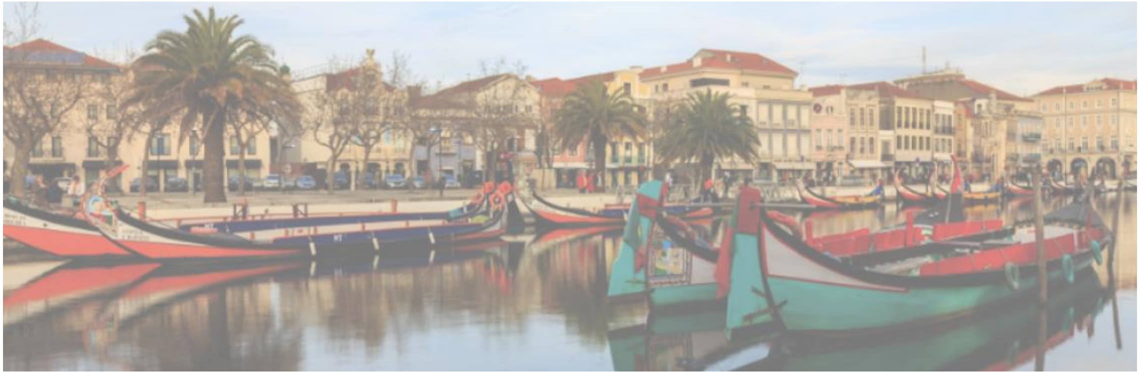
Keywords: dynamic geometry; GeoGebra Discovery; automated reasoning tools; mathematics education.

Acknowledgements

Second author partially supported by a grant PID2020-113192GB-I00 (Mathematical Visualization: Foundations, Algorithms and Applications) from the Spanish MICINN.

References

Ferrarello, D., Mammana, M.F., Pennisi, M., & Taranto, E. (2017). Teaching Intriguing Geometric Loci with DGS. In Aldon, G., Hitt, F., Bazzini, L., Gellert, U. (Eds.). *Mathematics and Technology. Advances in Mathematics Education*, pp. 579-605. Springer, Cham. https://doi.org/10.1007/978-3-319-51380-5_26



Why are so many mathematics teachers not using interactive resources?

Douglas Butler[1]
[1]iCT Training Centre, Cirencester, UK

Abstract

Mathematics teachers are incredibly fortunate these days to have a fantastic array of mature and pedagogically rich resources to help them in their classes, yet far too many are choosing not to build them into their lesson plans. Mathematics is a difficult subject to teach at the best of times, but the creative use of digital resources can make a material difference to the effectiveness of teaching and learning at all levels.

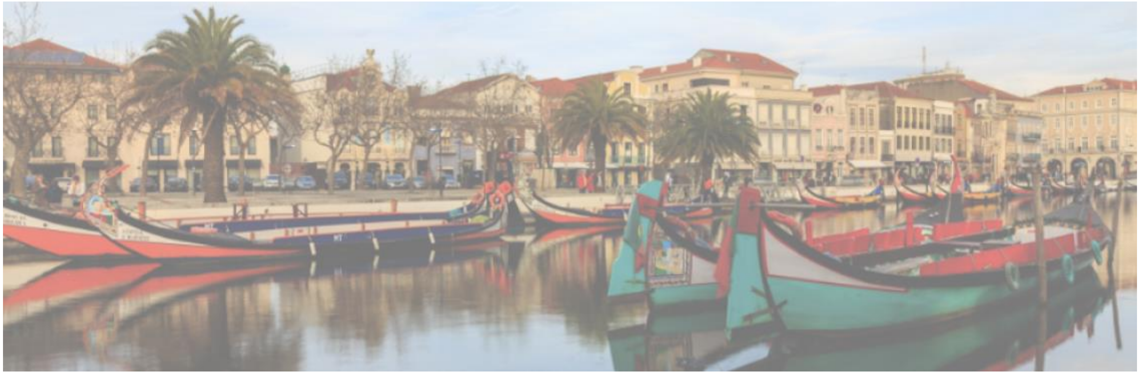
The most useful online resources are interactive, requiring intelligent inputs from teachers and students, including spreadsheets and dynamic graphing software. This sets Mathematics apart from most other school subjects where, by and large, online resources are static or otherwise require minimal input.

It is unfortunate that a significant majority of primary and secondary mathematics teachers choose to steer clear of using interactive resources at all. This session will attempt to explore some of the reasons for this, and conclude with some examples of best practice using different user-interfaces.

1. Lack of training.

Despite modern teachers being very IT-aware, learning how to incorporate interactive resources into mathematics lessons cannot be achieved by attending the odd 40-minute workshop at a conference. It is also given a low priority in most initial teacher training programs. When the TSM workshops were running in England (Technology for Secondary Mathematics) teachers spent three solid days learning Excel and dynamic graphing software.

These days, similar courses are available online, but teachers still need to commit the time. Following the TSM workshops a large collection of online resources has been created and is routinely updated:
https://www.tsm-resources.com



Spreadsheet

Another concern is the use of the spreadsheet, such an obvious tool in Mathematics, yet it fails to put in an appearance in most secondary or senior secondary mathematics curriculums.

2. Lack of confidence

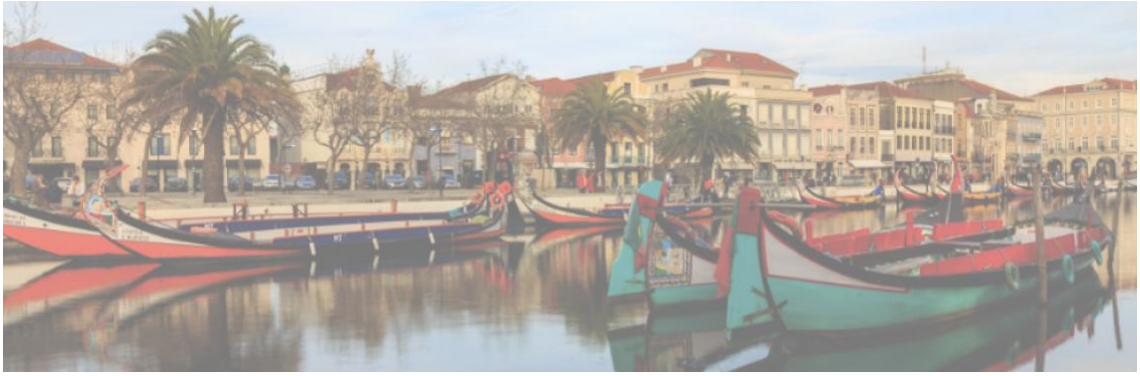
Many teachers steer clear of using interactive resources because they are not confident enough to make its use effective, and they worry about not coping if it goes wrong.

3. The user-interface is too complicated

When the major graphical tools were being developed there was a vital decision taken about the way objects and constructions were selected. GeoGebra asks users to select the operation first, then the objects to apply it to. This means, at the start, every conceivable operation must be available on screen. In GeoGebra's case this is achieved by an enormous hidden list of command-line options. This can be complicated and visually confusing. Many teachers come away from GeoGebra workshops excited by what is possible, but concerned that they may not remember how to do it in a few days. As a consequence, GeoGebra users tend to use finished files in their lessons.

The Geometer's Sketchpad and Autograph chose the other convention: select the objects first and then select the operation from a list that is appropriate to the selected objects. This is pedagogically more intuitive, allowing the build-up of a file from scratch to including important along the way.

This session will conclude with some examples of best practice: interactive resources that can make a significant difference to learning, using GeoGebra, Desmos and Autograph. This will include elementary geometry using images from Google Earth, exploring vectors and trig functions, and using the zooming feature to explore the mystery of calculus.



Possibilities of usage of inquiry-based learning in STE(A)M Education

Jan Guncaga[1], Mária Fuchsova[2], Vaclav Sykora [3]
[1]Comenius University in Bratislava, Faculty of Education
[2]Comenius University in Bratislava, Faculty of Education
[3]University of Ostrava, Faculty of Education

Abstract

The presentation will be oriented to practical outputs of two projects, VEGA 1/0033/22, Discovery-oriented teaching in mathematics, science and technology education, and also project KEGA 026UK-4/2022 "The concept of Constructionism and Augmented Reality in STEM Education (CEPENSAR)".

Our activities with students were oriented to using some inquiry-based strategies - for example, the Polya strategy, experiments in the field of science education, preparing results of the students' experiments, and work with applications of augmented reality (AR). The task for students was also to prepare their experiments, which they could evaluate. Through non-structured observation as a tool of the qualitative educational research the students – future teachers for primary level were observed about their argumentation in their answers/solutions of the tasks, in which it is possible to observe the logical steps in their thinking. The students' experiments, creativity, and evaluation will be discussed, and also, in the conclusions, our pedagogical experience limitations will be presented, and ideas for future research in this area will be addressed. The activities with future primary education students will be discussed, as some results of the qualitative study and suitable best practices from the school surrounding.

Grant KEGA Nr. 004KU-4/2022 "Prominent personalities of Slovak Mathematics II - idols for future generations" supported this work.

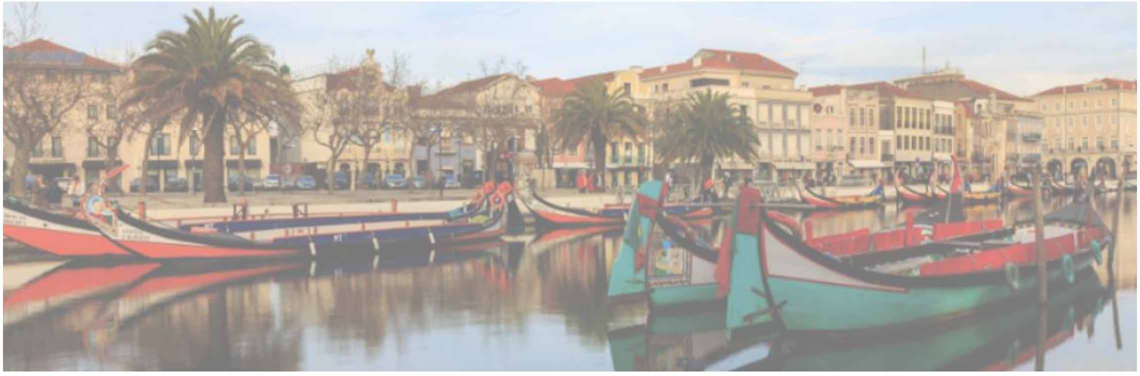
Keywords: STE(A)M education; inquiry-based learning; future primary students; qualitative research.

Acknowledgements

Supported by VEGA 1/0033/22, KEGA Nr. 004KU-4/2022

References

Pólya, G. (1945). *How to Solve It*; Princeton University Press: Princeton, NJ, USA; Oxford, UK, 1945.



STEM on initial teacher education: A drone delivery challenge

Neusa Branco[1], Bento Cavadas[1,2]
[1]Polytechnic Institute of Santarém
[2] Polytechnic Institute of Santarém / Lusófona University – Interdisciplinary Research Centre for Education and Development

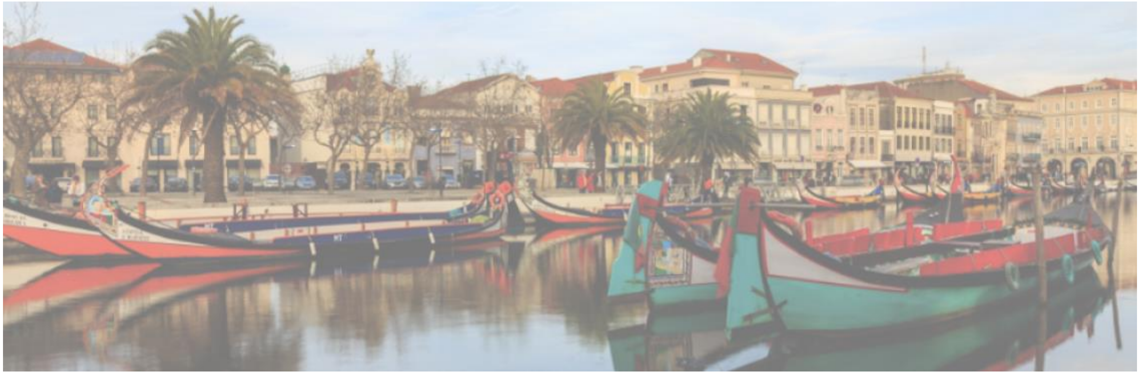
Abstract

Delivering products fast and safety is a goal of delivery companies. Some companies are studying new ways of delivering products, using drones. Vale et al. (2022) suggested that teacher education programs should include experiences that stimulate future teachers' knowledge, particularly solving and discussing tasks and using the teaching and learning principles, such as Engineering Design Process (EDP), that they are expected to use with their students. Considering that context, this work presents a STEM activity based on drone delivery, implemented with pre-service teachers (PSTs). In this activity, PSTs implemented cycles of engineering design to create a package to deliver objects using a drone equipped with a grabber.

Using a guide, they followed the NASA' model of EDP, presented in Hoban and Delaney (s.d.), and constituted by six steps: Ask, Imagine, Plan, Create, Experiment and Improve. The tasks of the six steps were adapted from Cavadas and Topçu (2022). In the "Ask" moment, PSTs had to define and understand the problem. After they had achieved consensus concerning the ideas about the design of the best package during the "Imagine" moment, they initiated the "Plan" step. In the planning moment, PSTs had to sketch their design having as a constraint the materials indicated in the guide. Each material had a unit cost and PSTs must calculate the cost of their package. For example, one square centimeter of paper sheet had one-unit cost and one small elastic rubber had two-unit cost. Then they moved on to the "Create" phase, making the package and adding a plasticine figure inside it. During the "Experiment" step, they had to video record the drone flight carrying the package. After the drone landing, PSTs had to unpack and look for possible damages on the plasticine figure. In that case, they had to "Improve" the design and test it again. Moreover, they could try to reduce the cost, upgrading the design of the package.

The PSTs productions and their explanations about their options were analysed. The analysis focuses are the mathematical ideas about the: i) geometric and measurement notions mobilized for the design, plan and construction of a safe package; ii) optimization of the costs, ensuring a safe delivery, and the improve of the package.

The EDP highlighted that simulating drone deliveries brought to light the challenge of designing an effective package. During the EDP, new issues related to the packaging surfaced, issues that PSTs couldn't anticipate before testing. These problems led some groups to reconsider the geometric shape of the package and the materials



used, aiming to enhance its resilience during flight for a secure delivery. Furthermore, material cost constraints sparked creativity among the participants, driving them to enhance the package structure while keeping costs down. Mathematical concepts, such as those related to the package's area and volume, played a crucial role. PSTs strategically reduced the surface area while ensuring sufficient volume to accommodate the object, resulting in a more cost-effective package.

The engagement of PSTs in the EDP process provide them a valuable experience about the STEM approach. They improved their understanding of learning experiences that could be important to propose to their future students, with the aim of exploring STEM competences for the 21st century.

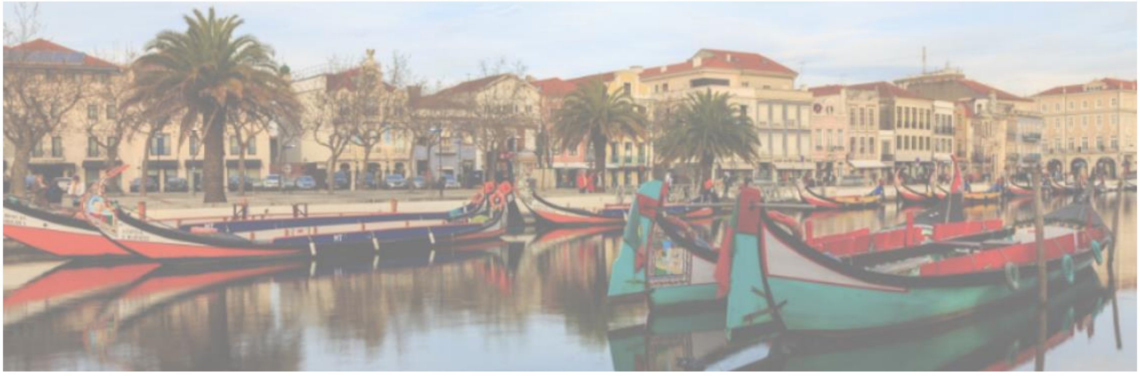
Keywords: drone-based learning; initial teacher education; STEM.

References

Cavadas, B., & Topçu, M. S. (2022). Innovation in preservice teacher education: Drone-based learning. In M. J. Loureiro, A. Loureiro & H. R. Gerber (Eds.), *Handbook of research on global education and the impact of institutional policies on educational technologies* (pp. 12-35). IGI Global. http://doi.org/10.4018/978-1-7998-8193-3.ch002

Hoban, S., & Delaney, M. (s.d.). NASA's Best students. Beginning Engineering, Science, and Technology. *An Educator's Guide to the Engineering Design Process Grades 6-8.* University of Maryland & NASA. https://www.nasa.gov/wp-content/uploads/2012/09/630754main_nasasbestactivityguide6-8.pdf

Vale, I., Barbosa, A., Peixoto, A., & Fernandes, F. (2022). Solving Problems through Engineering Design: An Exploratory Study with Pre-Service Teachers. *Education Sciences, 12*, 889. https://doi.org/10.3390/educsci12120889



Factors that influence engineering students' motivation to study mathematics

Timo Tossavainen[1], Evgeniya Burtseva[2]
[1]Department of Health, Education and Technology/Luleå University of Technology
[2]Department of Engineering Sciences and Mathematics/Luleå University of Technology

Abstract

Students' motivation plays a crucial role in studying mathematics. A recent Swedish study on engineering students' mathematical self-concept and its dependency on their views of mathematics and study habits showed that engineering students' motivation to study mathematics varies across the years (Tossavainen et al., 2022). In order to understand better this variation, we conducted interviews with eleven engineering students from a Swedish university. Initially, during their first year, all students reported that they felt motivated to study mathematics. However, by the second year, some of the students experienced a loss of motivation. Our primary focus is to understand the underlying reasons for these changes in engineering students' motivation throughout their university journey, as well as to explore their opinions on how to sustain motivation to study mathematics.

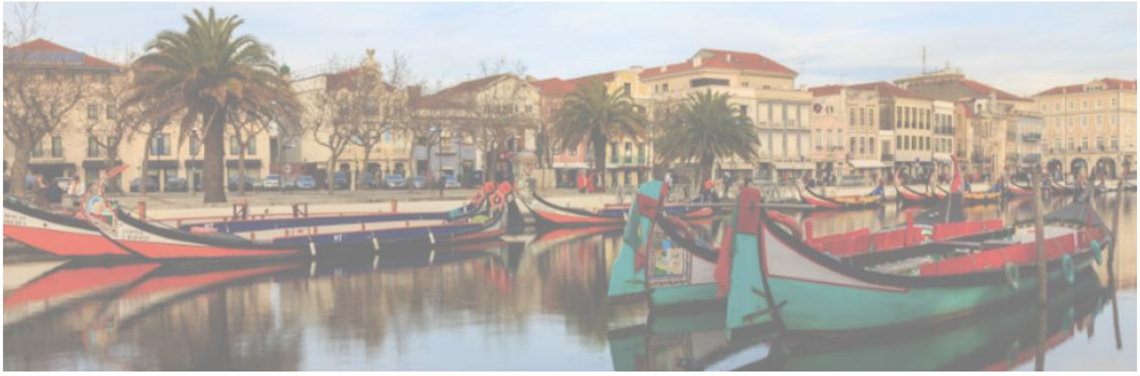
A majority of the interviewed students highlighted a negative impact on their motivation stemming from the sudden shift to distance education due to the COVID-19 pandemic. The lack of face-to-face communication with peers and teachers emerged as a significant challenge for students, especially at the beginning of their studies. Our main findings show that students are seeking for continuous feedback and engagement in active learning (Burtseva et al., 2023).

Moreover, we analyzed the connection between students' views of mathematics and their motivation. A "toolbox" view of mathematics is very common among engineering students that might also be a contributing factor to the observed decline in motivation. Including more proving in engineering students' learning process can develop their problem-solving thinking and as a result increase their motivation to study mathematics (cf. Tossavainen et al., 2022; Tossavainen & Burtseva, 2023).

Keywords: engineering student; motivation; online education; feedback.

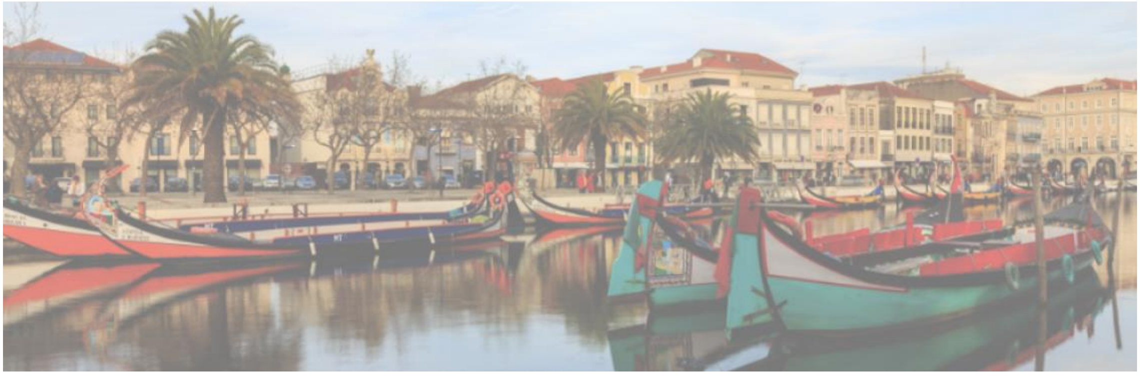
References

Tossavainen, T., Wall, P., & Sundhäll, M. (2022). Engineering students' mathematical self-concept and its dependence on their study habits and views about mathematics. *International Journal of Engineering Education*, *38*(5A), 1354-1365.



Tossavainen, T., & Burtseva, E. (2023). The perceived value of proving in learning engineering mathematics and its dependence on motivation and study habits. Submitted.

Burtseva, E., Sundhäll, M., Tossavainen, T., & Wall, P. (2024). Engineering students' varying motivation and self-concept in mathematics. *International Journal of Engineering Education*, *40*(1), 97-107.



Designing activity of exploratory data analysis through RStudio: an evaluation a posteriori

Carlotta Vielmo[1]
[1]University of Trento

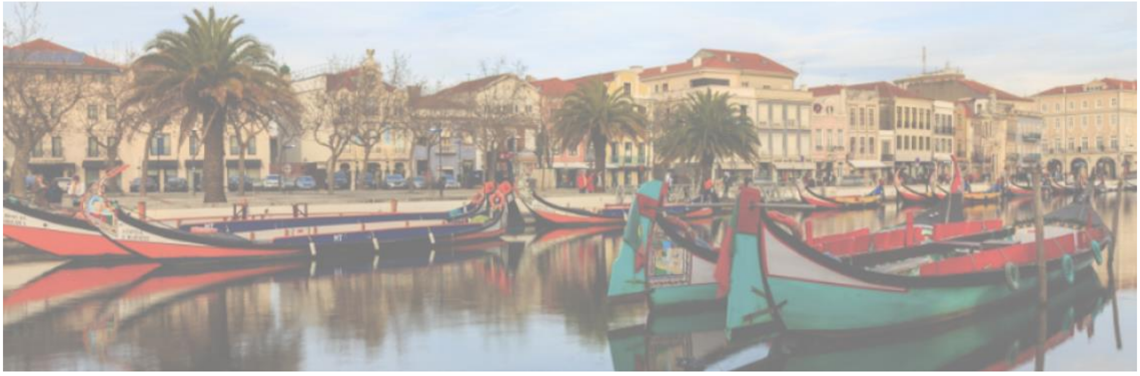
Abstract

Nowadays, informed citizens need to consciously make decisions based on data-driven arguments (OECD, 2018). Hence, research in statistics education faces the vast and current issue of ensuring the development of adequate statistical literacy, reasoning, and thinking (Ben-Zvi & Garfield, 2004). Exploratory Data Analysis (EDA) is pivotal for fostering statistical knowledge and informed decision-making in the modern era, and digital tools brought crucial changes in this field, speeding up operations and enabling faster and powerful representations. From an educational perspective, EDA may be a valuable way to approach statistics (Cobb & Moore, 1997) and technology gives students more time to focus on concepts and supports active learning, fostering the exploration of statistical reasoning through hands-on data analysis and interpretation of real-world data sets, that motivate deeper engagement (Libman, 2010), with representations becoming the object of the cognitive activity (Ben-Zvi, 2000).

R is a well-known statistical tool and ecosystem. There are several studies on its use at undergraduate level (e.g., Tucker et al., 2022), fewer at the secondary school one (e.g., Heinzman, 2022). Yet, R may be an interesting tool for EDA activities also in the latter: by requiring coding, statistical reasoning and data analysis are more visible and reproducible (Tucker et al., 2022). Students can customize the code by working on the command's arguments, helped by the RStudio interface. They must organize their thinking to produce a correct code and let data speak properly.

Statistical inquiry requires a constant dialogue between contextual and statistical aspects (Wild & Pfannkuch, 1999), and the design of EDA activities must balance these aspects with software. What to focus on when designing them for secondary school and how to do it is a fertile ground to investigate on. We built a one-week long project (30 hours) for grade 12 and 13 students to introduce EDA through RStudio, a first implementation allowed us to reflect on the design of EDA activities (see DiCoMat, 2023). The project begins with a basic introduction to R-RStudio (2 hours): short enough to reduce cognitive load (Biehler et al., 2013) and start working right away on data sets related to local environmental phenomena, a topic that can involve students while providing pedagogically rich data sets to explore. Teacher-guided tasks alternate with free exploration by students, thus stimulating awareness and autonomy in the use of the tool, encouraging students to search for ways to customize the final output.

For the last ten hours, students work in groups to produce a report from a new data set. This allows them



to choose a meaningful thesis, organize a discourse, and plan the communicative aspect.

We collected students' feedback as a daily logbook and a final questionnaire. They were expecting coding part being much more difficult and they perceived a potential in such a tool, and they recognize EDA as important. The final report also figured as a possible instrument for the expert to evaluate the effectiveness of the entire activity.

After this first exploratory implementation, several questions arose and will be addressed in future redesign: how to help students be aware in managing and choosing charts? How to develop clearer communication? How to make it a curricular activity instead of a week-long immersive one? How to make the report an effective tool to detect the learning progress?

Keywords: statistics education; statistical software; exploratory data analysis; ste(a)m.

Acknowledgements

This work is founded within the National Recovery and Resilience Plan (NRRP), the Italian plan part of Next Generation EU program, and the University of Trento (Department of Mathematics, and Department of Information Engineering and Computer Science).

References

Ben-Zvi, D. (2000). Toward understanding the role of technological tools in statistical learning. Mathematical Thinking and Learning, 2(1–2), 127–155. https://doi.org/10.1207/s15327833mtl0202_6

Ben-Zvi, D., & Garfield, J. (2004). The challenge of developing statistical literacy, reasoning and thinking. In *Springer eBooks*. https://doi.org/10.1007/1-4020-2278-6

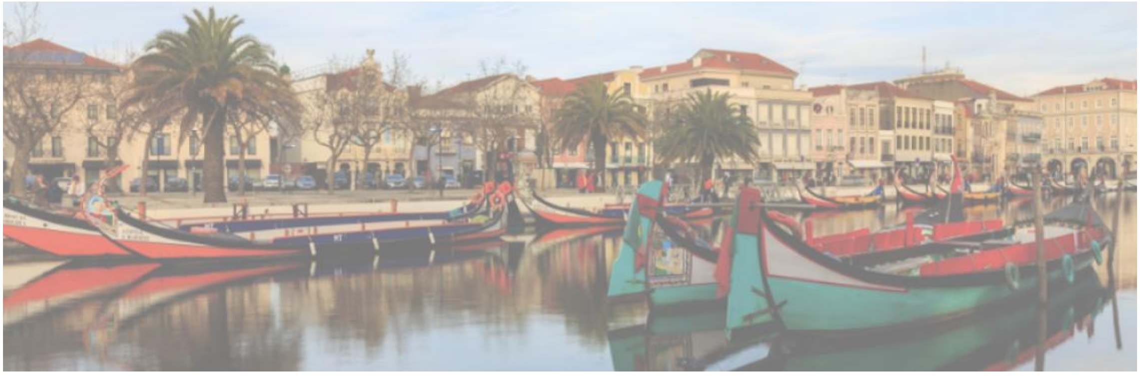
Biehler, R., Ben-Zvi, D., Bakker, A., & Makar, K. (2012). Technology for enhancing statistical reasoning at the school level. In *Springer eBooks* (pp. 643–689). https://doi.org/10.1007/978-1-4614-4684-2_21

Cobb, G. W., & Moore, D. S. (1997). Mathematics, statistics, and teaching. *The American Mathematical Monthly, 104*(9), 801. https://doi.org/10.2307/2975286

DiCoMat (2023). We live in a closed system. Data analysis useful for investigating environmental sustainability. Retrieved from: https://sites.google.com/view/we-live-in-a-closed-system/home-page

Heinzman, E. (2022). "I LOVE MATH ONLY IF IT'S CODING": a CASE STUDY OF STUDENT EXPERIENCES IN AN INTRODUCTION TO DATA SCIENCE COURSE. *Statistics Education Research Journal, 21*(2), 5. https://doi.org/10.52041/serj.v21i2.43

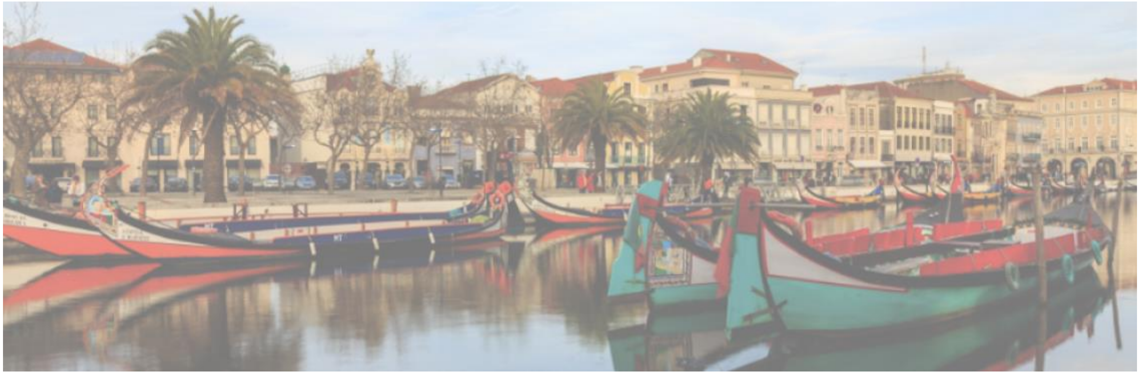
Libman, Z. (2010). Integrating Real-Life data analysis in Teaching descriptive Statistics: a constructivist approach. *Journal of Statistics Education, 18*(1). https://doi.org/10.1080/10691898.2010.11889477



OECD (2018). PISA 2021: Mathematics Framework (draft). Retrieved from: https://www.oecd.org/pisa/pisaproducts/pisa-2021-mathematics-framework-draft.pdf

Tucker, M. C., Shaw, S. T., Son, J. Y., & Stigler, J. W. (2022). Teaching Statistics and Data Analysis with R. *Journal of Statistics and Data Science Education, 31*(1), 18–32. https://doi.org/10.1080/26939169.2022.2089410

Wild, C. J., & Pfannkuch, M. (1999). Statistical thinking in empirical enquiry. *International Statistical Review, 67*(3), 223–248. https://doi.org/10.1111/j.1751-5823.1999.tb00442.x



Creativity

Calculus: Exploring the World of Open Middle Problems and GeoGebra

Nuno R. O. Bastos[1]

[1]Polythechnic Institute of Viseu & CIDMA – Center for Research and Development in Mathematics and Applications

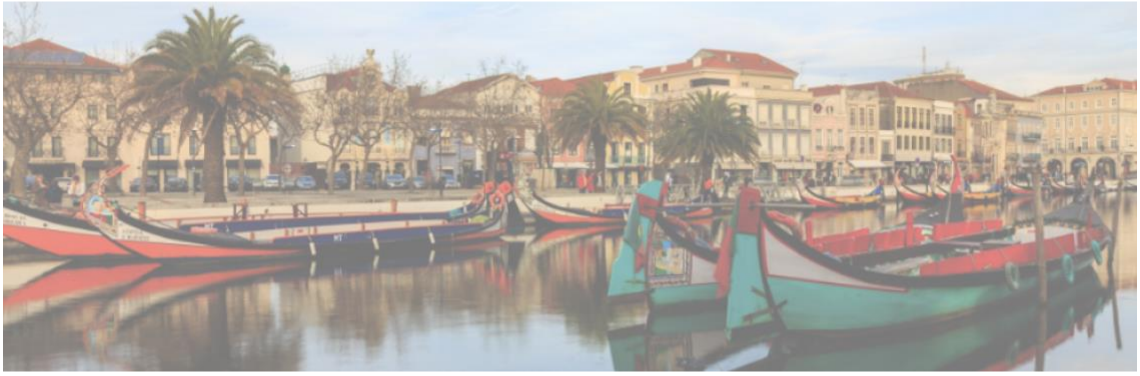
Abstract

Open Middle problems, a concept developed by Robert Kaplinsky in his book "Open Middle Math: Problems That Unlock Student Thinking, 6-12" are a recent addition to the realm of mathematical problem-solving. Open Middle problems are a type of math problem that aims to encourage students to think critically and creatively about math concepts. These problems have a specific structure involving a partially filled-in middle section, or “open middle”, which requires students to use their problem-solving skills to find the missing value(s). These problems often have multiple correct answers or even paths to a solution, which helps foster a growth mindset and helps students stick with challenges (Kaplinsky, 2020). Simply guessing the solutions to these problems proves to be more time-consuming and arduous than using reasoning skills. Students are encouraged to engage deeply with these problems, often requiring multiple attempts to arrive at a solution. Consequently, this process facilitates the development of mathematical understanding, the identification of misconceptions, and the expansion of mathematical knowledge.

The College Mathematics Practices and Professional Preparation Standards for Adult Education (Pimentel, 2013) elucidate the Mathematical Practices, providing a comprehensive definition of what it means to be a mathematically proficient student. According to the first standard, mathematically proficient students should not merely attempt solutions but also engage in making conjectures about the form and meaning of the solution, strategically planning a solution path. The Open Middle problems emerge as particularly effective tools in fostering this skill.

To illustrate it, I´ve selected a variety of intriguing Open Middle problems that are related to Calculus topics. These problems not only stimulate mathematical thinking but also foster creativity and independent thought. To enhance the interactive nature of these problems, I created and utilized dynamic versions using GeoGebra. These versions allow students to visualize mathematical concepts and explore various scenarios. Such approaches provide students with hands-on experience, reinforcing their comprehension and engagement with the subject.

Overall, the implementation of Open Middle problems, coupled with dynamic tools, has proven to be an effective method for promoting critical thinking, creativity, and a deeper understanding of mathematics



among students.

Keywords: problem-solving; GeoGebra; reasoning; critical thinking.

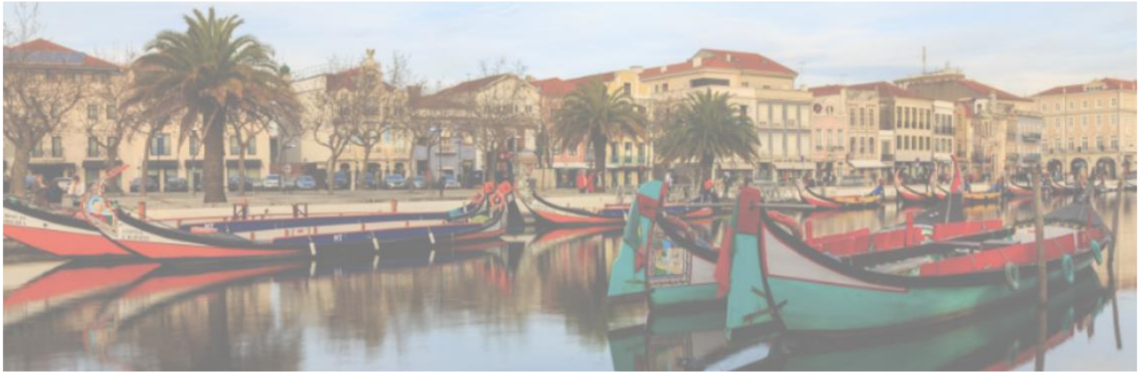
Acknowledgements

This work was supported by CIDMA and is funded by the Fundação para a Ciência e a Tecnologia, I.P. (FCT, Funder ID = 50110000187) under Grants UIDB/04106/2020 and UIDP/04106/2020.

References

Kaplinsky, R. (2020). *Open Middle Math : Problems That Unlock Student Thinking, 6-12*.

Pimentel, S. (2013). College and Career Readiness Standards for Adult Education. *Office of Vocational and Adult Education, US Department of Education*.



On some interpolation operators on triangles with curved sides

Teodora Cătinaş[1]
[1]Babeş-Bolyai University, Cluj-Napoca, Romania

Abstract

Teaching different results in multidimensional interpolation can be substantially improved when using graphical illustrations.

We consider here interpolation of functions defined on domains from the two dimensional Euclidean space.

Some original results we obtained on triangles having one curved side, are then extended by triangularization to more complex domains.

The interpolation operators can be applied for construction of surfaces that satisfy some given conditions, such as the roofs of the halls.

The operators defined on domains with curved sides permit essential boundary conditions to be satisfied exactly and they have important applications in: finite element methods for differential equations with given boundary conditions, the piecewise generation of surfaces in CAGD, in obtaining Bezier curves/surfaces in CAGD and in construction of surfaces that satisfy some given conditions.

Keywords: interpolation; triangles; roof surfaces; applications.

References

Barnhill, R. E. & Gregory, J. A. (1975) Compatible smooth interpolation in triangles, J. Approx. Theory, 15, 214-225.

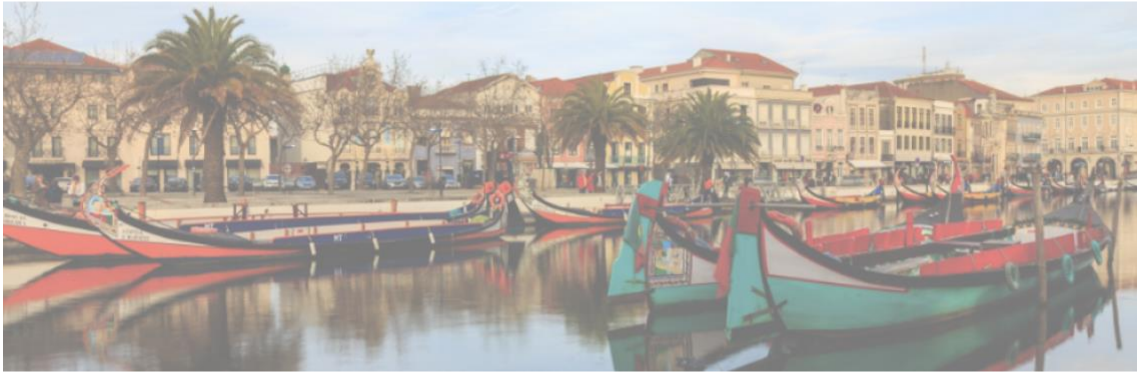
Bernardi, C., (1989) Optimal finite-element interpolation on curved domains, *SIAM J. Numer. Anal*., 26 (5), 1212-1240.

Blaga, P., Cătinaş, T., & Coman, G., (2011) Bernstein-type operators on triangle with all curved sides, *Appl. Math. Comput.,* 218, 3072--3082.

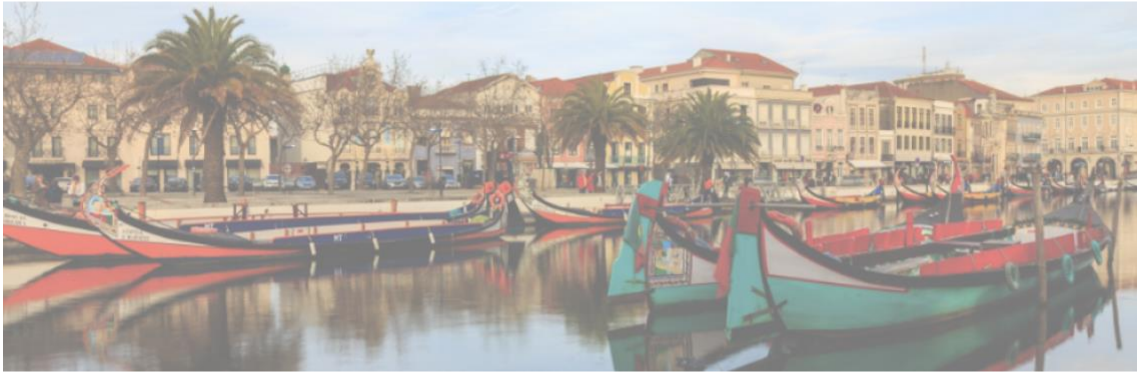
Cătinaş, T. (2017) Extension of some particular interpolation operators to a triangle with one curved side, *Appl. Math. Comput*., 315, 286--297.

Cătinaş, T., Blaga, P., & Coman, G. (2013) Surfaces generation by blending interpolation on a triangle with one curved side, *Results Math*., 64 (3-4), 343-355.

Coman, G., & Cătinaş, T. (2010) Interpolation operators on a triangle with one curved side, *BIT Numer. Math.*, 50 (2), 243-267.



Renka, R. J., & Cline, A. K. (1984) A triangle-based C^1 interpolation method, *Rocky Mountain J. Math*. 14, 223--237.



Assessment

SOLO taxonomy: Is it possible to measure the cognitive degree/difficulty of the exam?

Cristina Caridade[1,2] Verónica Pereira[1]
Polytechnic Institute of Coimbra, Coimbra Institute of Engineering, Coimbra, Portugal
CICGE, FCUP, 4430-146 Vila Nova de Gaia, Portugal

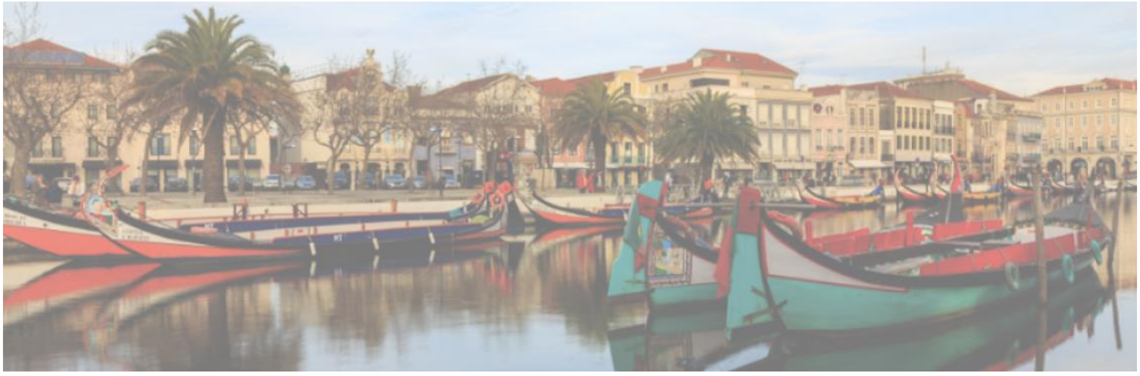
Abstract

Evaluation is a complex task that requires the definition of clear and transparent criteria, so that all stakeholders understand evaluation as a credible act, of responsibility and educational and social utility. However, evaluation is not an exact science, so subjectivity is an inherent feature of this entire process. In higher education, there are few studies on assessment and in mathematics they are almost non-existent. It is pertinent that a greater reflection on the knowledge of evaluation be initiated in higher education institutions.

The SOLO Taxonomy is a cognitive methodological tool developed by Biggs and Collis [1]. This methodology is based on a classification system composed of 5 levels of learning complexity, which consider aspects of quality and quantity of information: 1) prestructural; 2) unistructural; 3) multistructural; 4) relational; and 5) extended abstract. The contributions of the SOLO Taxonomy to educational evaluation are numerous. In the evaluation of exams, the taxonomy, in addition to contributing to the elaboration of items, provides more complete results (involving the level of cognitive complexity of students' thinking) to teachers. So, the SOLO Taxonomy can assist in the preparation of exam items, which allows teachers to balance superficial and deep issues, and can also be used to analyse test results (pedagogical interpretation).

In the Mathematical Analysis course of Electrical and Computer Engineering degree at Coimbra Institute of Engineering, the SOLO taxonomy was applied in the exams of the last 5 years.

In this paper, according to the work that has been carried out, intend to show the process followed for categorization an exam item according to the SOLO taxonomy. To categorize an item, the methodology proposed by [2] was followed according to 5 steps: Classification criteria; Proposed resolution; Topics, Procedures and Categorization. In the first step, the evaluation criteria are defined by grouping common references to be considered in the evaluation, by clearly describing their characteristics and assigning their score, so that the evaluation is consistent and impartial. The second step allows a rigorously description of the item resolution with the necessary justifications. Afterwards, all the topics and subtopics of the syllabus are identified. A set of procedures will have to be followed to reach the solution of the item. For this, conceptual (or mental) maps were developed to organize the topics in a coherent way and identifying the complexity of mathematical thinking, as a



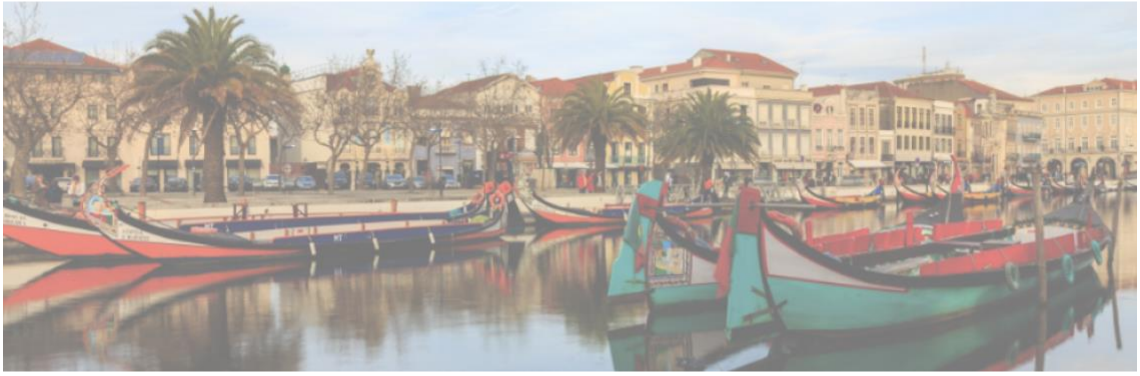
support and structure for the development of the resolution of the item. Finally, by analysing how topics and concepts are recognised, differentiated, and grouped in an item, it is possible to establish their categorization at a SOLO level.

Examples will be presented to categorize some items of an exam in levels of SOLO taxonomy. To measure the degree/cognitive difficulty of the proposed exam and whether it fits the skills that are required from mathematics students in an engineering course, the SOLO index [2] was calculated. It will also be presented, analysed, and discussed the difference between the exam with the highest and lowest score in the last 5 years, with the aim of better understanding how and in what way mathematics students in higher education should be evaluated, through exams, and how it is possible to increase their academic performance to prepare them for the job market.

Keywords: SOLO taxonomy; evaluation; higher education.

Acknowledgements

Cristina Caridade was partially supported by the project Centro de Investigação em Ciências Geo-Espaciais, reference UIDB/00190/2020, funded by COMPETE 2020 and FCT, Portugal.



Contribution of Digital Reflective Portfolio in initial training for Formative Assessment and Deepening the Mathematical Knowledge of Future Teachers

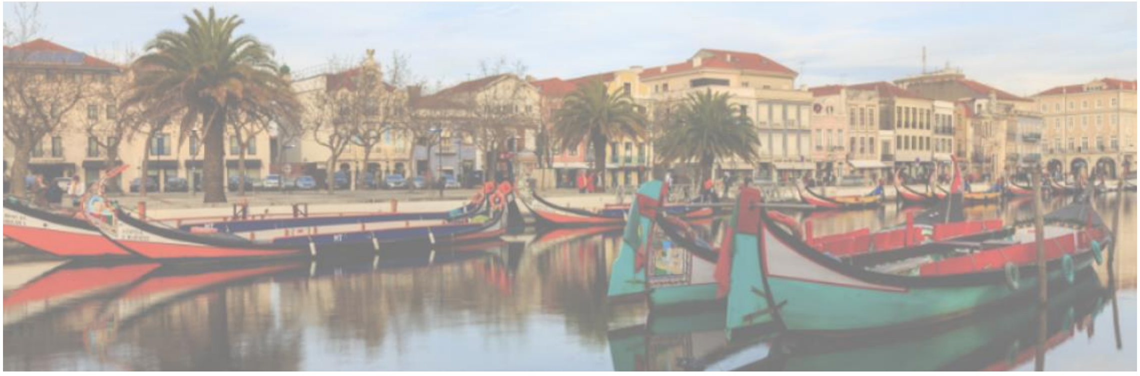
Ana Henriques[1]
[1]Instituto de Educação/ Universidade de Lisboa

Abstract

Mathematics is a fundamental discipline for students' learning in the international curriculums. However, in the Current curriculum documents the digital portfolio is salient as resource to be used as a diagnostic and guidance instrument in school teaching education (ME, 2018), both for its convenient report on the competences that students can demonstrate and for the nature and achievements of their self-regulated mathematical learning process, and based on the feedback of the formative assessment with portfolio that arises with the aim of providing methodological support to work in the classroom (Santos & Pinto, 2010). Thus, for the innovation teaching practices, as is not usual and teachers are not yet familiarized to carry out this portfolio in mathematics students teaching, is essential the use of portfolio in adequate initial teacher education, recognized as relevant to promote mathematical prospective teachers (PTs) knowledge of diverse current themes to support students´ learning and to integrate this resource in future practices (Dempsey & OShea, 2020).

The results of this qualitative and interpretative study (Bogdan & Biklen, 1994), focus on PTs initial training of the 3rd cycle of basic and secondary education, in the master degree of the mathematics teaching in Portugal, whose aims is to understand the contribution of using the digital reflective portfolio in mathematics PTs training for their formative assessment and the lessons learned from it with a view to its possible use in future professional practice.

The data collection and analysis take account the PTs individual carried work included in the reflective digital portfolio, based on some contents, presentations and organization and given feedback, considered aspects of their formative evaluation criteria (Delandshere & Arens, 2003): the variety of tasks they selected and worked on the topics studied in training (Mathematical and Statistical reasoning, Computational Thinking, STEM) and the procedures adopted; a final global reflection justifying choices and works carried out to contribute to their skills and learning developed for possible use in future practices. The results show their positive advantages in using the digital reflective portfolio in this training, highlighting and presenting evidence of helped them to develop understand of mathematical concepts and potential of the selected and solved tasks, and pedagogical knowledge for the diverse possibilities of



professional development.

Thus, this study was an effective innovation of this digital reflective portfolio contribute to identifying the necessary potential of encouragement and innovative their use for the development of mathematical learning to professional development in future students teaching and learning practices.

Keywords: digital reflective portfolio; mathematical and didactic knowledge; formative assessment; mathematics prospective teachers of initial training.

References

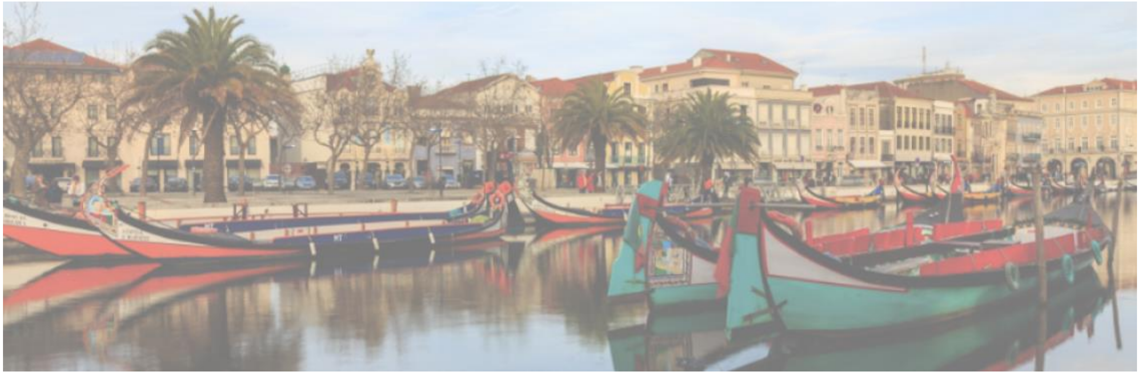
Bogdan, R. & Biklen, S. (1994). *Investigação qualitativa em educação*. Porto Editora.

Delandshere, G. & Arens, S. A. (2003). Examining the quality of the evidence in preservice teacher portfolios. *Journal of Teacher Education*, *54*(1), 57-73.

Dempsey, M. & OShea, A. (2020). The role of task classification and design in curriculum making for preservice teachers of mathematics. *The Curriculum Journal, 31*(3), 436-453. https://doi.org/10.1002/curj.18

Ministério da Educação (2018). *Aprendizagens Essenciais Matemática.* Lisboa: DGE.

Santos, L. & Pinto, J. (2010). The use of feedback in written reports and portfolio: an assessment for learning strategy. *Research in Mathematical Education, 14*(3), 281-297.



Minimizing academic dropout and maximizing success rates through collaborative work and MOODLE forums: A case study in Mathematics in higher education

Carla Martinho[1], Jaime Carvalho e Silva[2]
[1]Instituto Superior de Contabilidade e Administração de Lisboa, Politécnico de Lisboa, Portugal
[2]Universidade de Coimbra, Portugal

Abstract

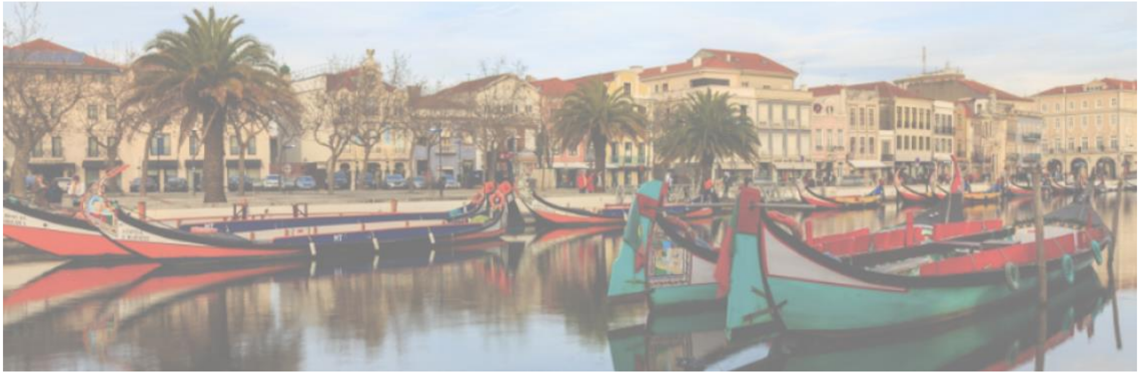
This study describes an experiment conducted to explore the effectiveness of using Moodle forums as a support for face-to-face classes and as a tool to promote student engagement and enhance success rates in a mathematics course at the higher education level. Three groups were selected: two groups used collaborative work and MOODLE forums for sharing these supervised assignments, guided by the instructor, while the third group did not employ this methodology. The results demonstrated that the integration of forums into the learning process significantly stimulated student participation, reduced attrition from assessments, and contributed to higher pass rates compared to the control group.

The increasing integration of information and communication technologies in higher education has led to a search for innovative strategies to enhance student engagement and, consequently, academic outcomes. In this context, virtual learning environments have provided opportunities to develop collaborative and interactive pedagogical approaches.

Three classes, totaling 144 students, from a mathematics course in a business sciences undergraduate program at a higher education institution, were selected for this study, all with similar aptitude levels and backgrounds. Two classes (Group A and Group B) were designated as experimental groups and used Moodle forums to share and discuss collaborative work done in the classroom under the supervision of the teacher. The other class (Group C) did not use this approach and served as the control group.

Groups A and B, which participated in the forum-based experiment, exhibited higher engagement in the classes, with success rates around 90% and consistent participation rates of around 80% in the continuous assessment scheme, as compared to Group C, which showed a rate of 59% for both situations. Collaborative work facilitated through forum-based sharing provided time, both inside and outside the classroom, for constructive discussions and exchange of ideas, encouraging students to express thoughts and doubts that might not have been addressed under a different approach.

The incorporation of forums within the Moodle virtual learning environment as a support for face-to-face teaching proved to be an effective strategy, enhancing student participation and fostering greater interaction among



students and between students and the instructor. Through collaborative assignment sharing, students were able to maintain a consistent involvement with the material, accessing all assignments completed within and outside of classes even after the end of the academic term. This dynamic led to lower attrition rates from assessments and, consequently, better pass rates for these two groups compared to the control group, within a first-year mathematics course in the Portuguese higher education system.

This study underscores the importance of integrating collaborative learning technologies, such as MOODLE forums, to encourage a more active and participatory approach in the teaching and learning process. Future investigations are recommended to explore alternative forms of interaction and assess the applicability of this approach in different academic contexts and for various curriculum units.

Keywords: academic success; collaborative work; mathematics; Moodle forums.

References

Abar, C. A. A. P. & Moraes, U. C. (2019). Flipped Classrooms and Moodle: Digital Technologies to Support Teaching and Learning Mathematics. *Acta Didactica Napocensia*, 12(2), 209-216, - DOI: 10.24193/adn.12.2.16.

Bigotte, M.E., Gomes, A., Branco, V. & Pessoa, T. (2016). The influence of educational learning paths in academic success of mathematics in engineering undergraduate," *2016 IEEE Frontiers in Education Conference (FIE)*, Erie, (pp. 1-6) PA, USA. doi: 10.1109/FIE.2016.7757453.

Blanco, M., & Ginovart, M. (2012). On How Moodle Quizzes Can Contribute to the Formative e-Assessment of First-Year Engineering Students in Mathematics Courses, In: "Mathematical e-learning" [online dossier]. *Universities and Knowledge Society Journal* (RUSC). Vol. 9, No 1. pp. 354-370 UOC.

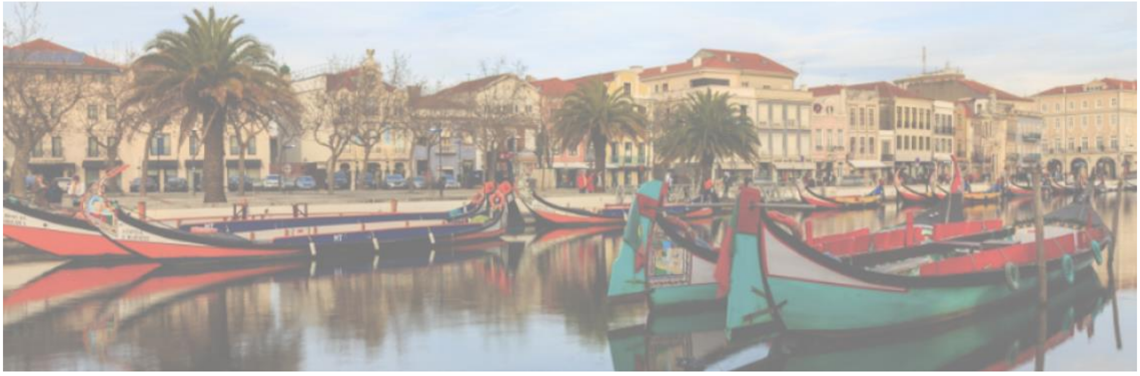
Gašević, D., Dawson, S., Rogers, T., & Gasevic, D. (2016). Learning analytics should not promote one size fits all: The effects of instructional conditions in predicting academic success. *Internet and Higher Education* vol. 28 (pp. 68-84)

Kadoić, N., & Oreški, D. (2018). Analysis of student behavior and success based on logs in Moodle *2018 41st International Convention on Information and Communication Technology, Electronics and Microelectronics (MIPRO)* (pp. 0654-0659), Opatija, Croatia. doi: 10.23919/MIPRO.2018.8400123.

Kerimbayev, N., Kultan, J., Abdykarimova, S., & Akramova, A. (2017). LMS Moodle: Distance international education in cooperation of higher education institutions of different countries. *Education and information technologies*, *22*(5), 2125-2139.

Lopes, A.P. (2011). Teaching with Moodle in Higher Education, *INTED2011 Proceedings*, (pp. 970-976). https://library.iated.org/view/LOPES2011TEA

Salvador, T., Pedrosa, S., Messias, I., Mendes, A., & Carvalho e Silva, J. (2016). ReM@t - a project to help students to improve mathematical skills. *Proceedings of the 6th e-Learning Conference*, Bratislava, Slovakia, 8-9 September, 2016.



Evaluation with didactic technologies in the classroom environment

Cristina Caridade[1]
[1]Polytechnic Institute of Coimbra, Coimbra Institute of Engineering, Coimbra, Portugal
CICGE, FCUP, 4430-146 Vila Nova de Gaia, Portugal

Abstract

Technologies have assumed a fundamental role in current education, including higher education, causing changes in the model of acquisition and dissemination of knowledge, modifying the classroom and the relationships between students and teacher.

In Mathematical Analysis of Electrical and Computer Engineering degree, the contents of numerical methods are explored on laboratories with the support of teaching technologies. In addition to the calculators and computers in these laboratories, computational tools are also used to offer students other types of skills and knowledge.

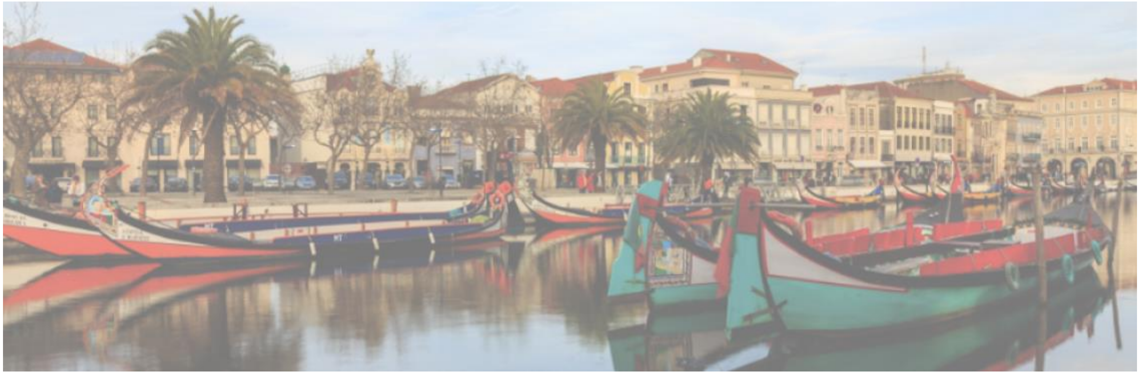
The Padlet is a Web Application that can be used as a digital wall for teaching, as a place to display projects, share productions, assign tasks, discuss, and send messages. It is, therefore, an interesting tool for teaching and learning.

During classes in the second semester of 2022/2023, the Padlet was used as a teaching/learning tool. Each Padlet class was constructed and planned according to table provided (Table 1). In agreement to the theme of the class, each student (group of 2) solved a problem using all the technological means available and uploaded it for Padlet. The problem was then made available to everyone and, afterwards, would serve as teaching material. Then the problems were randomly mixed, and each student was assigned a new problem, but this time to correct the problem's solution created by his/her colleagues. Here the reinforcement of learning is visible and allows the student to verify his learning. After classes, students continued their study by accessing the materials available on Padlet, and the following week they performed a new problem on the same topic individually.

Throughout this learning process, the teacher in the classroom has the role of advisor and tutor, assessing the interest, motivation and learning of his students by direct observation. At the end of the class, accessing the Padlet, the teacher obtains the evaluations of his students' learning through the analysis of the uploaded contents. Finally, in a face-to-face assessment, the teacher evaluates the student for completing a new problem the following week.

Table 1: Construction and Planning of each lesson with the Padlet.

Lesson	Description
Theme	Syllabus content.
Problem	Raffled among a set of exercises proposed by the teacher.
Objective	Problem solving as didactic material.
Strategies	In groups of 2 they solve 1 problem and correct another problem. The following week they solve a problem individually.

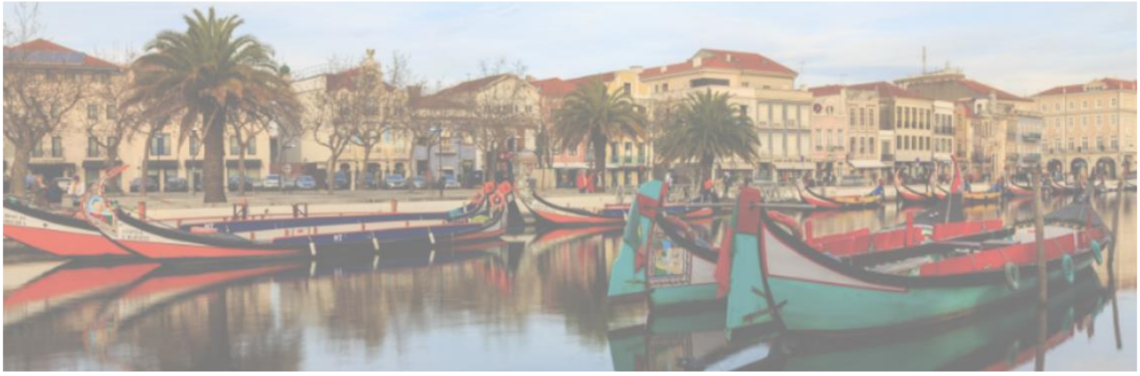


Technologies	Mathematics (calculators, Symbolab, Matlab, etc.), mathematical writing (pen and paper, tablets, computers, etc.), mathematical visualization (GeoGebra, Matlab, etc.), digitization (graphics tablets, mobile phones, etc.)
Evaluation descriptors	1. Learning the proposed content. 2. Learning mathematical skills. 3. Learning math writing skills. 4. Learning technological and digital skills. 5. Reinforcement of learning in correcting a problem.
Evaluation type	Observational (during classroom follow-up) Padlet (analyse the materials loaded there) Face-to-face (resolution in paper and delivery a problem)

As a final reflection, the classes planned with Padlet were very enriching for the teacher, as it allowed, on the one hand, to verify the positive feedback from students in using the Padlet during classes, and in its use outside of classes, especially before the assessments. The results of the learning carried out by the students and evaluated by the teacher in 3 moments (Observational, Padlet and Face-to-face), were very good and therefore this methodology will be applied in the next school year.

Keywords: Padlet; evaluation; didactic technologies.

Acknowled*gements*: Cristina Caridade was partially supported by the project Centro de Investigação em Ciências Geo-Espaciais, reference UIDB/00190/2020, funded by COMPETE 2020 and FCT, Portugal.



Learning problems in mathematics in secondary education: the contribution of an action research project

Israel Cardoso[1], Vanda Santos[2], Maria da Piedade Simões Santana Pessoa Vaz Rebelo Vaz[3], Maria Da Graça Amaro Bidarra [4]

[1]Instituto Federal de São Paulo,

[2]CIDTFF – Centro de Investigação em Didática e Tecnologia na Formação de Formadores/ Universidade de Aveiro,

[3]Universidade de Coimbra,

[4]Universidade de Coimbra

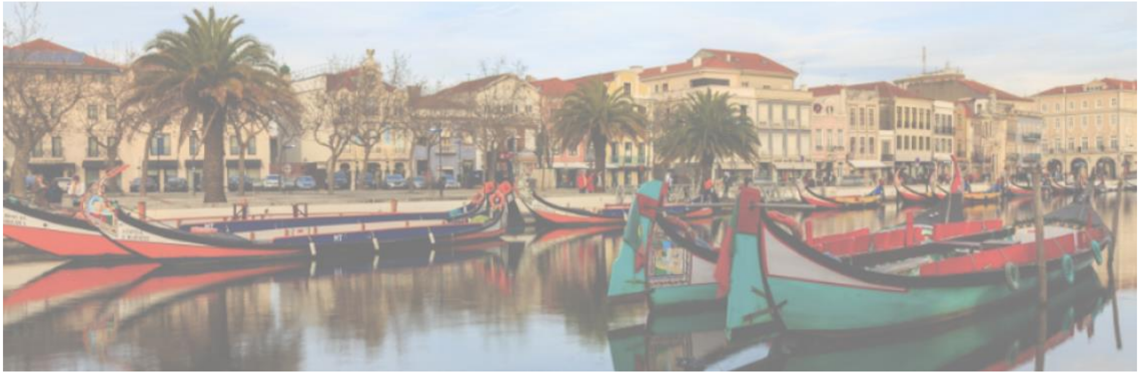
Abstract

Learning problems are understood as all the problems that prevent a student from learning, and mathematics is one of the subjects that is difficult to learn (Riesgo, Rotta & Ohlweiler, 2006). Learning problems in mathematics are a real problem, with significant consequences throughout school life and also for the students' entire lives, because the way human beings think is largely logical-mathematical (Pereira, 2013). And this ability to associate and relate is developed by the mathematical content learned at school. It is also noticeable that students who have learning problems in mathematics in the early years continue to have difficulties, worsening in the following years, remaining until the final years of high school and throughout their lives (Sousa, 2008).

In the theory of meaningful learning, meaningful knowledge is understood as an attempt to uncover a person's intellectual construction in relation to the use of pre-acquired knowledge. These previous concepts are considered to be the organizers of the new information, contributing to the consolidation and development of the existing cognitive structure. New knowledge is acquired through the interaction between new knowledge and existing prior knowledge (Ausubel, 2000).

The main objective of this work is to develop an action research project with students entering high school and math teachers that will contribute to the study of math learning problems in high school, as a result of the failure of the math teaching-learning process in elementary school, specifically regarding students' prior knowledge and teachers' pedagogical knowledge of math content.

The study aims to identify some problems in the learning of mathematics by students entering the first year of high school and teaching and learning strategies to overcome them, using the questionnaire survey methodology with students and teachers at this level of education, following which we will try to develop a training action with teachers, with a view to implementing the basic mathematics rescue course for students entering high school. Within the framework of an action research project, with the participation of students from the first-year classes of the integrated high school courses in Computer Networks,



Chemistry and Mechatronics at the IFSP Catanduva campus who attended the course (experimental group) and the three first-year classes of the integrated high school courses in Building, Informatics and Mechatronics at the IFSP Votuporanga campus, as a control group. The aim is to test the effectiveness of the rescue course implemented by monitoring the progress of the students in the experimental group, based on surveys, direct observation and product analysis, and using pre-test and post-test evaluations to calculate the differential in gains.

Finally, since this work is a contribution to the study of the problems of learning mathematics in secondary school, the conclusions will allow us to draw possible implications for curricular guidelines and teacher practice.

Keywords: action research; high school; assessment; meaningful learning.

Acknowledgements

The work of the second author is funded by national funds through FCT - Foundation for Science and Technology, I.P., within the scope of the Project UIDB/00194/2020 and in the scope of the framework contract foreseen in the numbers 4, 5 and 6 of the article 23, of the Decree-Law 57/2016, of August 29, changed by Law 57/2017, of July 19.

Este trabalho é financiado por Fundos Nacionais através da FCT – Fundação para a Ciência e a Tecnologia, I.P., no âmbito dos projetos UIDB/00194/2020, UIDP/00194/2020, UIDB/04106/2020 e UIDP/04106/2020.

fct Fundação para a Ciência e a Tecnologia

cidtff centro de investigação Didática e Tecnologia na Formação de Formadores

CIDMA Centro de I&D em Matemática e Aplicações / Center for R&D in Mathematics and Applications

universidade de aveiro

dep departamento de educação e psicologia

dmat departamento de matemática